I. Riešte nasledujúce diferenciálne rovnice

1. $y' = \dfrac{y-1}{x^2y^2}$.
2. $y' \operatorname{cotg} x + y = 2 \;\; y(0) = -1$.
3. $y' - xy^2 = 2xy$.
4. $xy' + y = y^2 \;\; y(0) = 0.5$.
5. $xy(1+y^2)\,\mathrm{d}x - (1+x^2)\,\mathrm{d}y = 0$.
6. $\sin x\,\mathrm{d}y - y\ln y\,\mathrm{d}x = 0$.
7. $(x+2y)\,\mathrm{d}x - x\,\mathrm{d}y = 0$.
8. $(y^2 - 2xy)\,\mathrm{d}x + x^2\,\mathrm{d}y = 0$.
9. $y^2 + x^2y' = xyy'$.
10. $(x+y+1)\,\mathrm{d}x = (2x+2y-1)\,\mathrm{d}y$.
11. $xy' - y = x\operatorname{tg}\dfrac{y}{x}$.
12. $(x^2+y^2)y' = 2xy$.
13. $y' = (x+y+2)^2$.
14. $y' = \dfrac{2x-y+1}{x-2y+1}$.
15. $y' = \dfrac{y+2}{x+y-1}$.
16. $y' = -y\operatorname{tg} x + \cos^{-1} x$.
17. $y' = 2x(x^2+y)$.
18. $x^2y' + xy + 1 = 0$.
19. $xy' + 2y + x^5y^3\,\mathrm{e}^x = 0$.
20. $y' = -2y + y^2\,\mathrm{e}^x$.
21. $y = \dfrac{x}{y'} + y'$.
22. $y = (y')^2\,(x+a)$.
23. $y = (y')^2\,x - 2\,(y')^3$.
24. $2y'x - y = \ln y'$.
25. $y = (y')^2 - \sqrt{a - (y')^2}, a > 0$.
26. $y = y'x + \dfrac{a}{y'}$.
27. $y = y'x - 3\,(y')^3$.
28. $y = y'x - 2 - y'$.
29. $y = y'x + a\,(y')^2$.
30. $y = y'x + |y'|$.
31. $y' + y^2 - 1 = 0$.
32. $xy' + 1 = \mathrm{e}^{x-y}$.
33. $y = xy' - x^2\,(y')^3$.
34. $x^2y' + xy + 1 = 0$.
35. $y = x(y' - x\cos x)$.
36. $2y' + x = 4\sqrt{y}$.
37. $xy' = y\cos\ln\dfrac{y}{x}$.
38. $x(y' - y) = \mathrm{e}^x$.
39. $(y')^2 - 2y' - 3 = 0$.
40. $y' = \mathrm{e}^{\frac{xy'}{y}}$.
41. $(y')^2 - 2xy' = 8x^2$.
42. $xy' + y = xy^2\ln x$.
43. $y' - \dfrac{xy}{2(x^2-1)} - \dfrac{x}{2y} = 0, y(0) = 1$.
44. $xy' - 4y = x^2\sqrt{y}$.
45. $x^2y' = y^2 + 2y + 5$.
46. $(3x+5y-7)\,\mathrm{d}y = (6x+10y+3)\,\mathrm{d}x$.
47. $xy' + y = y^2\ln x$.
48. $(y')^2 = xy$.
49. $x^2\,(y')^3 - y' = 0$.
50. $y = y'x + 4y'$.
51. $x\,(y')^2 - yy' + x = 0$.
52. $y' = 2x(x^2+y)$.
53. $xy' - 2x^2\sqrt{y} = 4y$.
54. $(2\,\mathrm{e}^y - x)y' = 1$.

