

Matematická analýza 1

2023/2024

2. Číselné postupnosti Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou  získate nápomoc.

Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

Obsah

- 1 Riešené limity 01–20
- 2 Riešené limity 21–35
- 3 Riešené limity 36–50
- 4 Riešené limity 51–65
- 5 Riešené limity 66–80

Zoznam riešených limit – príklady 01–80

- 01. $\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$. • 02. $\lim_{n \rightarrow \infty} n(\sqrt[3]{3} - \sqrt[3]{2})$. • 03. $\lim_{n \rightarrow \infty} \frac{n^3-2}{n^2+n}$. • 04. $\lim_{n \rightarrow \infty} \frac{n^2+n}{n^4-n^3}$. • 05. $\lim_{n \rightarrow \infty} (\frac{n^2}{n+2} - \frac{n^2}{n+3})$. • 06. $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$. • 07. $\lim_{n \rightarrow \infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$.
 • 08. $\lim_{n \rightarrow \infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$. • 09. $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{2n}$. • 10. $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{n^2+1}$. • 11. $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{\sqrt{n}+1}$. • 12. $\lim_{n \rightarrow \infty} (\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2})$. • 13. $\lim_{n \rightarrow \infty} \sqrt[3]{3^n - 2^n}$.
 • 14. $\lim_{n \rightarrow \infty} \sqrt[3]{2^n + 1}$. • 15. $\lim_{n \rightarrow \infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$. • 16. $\lim_{n \rightarrow \infty} n(\sqrt[3]{2} - \sqrt[3]{3})$. • 17. $\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$. • 18. $\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$. • 19. $\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$.
 • 20. $\lim_{n \rightarrow \infty} (\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2})$. • 21. $\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4}}$. • 22. $\lim_{n \rightarrow \infty} (\frac{2n-1}{2n+3})^6$. • 23. $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$. • 24. $\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$.
 • 25. $\lim_{n \rightarrow \infty} \frac{1^2+2^2+3^2+\dots+n^2}{n^3}$. • 26. $\lim_{n \rightarrow \infty} \frac{1^2+3^2+5^2+\dots+(2n-1)^2}{n^3}$. • 27. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$. • 28. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$. • 29. $\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$.
 • 30. $\lim_{n \rightarrow \infty} n(\ln(n+3) - \ln n)$. • 31. $\lim_{n \rightarrow \infty} (\sqrt{n^2+n+1} - \sqrt{n^2+n+2})$. • 32. $\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4.5^{-n}}{3n+2-n.2^{-n}}$. • 33. $\lim_{n \rightarrow \infty} (\sqrt{n^2-n+1} - \sqrt{n^2-3n+2})$.
 • 34. $\lim_{n \rightarrow \infty} (\sqrt[3]{n+1} - \sqrt[3]{n+2})$. • 35. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$. • 36. $\lim_{n \rightarrow \infty} (\sqrt[4]{n^4-1} - \sqrt[4]{n^4+1})$. • 37. $\lim_{n \rightarrow \infty} (\sqrt{n^2+4n+1} - n + 1)$.
 • 38. $\lim_{n \rightarrow \infty} (\sqrt{n^2+n+1} - \sqrt{n-1})$. • 39. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$. • 40. $\lim_{n \rightarrow \infty} (\sqrt[4]{n^4+1} - \sqrt[4]{n+1})$. • 41. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$. • 42. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$.
 • 43. $\lim_{n \rightarrow \infty} (\sqrt[3]{n^3+1} - n + 1)$. • 44. $\lim_{n \rightarrow \infty} (\sqrt[3]{n^3+1} - n + 1)$. • 45. $\lim_{n \rightarrow \infty} (\frac{2n-1}{2n+3})^{n+6}$. • 46. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2.5^n}{2n^2-n^3+3.5^{n+1}}$. • 47. $\lim_{n \rightarrow \infty} (\frac{2n-1}{2n+3})^{n^6+6}$.
 • 48. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3.5^{n+1}}$. • 49. $\lim_{n \rightarrow \infty} (\frac{2n-1}{2n-3})^{n^6+6}$. • 50. $\lim_{n \rightarrow \infty} \sqrt[5]{\frac{5n+1}{n+5}}$. • 51. $\lim_{n \rightarrow \infty} (\frac{2n^6-1}{2n^6+3})^{n+6}$. • 52. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$. • 53. $\lim_{n \rightarrow \infty} (\frac{2n^6-1}{2n^6+3})^{n^6+6}$.
 • 54. $\lim_{n \rightarrow \infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$. • 55. $\lim_{n \rightarrow \infty} (\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}+3})^{\sqrt[n]{n}+6}$. • 56. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$. • 57. $\lim_{n \rightarrow \infty} (\frac{2n-1}{2n+3})^{\sqrt[n]{n}+6}$. • 58. $\lim_{n \rightarrow \infty} \frac{2^n n!}{n^2}$. • 59. $\lim_{n \rightarrow \infty} (\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}+3})^{n+6}$. • 60. $\lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$.
 • 61. $\lim_{n \rightarrow \infty} (\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}-3})^{n+6}$. • 62. $\lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n}$. • 63. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2-2.5^n}{2n^2-n^3}$. • 64. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2.5^n}{2n^2-n^3+6.7^n}$. • 65. $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt[n]{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt[n]{n-1}}$. • 66. $\lim_{n \rightarrow \infty} (\frac{1}{5})^n - (\frac{1}{4})^n$.
 • 67. $\lim_{n \rightarrow \infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n - (\frac{1}{3})^n}$. • 68. $\lim_{n \rightarrow \infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n - (\frac{1}{3})^n}$. • 69. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2.5^n}{2n^2-n^3}$. • 70. $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n}+\sqrt{n}}$. • 71. $\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[n]{n+3}}$. • 72. $\lim_{n \rightarrow \infty} (\frac{\sqrt[3]{2}+\sqrt[3]{3}}{2})^n$.
 • 73. $\lim_{n \rightarrow \infty} n(\sqrt[3]{2} - n^{+1/2})$. • 74. $\lim_{n \rightarrow \infty} n^2(\sqrt[3]{2} - n^{+1/2})$. • 75. $\lim_{n \rightarrow \infty} \frac{1}{n} \ln(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n})$. • 76. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2.5^n}{2n^2-n^3+6.4^n}$.
 • 77. $\lim_{n \rightarrow \infty} a_n$, ak $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$. • 78. $\lim_{n \rightarrow \infty} a_n$, ak $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots\}$.
 • 79. $\lim_{n \rightarrow \infty} a_n$, ak rekurentne $a_1 = 2$, $a_{n+1} = \sqrt{2a_n+3}$, $n \in \mathbb{N}$. • 80. Číslo 32,1771 vyjadrite ako zlomok.

Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^2 - 2}$$



$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$



Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \sqrt[n]{2}$$

Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2} \cdot \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right) = \lim_{n \rightarrow \infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1) \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \sqrt[n]{2} = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2}$$

Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2} \cdot \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right) = \lim_{n \rightarrow \infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1) \right) \\ &= \lim_{n \rightarrow \infty} n(\sqrt[n]{3} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \sqrt[n]{2} = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1$$

Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2} \cdot \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right) = \ln 3 - \ln 2 = \ln \frac{3}{2}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right) = \lim_{n \rightarrow \infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1) \right) \\ &= \lim_{n \rightarrow \infty} n(\sqrt[n]{3} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) = \ln 3 - \ln 2 = \ln \frac{3}{2}. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \sqrt[n]{2} = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$$

Riešené limity – 01, 02

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2} \cdot \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right) = \ln 3 - \ln 2 = \ln \frac{3}{2}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right) = \lim_{n \rightarrow \infty} \left(n(\sqrt[n]{3} - 1) - n(\sqrt[n]{2} - 1) \right) \\ &= \lim_{n \rightarrow \infty} n(\sqrt[n]{3} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) = \ln 3 - \ln 2 = \ln \frac{3}{2}. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \sqrt[n]{2} = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}} = \frac{\infty - 0}{1 + 0}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{0 + 0}{1 - 0}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}} = \frac{\infty - 0}{1 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{0 + 0}{1 - 0} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \\ &= \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{5}{n} + \frac{6}{n^2}} \end{aligned}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}} = \frac{\infty - 0}{1 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{0 + 0}{1 - 0} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \\ &= \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{5}{n} + \frac{6}{n^2}} = \frac{1}{1 + \frac{5}{\infty} + \frac{6}{\infty}} \end{aligned}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}} = \frac{\infty - 0}{1 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{0 + 0}{1 - 0} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \\ &= \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}} = \frac{1}{1+0+0} \end{aligned}$$

Riešené limity – 03, 04, 05

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n - \frac{2}{n^2}}{1 + \frac{1}{n}} = \frac{\infty - \frac{2}{\infty}}{1 + \frac{1}{\infty}} = \frac{\infty - 0}{1 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}} = \frac{\frac{1}{\infty} + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{0 + 0}{1 - 0} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{n+2} - \frac{n^2}{n+3} \right) = 1$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \\ &= \lim_{n \rightarrow \infty} \frac{n^2}{n^2+5n+6} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}} = \frac{1}{1+0+0} = 1. \end{aligned}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\bullet = \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{1}{\frac{1}{3^n}}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{1}{\frac{1}{(-3)^n}}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\bullet = \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{1}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{1}{(-3)^n} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n}}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\bullet = \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{1}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^{n+1}}{2(\frac{2}{3})^{n+1} + 3}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{1}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^{n+1}}{-2 \cdot (\frac{-2}{-3})^{n+1} - 3 \cdot (\frac{-3}{-3})^{n+1}}$$

$$= \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^{n+1}}{-2(\frac{2}{3})^{n+1} - 3}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} \\ &= \frac{1}{\infty + \infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3} = \frac{0+1}{2 \cdot 0 + 3}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} \\ &= \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3} = \frac{0+1}{-2 \cdot 0 - 3} \end{aligned}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} \\ &= \frac{1}{\infty + \infty} = \frac{1}{\infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} = \frac{1}{3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^{n+1}} + \frac{3^{n+1}}{3^{n+1}}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3} = \frac{0+1}{2 \cdot 0 + 3} = \frac{1}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} = -\frac{1}{3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^{n+1}} + \frac{(-3)^{n+1}}{(-3)^{n+1}}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} \\ &= \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3} = \frac{0+1}{-2 \cdot 0 - 3} = -\frac{1}{3}. \end{aligned}$$

Riešené limity – 06, 07, 08

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} \\ &= \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} = \frac{1}{3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^{n+1}} + \frac{3^{n+1}}{3^{n+1}}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3} = \frac{0+1}{2 \cdot 0 + 3} = \frac{1}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} = -\frac{1}{3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^{n+1}} + \frac{(-3)^{n+1}}{(-3)^{n+1}}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} \\ &= \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3} = \frac{0+1}{-2 \cdot 0 - 3} = -\frac{1}{3}. \end{aligned}$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^2$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{n^2+1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{\sqrt{n}+1}{n}}$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^2 = \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right)^2$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{n^2+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{n + \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{1}{\sqrt{n}} + \frac{1}{n}}$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n} = e^2$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^2 = \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right)^2 = e^2.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{n^2+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{n + \frac{1}{n}} = e^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{1}{\sqrt{n}} + \frac{1}{n}} = e^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n} = e^2$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^2 = \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right)^2 = e^2.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{n^2+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{n + \frac{1}{n}} = e^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \quad \bullet \lim_{n \rightarrow \infty} \left(n + \frac{1}{n}\right) = \infty + 0 = \infty.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{1}{\sqrt{n}} + \frac{1}{n}} = e^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \quad \bullet \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{n}\right) = 0 + 0 = 0.$$