II. Nájdite všeobecné riešenie nasledujúcich diferenciálnych rovníc

1. $y'' = 6x - x^{-2}$.
2. $y'' = \dfrac{1}{2y'}, y' > 0$.
3. $y''' = \dfrac{2}{x}$.
4. $y'' = \dfrac{y'}{x} + x^2$.
5. $y'' = \dfrac{(y')^2+1}{x^2+1}$.
6. $y'' - 9y = 0$.
7. $y'' + 3y' - 4y = 0$.
8. $y'' + 5y' = 0$.
9. $2y'' - 5y' + 2y = 0$.
10. $y'' + 6y' + 9y = 0$.
11. $4y'' + 12y' + 9y = 0$.
12. $y'' - 2a^2y' + a^4y = 0$.
13. $y'' + 16y = 0$.
14. $y'' - 4y' + 13y = 0$.
15. $y'' + y' + 2y = 0$.
16. $y'' + y' - 2y = 0$.
17. $3y'' - 2y' - 8y = 0$.
18. $y'' - 2y' - y = 0$.
19. $y'' + 6y' + 13y = 0$.
20. $4y'' - 8y' + 5y = 0$.
21. $4y'' - 20y' + 25y = 0$.
22. $y''' - y' = 0$.
23. $y'''' - 5y'' + 4y = 0$.
24. $y'''' - 2y''' + y'' = 0$.
25. $y''' - y = 0$.
26. $y'''' + 4y = 0$.
27. $y'''' + 5y'' + 4 = 0$.
28. $y^{(5)} + 2y''' + y' = 0$.
29. $y''''13y'' + 36y = 0$.

III. Nájdite všeobecné riešenie nasledujúcich diferenciálnych rovníc

1. $y'' - 7y' + 10y = 40$.
2. $y'' - 7y' + 10y = 6\,\mathrm{e}^{2x}$.
3. $y'' + 4y = \cos 2x$.
4. $y'' - 7y' + 10y = 20x^2 - 28x + 14$.
5. $y'' - 7y' + 10y = 8\,\mathrm{e}^{2x}\sin x$.
6. $y'' - 4y' + 5y = 2x^2$.
7. $y'' - 7y' + 10y = -(6x+7)\,\mathrm{e}^{2x}$.
8. $y'' + 4y = x^4 - 2x$.
9. $y'' + 4y = \cos 3x$.
10. $y'' - 7y' + 10y = -12\,\mathrm{e}^{3x}$.
11. $y'' + 4y = \mathrm{e}^{-2x}$.
12. $y'' + 4y = 2x\sin 2x$.
13. $y'' + 4y = x\,\mathrm{e}^{2x}\sin 2x$.
14. $y'' - 4y' + 5y = \mathrm{e}^{2x}$.
15. $y'' - 4y' + 5y = \sin x$.
16. $y'' - 7y' + 10y = 65\sin 2x$.
17. $y'' - 4y' + 5y = \mathrm{e}^{2x}\sin x$.
18. $y'' + y' = 4x\,\mathrm{e}^x$.
19. $y'' - 4y' + 5y = x\,\mathrm{e}^{2x}\cos x$.
20. $y'' + y' = x$.
21. $y'' - 2y' - 3y = \mathrm{e}^{4x}$.
22. $y'' - 4y' + 5y = \mathrm{e}^{2x}\sin 2x$.
23. $y'' + y' - 2y = 3x\,\mathrm{e}^x$.
24. $y'' - 5y' + 4y = 4x^2\,\mathrm{e}^{2x}$.
25. $y'' - 3y' + 2y = x\cos x$.
26. $y'' - 2y' + y = 6x\,\mathrm{e}^x$.
27. $y'' + 4y' + 4y = x\,\mathrm{e}^{2x}$.

28. $y'' + 4y = 5\sin 3x + \cos 3x + \sin 2x$. **29.** $y'' - 2y' + 2y = x^2 + \sin 2x$. **30.** $y'' - y = 2\,\mathrm{e}^x - x^2$.

31. $y'' + 3y' - 4y = \mathrm{e}^{-4x} + x\,\mathrm{e}^{-x}$. **32.** $y'' + y' - 6y = x + \mathrm{e}^{2x}$. **33.** $y'' + 2y' - 3y = x^2\,\mathrm{e}^x$.

34. $y'' + 2y' + 5y = -\dfrac{17}{2}\cos 2x$. **35.** $2y'' + y' - y = 2\,\mathrm{e}^x$. **36.** $y'' + 4y' - 5y = 1$.

37. $y''' + 3y'' + 3y' + y = \mathrm{e}^{-x}\sin x$. **38.** $y'' + 2y' + y = \mathrm{e}^{-x} + \mathrm{e}^x$. **39.** $y'' + a^2 y = \mathrm{e}^x$.

40. $y''' - 3y' + 2y = (4x^2 + 4x - 10)\,\mathrm{e}^{-x}$. **41.** $y''' + y'' = \sin x + x\cos x$. **42.** $y''' - y = x^3 - 1$.

IV. Riešte začiatočné úlohy

1. $y'' = 2x^3, y(0) = 2, y'(0) = 1$.
2. $y''' = x^{-2}, y(1) = 0, y'(1) = y''(1) = 1$.
3. $y'' = \dfrac{y'}{x} + x\,\mathrm{e}^x, y(0) = 2, y'(0) = 1$.
4. $y'' = \dfrac{1}{2y'}, y(0) = 0, y'(0) = 2$.
5. $y'' = \sqrt{a^2 - (y')^2}, y(0) = -1, y'(0) = 0, a > 0$.
6. $y'' - 4y' = 0, y(0) = 1, y'(0) = 3$.
7. $y'' - 2y' + y = 0, y(2) = 1, y'(2) = -2$.
8. $4y'' + y = 0, y(\pi) = 2, y'(\pi) = 3$.
9. $y''' - y' = 0, y(0) = 3, y'(0) = -1, y''(0) = 1$.
10. $y'' + 2y' + 5y = 0, y(0) = 2, y'(0) = 0$.
11. $y'''' + 4y = 0, y(0) = y'(0) = y''(0) = 0, y'''(0) = 1$.
12. $4y'' + y = 0, y(\pi) = 2, y'(\pi) = 3$.
13. $y'' - 4y' + 3y = 0, y(0) = 6, y'(0) = 10$.
14. $y'' + 4y' + 29y = 0, y(0) = 0, y'(0) = 15$.
15. $4y'' + 4y' + y = 0, y(0) = 2, y'(0) = 0$.
16. $y'' + y = \sin x, y(0) = 1, y'(0) = 0$.
17. $y'' + y = 4\,\mathrm{e}^x, y(0) = 4, y'(0) = -3$.
18. $y'' - 2y' = 2\,\mathrm{e}^x, y(1) = -1, y'(1) = 0$.
19. $4y'' + 16y' + 15y = 4\,\mathrm{e}^{-3x/2}, y(0) = 3, y'(0) = -\dfrac{11}{2}$.
20. $y'' + 2y' + 2y = x\,\mathrm{e}^{-x}, y(0) = y'(0) = 0$.
21. $y'' - y' = 2(1 - x), y(0) = y'(0) = 1$.
22. $y'' - 2y' = (x^2 + x - 3)\,\mathrm{e}^x, y(0) = y'(0) = 2$.
23. $y''' - 3y' - 2y = 9\,\mathrm{e}^{2x}, y(0) = 0, y'(0) = -3, y''(0) = 3$.
24. $y'' - y = 2x, y(0) = y'(0) = 0$.
25. $y''' + 2y'' + y' = -2\,\mathrm{e}^{-2x}, y(0) = 2, y'(0) = y''(0) = 1$.
26. $y'' + y = \sin x, y(0) = 1, y'(0) = 0$.
27. $y''' - y' = 3(2 - x^2), y(0) = y'(0) = y''(0) = 1$.
28. $y'' - 5y' + 6y = x + \mathrm{e}^x, y(0) = 0, y'(0) = 1$.
29. $y'''' + y'' = 2\cos x,\ y(0) = -2,\ y'(0) = 1,\ y''(0) = 0,\ y'''(0) = 0$.