Riešené limity – 09, 10, 11

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n} = e^2$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^2 = \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right)^2 = e^2.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{n^2+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{n + \frac{1}{n}} = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \quad \bullet \lim_{n \rightarrow \infty} \left(n + \frac{1}{n}\right) = \infty + 0 = \infty.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{\sqrt{n}+1}{n}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n}\right)^n \right)^{\frac{1}{\sqrt{n}} + \frac{1}{n}} = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \quad \bullet \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{n}\right) = 0 + 0 = 0.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$



$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\bullet = \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)}$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right)$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\bullet = \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right)$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

$$\Rightarrow \bullet \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \sqrt[n]{1}.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n. \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n. \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

$$\Rightarrow \bullet \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \sqrt[n]{1}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1} = 1.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right)$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

$$\Rightarrow \bullet \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \sqrt[n]{1}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1} = 1. \Rightarrow \bullet \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 1.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right) = \frac{1}{2}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} = \frac{1}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \cdot 1$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n. \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n. \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

$$\Rightarrow \bullet \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \sqrt[n]{1}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1} = 1. \Rightarrow \bullet \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 1.$$

Riešené limity – 12, 13

$$\lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right) = \frac{1}{2}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{2} - \frac{n(n+1)}{2(n+2)} \right) = \lim_{n \rightarrow \infty} \frac{n(n+2) - n(n+1)}{2(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 + 2n - n^2 - n}{2(n+2)} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} = \frac{1}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n} = 3$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \cdot 1 = 3.$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \frac{2}{3} \geq \left(\frac{2}{3}\right)^n. \Rightarrow \bullet -\frac{2}{3} \leq -\left(\frac{2}{3}\right)^n. \Rightarrow \bullet \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n \leq 1.$$

$$\Rightarrow \bullet \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \sqrt[n]{1}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1} = 1. \Rightarrow \bullet \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 1.$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n]{3})$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

Pre všetky $n \in \mathbb{N}$ platí:

- $2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n$.

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

- $= \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{1}{\frac{1}{\sqrt{n}}}$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n]{3} \right)$$

- $= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1 \right)$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \ 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow \bullet \ 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n + 1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n]{3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1 \right) = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) - \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 \right)$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \quad 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow \bullet \quad 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n + 1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$$

$$\Rightarrow \bullet \quad 2 = \lim_{n \rightarrow \infty} 2 \leq \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} \leq \lim_{n \rightarrow \infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2.$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n]{3} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1 \right) = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) - \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 \right)$$

$$= \left[\text{Pre všetky } a > 0 \text{ platí } \lim_{n \rightarrow \infty} n \left(\sqrt[n]{a} - 1 \right) = \ln a. \right]$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \quad 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow \bullet \quad 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n + 1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$$

$$\Rightarrow \bullet \quad 2 = \lim_{n \rightarrow \infty} 2 \leq \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} \leq \lim_{n \rightarrow \infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow \bullet \quad \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2.$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{1 - 1}{1 + 1}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n]{3} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1 \right) = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) - \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 \right) \\ &= \left[\text{Pre všetky } a > 0 \text{ platí } \lim_{n \rightarrow \infty} n \left(\sqrt[n]{a} - 1 \right) = \ln a. \right] = \ln 2 - \ln 3 \end{aligned}$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \quad 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow \bullet \quad 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n + 1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$$

$$\Rightarrow \bullet \quad 2 = \lim_{n \rightarrow \infty} 2 \leq \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} \leq \lim_{n \rightarrow \infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow \bullet \quad \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2.$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{0 - 1}{0 + 1}$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n]{3}) = \ln \frac{2}{3}$$

$$\bullet = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n]{3} - 1)$$

$$= \left[\text{Pre všetky } a > 0 \text{ platí } \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a. \right] = \ln 2 - \ln 3 = \ln \frac{2}{3}.$$

Riešené limity – 14, 15, 16

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet \quad 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow \bullet \quad 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n + 1} \leq \sqrt[n]{2 \cdot 2^n} = 2 \cdot \sqrt[n]{2}.$$

$$\Rightarrow \bullet \quad 2 = \lim_{n \rightarrow \infty} 2 \leq \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} \leq \lim_{n \rightarrow \infty} 2 \cdot \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow \bullet \quad \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2.$$

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} = -1$$

$$\bullet \quad = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{0 - 1}{0 + 1} = -1.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n]{3}) = \ln \frac{2}{3}$$

$$\begin{aligned} \bullet \quad &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n]{3} - 1) \\ &= \left[\text{Pre všetky } a > 0 \text{ platí } \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a. \right] = \ln 2 - \ln 3 = \ln \frac{2}{3}. \end{aligned}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right)$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{1}{3^n}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{1}{n}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right) = \infty \cdot \left(\sqrt{1 + \frac{1}{\infty}} - \infty \right)$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right) = \infty \cdot \left(\sqrt{1 + \frac{1}{\infty}} - \infty \right) = \infty \cdot (\sqrt{1+0} - \infty)$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1}$$

Označme $a_n = \frac{n}{3^n}$, $n \in \mathbb{N}$.