V. Riešte homogénne systémy lineárnych diferenciálnych rovníc

1. $y_1' = 4y_1 - 3y_2$, $y_2' = 5y_1 - 4y_2$.
2. $y_1' = 7y_1 + 6y_2$, $y_2' = 2y_1 + 6y_2$.
3. $y_1' = y_1 + y_2$, $y_2' = 8y_1 - y_2$.
4. $y_1' = y_1 + y_2$, $y_2' = -5y_1 - y_2$.

5. $y_1' = y_1 + 3y_2$, $y_2' = -3y_1 + y_2$.
6. $y_1' = 2y_1 + y_2$, $y_2' = 3y_1 + 4y_2$.
7. $y_1' = y_1 - y_2$, $y_2' = -4y_1 + y_2$.
8. $y_1' = y_1 + y_2$, $y_2' = -2y_1 + 3y_2$.

9. $y_1' = -7y_1 + y_2$, $y_2' = -2y_1 - 5y_2$.
10. $y_1' = y_1 - 3y_2$, $y_2' = 3y_1 + y_2$.
11. $y_1' = 2y_1 + y_2$, $y_2' = -y_1 + 4y_2$.
12. $y_1' = 3y_1 - y_2$, $y_2' = 4y_1 - y_2$.

13. $y_1' = y_1 - y_2 + y_3$, $y_2' = y_1 + y_2 - y_3$, $y_3' = 2y_1 - y_2$.
14. $y_1' = y_1 - 2y_2 - y_3$, $y_2' = -y_1 + y_2 + y_3$, $y_3' = y_1 - y_3$.
15. $y_1' = y_1 - y_2 - y_3$, $y_2' = y_1 + y_2$, $y_3' = 3y_1 + y_3$.
16. $y_1' = 2y_1 - y_2 + 2y_3$, $y_2' = y_1 + 2y_3$, $y_3' = -2y_1 + y_2 - y_3$.

17. $y_1' = 4y_1 - y_2 - y_3$, $y_2' = y_1 + 2y_2 - y_3$, $y_3' = y_1 - y_2 + 2y_3$.
18. $y_1' = y_1 - y_2 + y_3$, $y_2' = y_1 + y_2 - y_3$, $y_3' = -y_2 + 2y_3$.
19. $y_1' = 3y_1 - y_2 + y_3$, $y_2' = y_1 + y_2 + y_3$, $y_3' = 4y_1 - y_2 + 4y_3$.
20. $y_1' = 2y_1 + y_2$, $y_2' = y_1 + 3y_2 - y_3$, $y_3' = -y_1 + 2y_2 + 3y_3$.

VI. Riešte začiatočné úlohy

1. $y_1' = -5y_1 + 2y_2$, $y_1(0) = 1$; $y_2' = -y_1 - 7y_2$, $y_2(0) = -1$
2. $y_1' = y_1 - 3y_2$, $y_1(0) = 1$; $y_2' = 4y_1 - 6y_2$, $y_2(0) = 0$
3. $y_1' = y_1 + 2y_2$, $y_1(0) = 2$; $y_2' = 4y_1 + 3y_2$, $y_2(0) = 1$.

4. $y_1' = y_2$, $y_1(0) = 3$; $y_2' = -2y_1 + 2y_2$, $y_2(0) = 1$
5. $y_1' = y_2$, $y_1(0) = 2$; $y_2' = 12y_1 - y_2$, $y_2(0) = -1$
6. $y_1' = -3y_1 - y_2$, $y_1(0) = 2$; $y_2' = y_1 - y_2$, $y_2(0) = -1$.

7. $y_1' = y_2$, $y_1(0) = 3$; $y_2' = y_3$, $y_2(0) = -1$; $y_3' = y_2$, $y_3(0) = 3$.
8. $y_1' = 5y_1 + 3y_2$, $y_1(0) = 0$; $y_2' = -3y_1 - y_2$, $y_2(0) = 2$.
9. $y_1' = y_2$, $y_1(0) = 0$; $y_2' = y_3$, $y_2(0) = -1$; $y_3' = -y_2$, $y_3(0) = 1$.

10. $y_1' = y_3 \qquad y_1(0) = -1$
$y_2' = -y_1 + y_3 \qquad y_2(0) = 2$
$y_3' = -y_2 + y_3 \qquad y_3(0) = 1.$

11. $y_1' = -y_1 + y_2 + y_3 \qquad y_1(0) = 1$
$y_2' = y_1 - y_2 + y_3 \qquad y_2(0) = 0$
$y_3' = y_1 + y_2 + y_3 \qquad y_3(0) = 0.$

12. $y_1' = y_2 + y_3 \qquad y_1(0) = -1$
$y_2' = y_1 + y_3 \qquad y_2(0) = 1$
$y_3' = y_1 + y_2 \qquad y_3(0) = 0.$

VII. Nájdite štandartnú fundamentálnu maticu $\mathrm{e}^{\boldsymbol{A}x}$ a použite ju na riešenie počiatočných úloh $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{b}$, pričom tucnemat $\boldsymbol{A}$ a vektor $\boldsymbol{b}$ sú zadané

1. $\begin{pmatrix} 7 & -18 \\ 3 & -8 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$ **2.** $\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$ **3.** $\begin{pmatrix} -1 & 5 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$

4. $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$ **5.** $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$ **6.** $\begin{pmatrix} 2 & 1 & -2 \\ -1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$

VIII. Riešte nehomogénne systémy lineárnych diferenciálnych rovníc

1. $y_1' = 3y_1 - 2y_2$
$y_2' = 2y_1 - y_2 + 1.$

2. $y_1' = -y_1 + 5y_2$
$y_2' = -y_1 + y_2 + 8x.$

3. $y_1' = y_2 + x^2$
$y_2' = y_1 + 2\,\mathrm{e}^x.$

4. $y_1' = 2y_1 + 3y_2 + 8\,\mathrm{e}^x$
$y_2' = 3y_1 + 2y_2 + 5x.$

5. $y_1' = -5y_1 + 2y_2 + \mathrm{e}^x$
$y_2' = y_1 - y_2 + \mathrm{e}^{2x}.$

6. $y_1' = y_1 - y_2 + 2\sin x$
$y_2' = 2y_1 - y_2.$

7. $y_1' = 2y_1 - y_2$
$y_2' = y_1 + 2\,\mathrm{e}^x.$

8. $y_1' = 2y_1 + 4y_2 + \cos x$
$y_2' = -y_1 - 2y_2 + \sin x.$

9. $y_1' = -y_1 + 2y_2$
$y_2' = -y_1 + y_2 + \dfrac{1}{\cos x}.$

10. $y_1' = -y_1 + 2y_2$
$y_2' = -2y_1 + 3y_2 + 15\sqrt{x}\,\mathrm{e}^x.$

11. $y_1' = y_2$
$y_2' = y_1 + \mathrm{e}^x + \mathrm{e}^{-x}.$

12. $y_1' = -5y_1 + 2y_2 + \mathrm{e}^x$
$y_2' = -2y_1 - 3y_2 + \mathrm{e}^{-2x}.$

13. $y_1' = y_1 + y_2 + 3y_3 + 8\,\mathrm{e}^{-x}$
$y_2' = -y_1 + y_2$
$y_3' = -y_1 + y_3.$

14. $y_1' = 2y_1 + y_2 - 2y_3$
$y_2' = -y_1 + x$
$y_3' = y_1 + y_2 - y_3.$

IX. Riešte počiatočné úlohy

1. $y_1' = y_2 - 5\cos x$
$y_2' = 2y_1 + y_2$
$y_1(0) = y_2(0) = 0.$

2. $y_1' = 3y_1 + 2y_2 + 4\,\mathrm{e}^x$
$y_2' = y_1 + 2y_2$
$y_1(0) = 1 \;\; y_2(0) = -1.$

3. $y_1' = y_1 - y_2 + \dfrac{1}{\cos x}$
$y_2' = 2y_1 - y_2$
$y_1(0) = 2 \;\; y_2(0) = 0.$