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \\ \text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}} = \frac{1 + \frac{1}{\infty \cdot \infty}}{1 - \frac{1}{\infty \cdot \infty}}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right) = \infty \cdot \left(\sqrt{1 + \frac{1}{\infty}} - \infty \right) = \infty \cdot (\sqrt{1+0} - \infty) \\ &= \infty \cdot (1 - \infty) \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1} = \frac{0+1}{0-1}$$

Označme $a_n = \frac{n}{3^n}$, $n \in \mathbb{N}$.

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \\ \text{Resp.} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}} = \frac{1 + \frac{1}{\infty \cdot \infty}}{1 - \frac{1}{\infty \cdot \infty}} = \frac{1 + \frac{1}{\infty}}{1 - \frac{1}{\infty}}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n) = -\infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right) = \infty \cdot \left(\sqrt{1 + \frac{1}{\infty}} - \infty \right) = \infty \cdot (\sqrt{1+0} - \infty) \\ &= \infty \cdot (1 - \infty) = \infty \cdot (-\infty) = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} = -1$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1} = \frac{0+1}{0-1} = -1.$$

Označme $a_n = \frac{n}{3^n}$, $n \in \mathbb{N}$.

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \\ \text{Resp.} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}} = \frac{1 + \frac{1}{\infty \cdot \infty}}{1 - \frac{1}{\infty \cdot \infty}} = \frac{1 + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{1+0}{1-0}$$

Riešené limity – 17, 18, 19

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - n) = -\infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right) = \infty \cdot \left(\sqrt{1 + \frac{1}{\infty}} - \infty \right) = \infty \cdot (\sqrt{1+0} - \infty) \\ &= \infty \cdot (1 - \infty) = \infty \cdot (-\infty) = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} = -1$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1} = \frac{0+1}{0-1} = -1.$$

Označme $a_n = \frac{n}{3^n}$, $n \in \mathbb{N}$.

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \\ \text{Resp.} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}} = \frac{1 + \frac{1}{\infty \cdot \infty}}{1 - \frac{1}{\infty \cdot \infty}} = \frac{1 + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{1+0}{1-0} = 1.$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \end{array} \right] \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \frac{n}{\sqrt[3]{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right. \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \\ &= \frac{2 - 0}{\infty \cdot \left(\sqrt[3]{(1 + 0 + 0)^2 + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right)} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right. \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \\ &= \frac{2 - 0}{\infty \cdot \left(\sqrt[3]{(1 + 0 + 0)^2 + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right)} = \frac{2}{\infty \cdot (1 + 1 \cdot 1 + 1)} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right. \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \\ &= \frac{2 - 0}{\infty \cdot \left(\sqrt[3]{(1 + 0 + 0)^2 + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right)} = \frac{2}{\infty \cdot (1 + 1 \cdot 1 + 1)} = \frac{2}{\infty \cdot 3} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \\ &= \frac{2 - 0}{\infty \cdot \left(\sqrt[3]{(1 + 0 + 0)^2 + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right)} = \frac{2}{\infty \cdot (1 + 1 \cdot 1 + 1)} = \frac{2}{\infty \cdot 3} = \frac{2}{\infty} \end{aligned}$$

Riešené limity – 20

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) = 0$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right) \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 3n + 1) - (n^2 + n + 2)}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2 + \sqrt[3]{n^2 + 3n + 1} \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2 \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)}} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \left[\sqrt[3]{\frac{n}{n^2 \cdot n^2}} = \sqrt[3]{\frac{n}{n^3 \cdot n}} = \frac{1}{\sqrt[3]{n}} \right] \right. \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \cdot \left(\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2 + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right)} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \cdot \left(\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2 + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right)} \\ &= \frac{2 - 0}{\infty \cdot \left(\sqrt[3]{(1 + 0 + 0)^2 + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right)} = \frac{2}{\infty \cdot (1 + 1 \cdot 1 + 1)} = \frac{2}{\infty \cdot 3} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

- $= 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$

- $= 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$

- $\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6$

- $= \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{1}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6 = \left(1 - \frac{4}{\infty} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6 = \left(1 - \frac{4}{\infty} \right)^6 \\ = (1 - 0)^6 = 1^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{1}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6 = 1^6$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^2$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2.$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6 = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6 = \left(1 - \frac{4}{\infty} \right)^6 = (1-0)^6 = 1^6 = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{1}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6 = 1^6 = 1.$$

Riešené limity – 21, 22

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}} = 16$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2 = 16.$$

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^2 = 16.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} = \lim_{n \rightarrow \infty} \frac{8n+1}{4n-3} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2.$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6 = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6 = \left(1 - \frac{4}{\infty} \right)^6 = (1-0)^6 = 1^6 = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{1}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6 = 1^6 = 1.$$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

- = $\left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2}. \end{array} \right]$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

- = $\lim_{n \rightarrow \infty} \frac{n^2}{n^2}$

Pre konečné aritmetické rady platí:

- $1+3+5+\dots+(2n-1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] \stackrel{?}{=} \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1 + \frac{1}{n})}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n \cdot n}}{\frac{1}{\sqrt{n^4}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1 + \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1 + \frac{1}{n}}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n \cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1 + \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1 + \frac{1}{n}} = \frac{2}{1 + \frac{1}{\infty}}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n \cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0}$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{\sqrt{n^4}}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n} = 2$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Pre konečné aritmetické rady platí:

- $1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{\sqrt{n^4}}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}} \\ &= \frac{1+0}{2\sqrt{9+0}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n} = 2$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Pre konečné aritmetické rady platí:

- $1+3+5+\dots+(2n-1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2.$
- $1+2+3+\dots+n = \frac{(1+n)n}{2}.$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{\sqrt{n^4}}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2 \cdot 3}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n} = 2$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n^2}{2}}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Pre konečné aritmetické rady platí:

$$\bullet 1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2. \rightarrow$$

$$\bullet 1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}. \rightarrow$$

Riešené limity – 23, 24

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}} = \frac{1}{6}$$

$$\bullet = \left[\begin{array}{l} \text{Aritmetický rad} \\ 1+2+\dots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt{n^4} \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{\sqrt{n^4}}}{\frac{1}{\sqrt{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2 \cdot 3} = \frac{1}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n} = 2$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{n \cdot 2}{n \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Pre konečné aritmetické rady platí:

$$\bullet 1 + 3 + 5 + \dots + (2n - 1) = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + \dots + (2n - 1) = \frac{(1+2n-1)n}{2} = n^2. \rightarrow$$

$$\bullet 1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2}. \rightarrow$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$



$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$



Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n(4n^2-1)}{3}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} \\ &= \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \left(2 + \frac{1}{n}\right) \end{aligned}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right)$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} \\ &= \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \left(2 + \frac{1}{n}\right) = \frac{1}{6} \left(1 + \frac{1}{\infty}\right) \cdot \left(2 + \frac{1}{\infty}\right) \end{aligned}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} \\ &= \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \left(2 + \frac{1}{n}\right) = \frac{1}{6} \left(1 + \frac{1}{\infty}\right) \cdot \left(2 + \frac{1}{\infty}\right) = \frac{1}{6} (1 + 0) \cdot (2 + 0) \end{aligned}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} \\ &= \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \left(2 + \frac{1}{n}\right) = \frac{1}{6} \left(1 + \frac{1}{\infty}\right) \cdot \left(2 + \frac{1}{\infty}\right) = \frac{1}{6} (1 + 0) \cdot (2 + 0) = \frac{1}{6} \cdot 1 \cdot 2 \end{aligned}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3} = \frac{4}{3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 25, 26

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} = \frac{1}{3}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} \\ &= \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \cdot \left(2 + \frac{1}{n}\right) = \frac{1}{6} \left(1 + \frac{1}{\infty}\right) \cdot \left(2 + \frac{1}{\infty}\right) = \frac{1}{6} (1 + 0) \cdot (2 + 0) = \frac{1}{6} \cdot 1 \cdot 2 = \frac{1}{3}. \end{aligned}$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3} = \frac{4}{3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$$

Pre konečný číselný rad platí:

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1} - \sqrt{n^2+n+2}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1} - \sqrt{n^2-n-1}}$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} \end{aligned}$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left(\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} \right)$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1} \\ &= -\sqrt{\infty} - \sqrt{\infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left(\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} \right) \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1} \\ &= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left(\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} \right) \\ &= \frac{1}{2} \cdot \infty \cdot (\sqrt{1-0+0} + \sqrt{1-0-0}) \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1} \\ &= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left(\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} \right) \\ &= \frac{1}{2} \cdot \infty \cdot (\sqrt{1-0+0} + \sqrt{1-0-0}) = \frac{1}{2} \cdot \infty \cdot 2 \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$$

Riešené limity – 27, 28

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1} \\ &= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} = \infty$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left(\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^2}} \right) \\ &= \frac{1}{2} \cdot \infty \cdot (\sqrt{1-0+0} + \sqrt{1-0-0}) = \frac{1}{2} \cdot \infty \cdot 2 = \infty. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2}$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2}\right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n}\right)^{-n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2}\right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2}\right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n}\right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n}\right)^{-n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2}\right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2}\right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{2}{n} \right)^n \right)^{-1} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{-2}{n+2} \right)^{n+2} \right)^{\frac{n}{n+2}} \end{aligned}$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{2}{n} \right)^n \right)^{-1} = \ln(e^2)^{-1} \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{-2}{n+2} \right)^{n+2} \right)^{\frac{n}{n+2}} = \ln(e^{-2})^1 \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{2}{n} \right)^n \right)^{-1} = \ln(e^2)^{-1} = \ln e^{-2} \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

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$$\bullet \lim_{n \rightarrow \infty} \frac{n}{n+2} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot \left(1 + \frac{2}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{2}{n}} = \frac{1}{1 + \frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{2}{n} \right)^n \right)^{-1} = \ln(e^2)^{-1} = \ln e^{-2} = -2. \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

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$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n}{n+2} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot \left(1 + \frac{2}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{2}{n}} = \frac{1}{1 + \frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{2}{n} \right)^n \right)^{-1} = \ln(e^2)^{-1} = \ln e^{-2} = -2. \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{-2}{n+2} \right)^{n+2} \right)^{\frac{n}{n+2}} = \ln(e^{-2})^1 = \ln e^{-2} = -2. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n}{n+2} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot \left(1 + \frac{2}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{2}{n}} = \frac{1}{1 + \frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2}$$

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Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n(\ln n - \ln(n+2))$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n$$

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Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^{n+2-2} \end{aligned}$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 = -2. \end{aligned}$$

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Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 = -2. \end{aligned}$$

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$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 = -2. \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

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$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{-2} = \left(1 + \frac{-2}{\infty} \right)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 = -2. \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^{n+2-2} = \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{-2}{n+2} \right)^{n+2} \cdot \left(1 + \frac{-2}{n+2} \right)^{-2} \right) \\ &= \ln \left(e^{-2} \cdot 1 \right) = \ln e^{-2} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{-2} = \left(1 + \frac{-2}{\infty} \right)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 29

$$\lim_{n \rightarrow \infty} n \left(\ln n - \ln(n+2) \right) = -2$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \left(-n \cdot \ln \frac{n+2}{n} \right) = - \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+2}{n} = - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \\ &= - \ln e^2 = -2. \end{aligned}$$

Pre všetky $a \in \mathbb{R}$ platí:

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \cdot \ln \frac{n}{n+2} = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^n \\ &= \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^{n+2-2} = \lim_{n \rightarrow \infty} \ln \left(\left(1 + \frac{-2}{n+2} \right)^{n+2} \cdot \left(1 + \frac{-2}{n+2} \right)^{-2} \right) \\ &= \ln \left(e^{-2} \cdot 1 \right) = \ln e^{-2} = -2. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = n+2 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-2}{m} \right)^m = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{-2} = \left(1 + \frac{-2}{\infty} \right)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right)$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n}$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = \ln e^3$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \end{aligned}$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n} = 3$$

$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = \ln e^3 = 3.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} = \frac{-1}{\sqrt{\infty} + \sqrt{\infty}} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a \in \mathbb{R}.$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n} = 3$$

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$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} = \frac{-1}{\sqrt{\infty} + \sqrt{\infty}} = \frac{-1}{\infty} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \\ &= \frac{-1}{\infty \cdot (\sqrt{1-0+0} + \sqrt{1-0+0})} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a \in \mathbb{R}.$$

Riešené limity – 30, 31

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$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = \ln e^3 = 3.$$

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$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} = \frac{-1}{\sqrt{\infty} + \sqrt{\infty}} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \\ &= \frac{-1}{\infty \cdot (\sqrt{1-0+0} + \sqrt{1-0+0})} = \frac{-1}{\infty \cdot 2} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a \in \mathbb{R}.$$

Riešené limity – 30, 31

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$$\bullet = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right) = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = \ln e^3 = 3.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} = \frac{-1}{\sqrt{\infty} + \sqrt{\infty}} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \\ &= \frac{-1}{\infty \cdot (\sqrt{1-0+0} + \sqrt{1-0+0})} = \frac{-1}{\infty \cdot 2} = \frac{-1}{\infty} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a \in \mathbb{R}.$$

Riešené limity – 30, 31

$$\lim_{n \rightarrow \infty} n \left(\ln(n+3) - \ln n \right) = \lim_{n \rightarrow \infty} n \cdot \ln \frac{n+3}{n} = 3$$

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$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty^2 + \infty + 1} + \sqrt{\infty^2 + \infty + 2}} = \frac{-1}{\sqrt{\infty} + \sqrt{\infty}} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right) \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 + n + 1) - (n^2 + n + 2)}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2} \cdot \left(\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 + \frac{1}{n} + \frac{2}{n^2}} \right)} \\ &= \frac{-1}{\infty \cdot (\sqrt{1-0+0} + \sqrt{1-0+0})} = \frac{-1}{\infty \cdot 2} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \sqrt{n^2} = \lim_{n \rightarrow \infty} n = \infty. \quad \bullet \lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0. \quad \bullet \lim_{n \rightarrow \infty} \frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a \in \mathbb{R}.$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\bullet = \frac{\quad}{-0}$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \left. \begin{array}{l} \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \\ \text{Resp. } \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0}$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$$

Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \frac{1}{2} < 1. \\ \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{1}{2} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2-0}$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$$

Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \frac{1}{2} < 1. \\ \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{1}{2} < 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty}$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$$

Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2}. \end{array} \right]$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$$

Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1.$$

Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

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$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

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Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

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Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

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$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}} = \frac{2 - 0}{\sqrt{1 - 0 + 0} + \sqrt{1 - 0 + 0}}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

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Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

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$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}} = \frac{2 - 0}{\sqrt{1 - 0 + 0} + \sqrt{1 - 0 + 0}} = \frac{2}{1 + 1}$$

Riešené limity – 32, 33

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$\bullet = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

Označme $a_n = \frac{n}{2^n}$, $n \in \mathbb{N}$.

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Resp. $\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1.$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right) \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \lim_{n \rightarrow \infty} \frac{(n^2 - n + 1) - (n^2 - 3n + 2)}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}} = \frac{2 - 0}{\sqrt{1 - 0 + 0} + \sqrt{1 - 0 + 0}} = \frac{2}{1 + 1} = 1.$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1}\sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1}\sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \frac{-1}{\sqrt[3]{(\infty+1)^2} + \sqrt[3]{\infty+1} \sqrt[3]{\infty+2} + \sqrt[3]{(\infty+2)^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \\ &= \frac{\sqrt[3]{(\infty-1)^2} + \sqrt[3]{\infty-1} \sqrt[3]{\infty-2} + \sqrt[3]{(\infty-2)^2}}{1} \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \frac{-1}{\sqrt[3]{(\infty+1)^2} + \sqrt[3]{\infty+1} \sqrt[3]{\infty+2} + \sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty + \infty \cdot \infty + \infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \\ &= \frac{\sqrt[3]{(\infty-1)^2} + \sqrt[3]{\infty-1} \sqrt[3]{\infty-2} + \sqrt[3]{(\infty-2)^2}}{1} = \infty + \infty \cdot \infty + \infty \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \frac{-1}{\sqrt[3]{(\infty+1)^2} + \sqrt[3]{\infty+1} \sqrt[3]{\infty+2} + \sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty + \infty \cdot \infty + \infty} = \frac{-1}{\infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} = \infty$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \\ &= \frac{\sqrt[3]{(\infty-1)^2} + \sqrt[3]{\infty-1} \sqrt[3]{\infty-2} + \sqrt[3]{(\infty-2)^2}}{1} = \infty + \infty \cdot \infty + \infty = \infty. \end{aligned}$$

Riešené limity – 34, 35

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt[3]{n+1} - \sqrt[3]{n+2} \right) \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1) - (n+2)}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \frac{-1}{\sqrt[3]{(\infty+1)^2} + \sqrt[3]{\infty+1} \sqrt[3]{\infty+2} + \sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty + \infty \cdot \infty + \infty} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{(n-1) - (n-2)} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \\ &= \frac{\sqrt[3]{(\infty-1)^2} + \sqrt[3]{\infty-1} \sqrt[3]{\infty-2} + \sqrt[3]{(\infty-2)^2}}{1} = \infty + \infty \cdot \infty + \infty = \infty. \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \\ &= \frac{1}{(\sqrt[4]{\infty-1} + \sqrt[4]{\infty+1}) \cdot (\sqrt{\infty-1} + \sqrt{\infty+1})} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \\ &= \frac{1}{(\sqrt[4]{\infty-1} + \sqrt[4]{\infty+1}) \cdot (\sqrt{\infty-1} + \sqrt{\infty+1})} = \frac{1}{(\infty + \infty) \cdot (\infty + \infty)} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \\ &= \frac{1}{(\sqrt[4]{\infty-1} + \sqrt[4]{\infty+1}) \cdot (\sqrt{\infty-1} + \sqrt{\infty+1})} = \frac{1}{(\infty + \infty) \cdot (\infty + \infty)} = \frac{1}{\infty \cdot \infty} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4 - 1)^3} + \sqrt[4]{(n^4 - 1)^2} \sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \sqrt[4]{(n^4 + 1)^2} + \sqrt[4]{(n^4 + 1)^3}}{\sqrt[4]{(n^4 - 1)^3} + \sqrt[4]{(n^4 - 1)^2} \sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \sqrt[4]{(n^4 + 1)^2} + \sqrt[4]{(n^4 + 1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4 - 1) - (n^4 + 1)}{\sqrt[4]{(n^4 - 1)^3} + \sqrt[4]{(n^4 - 1)^2} \sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \sqrt[4]{(n^4 + 1)^2} + \sqrt[4]{(n^4 + 1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4 - 1)^3} + \sqrt[4]{(n^4 - 1)^2} \sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \sqrt[4]{(n^4 + 1)^2} + \sqrt[4]{(n^4 + 1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}}{\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4 - 1)^2} - \sqrt[4]{(n^4 + 1)^2}}{\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 - 1} - \sqrt{n^4 + 1}}{\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 - 1} - \sqrt{n^4 + 1}}{\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}} \cdot \frac{\sqrt{n^4 - 1} + \sqrt{n^4 + 1}}{\sqrt{n^4 - 1} + \sqrt{n^4 + 1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4 - 1) - (n^4 + 1)}{(\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}) \cdot (\sqrt{n^4 - 1} + \sqrt{n^4 + 1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1}) \cdot (\sqrt{n^4 - 1} + \sqrt{n^4 + 1})} \\ &= \frac{1}{(\sqrt[4]{\infty - 1} + \sqrt[4]{\infty + 1}) \cdot (\sqrt{\infty - 1} + \sqrt{\infty + 1})} = \frac{1}{(\infty + \infty) \cdot (\infty + \infty)} = \frac{1}{\infty \cdot \infty} = \frac{1}{\infty} \end{aligned}$$

Riešené limity – 36

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) = 0$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^4-1)^3} + \sqrt[4]{(n^4-1)^2} \sqrt[4]{n^4+1} + \sqrt[4]{n^4-1} \sqrt[4]{(n^4+1)^2} + \sqrt[4]{(n^4+1)^3}} = \left[\begin{array}{l} \text{Pre } k \in \mathbb{N} \text{ platí} \\ \lim_{n \rightarrow \infty} (n^4 \pm 1)^k = \infty. \end{array} \right] \\ &= \frac{1}{\sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty} \sqrt[4]{\infty} + \sqrt[4]{\infty}} = \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 - 1} - \sqrt[4]{n^4 + 1} \right) \cdot \frac{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4-1)^2} - \sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4-1} - \sqrt{n^4+1}}{\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}} \cdot \frac{\sqrt{n^4-1} + \sqrt{n^4+1}}{\sqrt{n^4-1} + \sqrt{n^4+1}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^4-1) - (n^4+1)}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} = \lim_{n \rightarrow \infty} \frac{1}{(\sqrt[4]{n^4-1} + \sqrt[4]{n^4+1}) \cdot (\sqrt{n^4-1} + \sqrt{n^4+1})} \\ &= \frac{1}{(\sqrt[4]{\infty-1} + \sqrt[4]{\infty+1}) \cdot (\sqrt{\infty-1} + \sqrt{\infty+1})} = \frac{1}{(\infty + \infty) \cdot (\infty + \infty)} = \frac{1}{\infty \cdot \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - n + 1)$$

$$\bullet = \lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - (n - 1)) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)}$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - n)$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - n + 1)$$

$$\bullet = \lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - (n - 1)) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1}$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - n) = 1 + \lim_{n \rightarrow \infty} (\sqrt{n^2 + 4n + 1} - n) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2}. \end{array} \right] \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2}. \end{array} \right] \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{6}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} = 1 + \frac{4 + \frac{1}{\infty}}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{6}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{6}{\sqrt{1 + 0 + 0} + 1 - 0} \end{aligned}$$

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} = 1 + \frac{4 + \frac{1}{\infty}}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{6}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{6}{\sqrt{1 + 0 + 0} + 1 - 0} \\ &= \frac{6}{2} \end{aligned}$$

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} = 1 + \frac{4 + \frac{1}{\infty}}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} \\ &= 1 + \frac{4}{2} \end{aligned}$$

Riešené limity – 37

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n + 1 \right) = 3$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - (n - 1) \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n - 1)^2}{\sqrt{n^2 + 4n + 1} + n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - (n^2 - 2n + 1)}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{6}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{6}{\sqrt{1 + 0 + 0} + 1 - 0} \\ &= \frac{6}{2} = 3. \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n + 1} - n \right) \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^2 + 4n + 1) - n^2}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \cdot \frac{1}{n} \\ &= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n = \sqrt{n^2} \end{array} \right] = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} = 1 + \frac{4 + \frac{1}{\infty}}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} \\ &= 1 + \frac{4}{2} = 3. \end{aligned}$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right)$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left(\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \cdot \left(\sqrt{n^2 + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right)}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left(\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right)$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left(\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right) = \infty \cdot \left(\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \cdot \left(\sqrt[3]{n^2 + 1 + \frac{1}{n}} - \sqrt[3]{1 - \frac{1}{n}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[3]{\infty + 1 + \frac{1}{\infty}} - \sqrt[3]{1 - \frac{1}{\infty}} \right)}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left(\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right) = \infty \cdot \left(\sqrt[4]{\infty + \frac{1}{\infty}} - \sqrt[4]{1 + \frac{1}{\infty}} \right)$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left(\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right) = \infty \cdot \left(\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt{\infty + 1 + 0} - \sqrt{1 - 0}) \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \cdot \left(\sqrt[3]{n^2 + 1 + \frac{1}{n}} - \sqrt[3]{1 - \frac{1}{n}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[3]{\infty + 1 + \frac{1}{\infty}} - \sqrt[3]{1 - \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[3]{\infty + 1 + 0} - \sqrt[3]{1 - 0})}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left(\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right) = \infty \cdot \left(\sqrt[4]{\infty + \frac{1}{\infty}} - \sqrt[4]{1 + \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt[4]{\infty + 0} - \sqrt[4]{1 + 0}) \end{aligned}$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left(\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right) = \infty \cdot \left(\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt{\infty + 1 + 0} - \sqrt{1 - 0}) = \infty \cdot (\infty - 1) \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \cdot \left(\sqrt[3]{n^2 + 1 + \frac{1}{n}} - \sqrt[3]{1 - \frac{1}{n}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[3]{\infty + 1 + \frac{1}{\infty}} - \sqrt[3]{1 - \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[3]{\infty + 1 + 0} - \sqrt[3]{1 - 0})} \\ &= \frac{1}{\infty \cdot (\infty - 1)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left(\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right) = \infty \cdot \left(\sqrt[4]{\infty + \frac{1}{\infty}} - \sqrt[4]{1 + \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt[4]{\infty + 0} - \sqrt[4]{1 + 0}) = \infty \cdot (\infty - 1) \end{aligned}$$

Riešené limity – 38, 39, 40

$$\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right) = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left(\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right) = \infty \cdot \left(\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt{\infty + 1 + 0} - \sqrt{1 - 0}) = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}} = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \cdot \left(\sqrt[3]{n^2 + 1 + \frac{1}{n}} - \sqrt[3]{1 - \frac{1}{n}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[3]{\infty + 1 + \frac{1}{\infty}} - \sqrt[3]{1 - \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[3]{\infty + 1 + 0} - \sqrt[3]{1 - 0})} \\ &= \frac{1}{\infty \cdot (\infty - 1)} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right) = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left(\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right) = \infty \cdot \left(\sqrt[4]{\infty + \frac{1}{\infty}} - \sqrt[4]{1 + \frac{1}{\infty}} \right) \\ &= \infty \cdot (\sqrt[4]{\infty + 0} - \sqrt[4]{1 + 0}) = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}} \right)}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{(n^3+1)-n^3}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}} \right)}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{(n^3+1)-n^3} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[5]{\infty+0} - \sqrt[5]{1+0})}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{(n^3+1)-n^3} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right] \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[5]{\infty+0} - \sqrt[5]{1+0})} \\ &= \frac{1}{\infty \cdot (\infty-1)} \end{aligned}$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{(n^3+1)-n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot (\sqrt[5]{\infty+0} - \sqrt[5]{1+0})}$$

$$= \frac{1}{\infty \cdot (\infty - 1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right) = \infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + 0} - \sqrt[5]{1 + 0} \right)}$$

$$= \frac{1}{\infty \cdot (\infty - 1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right) = \infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)$$

$$= \infty \cdot \left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + 0} - \sqrt[5]{1 + 0} \right)}$$

$$= \frac{1}{\infty \cdot (\infty - 1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}}+n^2}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right) = \infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)$$

$$= \infty \cdot \left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right) = \infty(1+1+1)$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + 0} - \sqrt[5]{1 + 0} \right)}$$

$$= \frac{1}{\infty \cdot (\infty - 1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{(n^3+1)-n^3}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{1} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} n^2 \cdot \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right) = \infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)$$

$$= \infty \cdot \left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right) = \infty(1+1+1) = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left(\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right)} = \frac{1}{\infty \cdot \left(\sqrt[5]{\infty + 0} - \sqrt[5]{1 + 0} \right)}$$

$$= \frac{1}{\infty \cdot (\infty - 1)} = 0.$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3. \end{array} \right]$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3 \end{array} \right] = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3 \end{array} \right] = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}$$

$$= \frac{3 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right) + \left(1 - \frac{1}{\infty}\right)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3 \end{array} \right] = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}$$

$$= \frac{3 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right) + \left(1 - \frac{1}{\infty}\right)^2} = \frac{3 - 0 + 0}{\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} \cdot (1-0) + (1-0)^2}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3 \end{array} \right] = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}$$

$$= \frac{3 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right) + \left(1 - \frac{1}{\infty}\right)^2} = \frac{3 - 0 + 0}{\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} \cdot (1-0) + (1-0)^2} = \frac{3}{1+1 \cdot 1+1}$$

Riešené limity – 43

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right) = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - (n - 1) \right) \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n-1)^3}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{(n^3+1) - (n^3 - 3n^2 + 3n - 1)}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \cdot n = \sqrt[3]{n^3} \cdot n = \sqrt[3]{n^3} \cdot n^3 \end{array} \right] = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}$$

$$= \frac{3 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right) + \left(1 - \frac{1}{\infty}\right)^2} = \frac{3 - 0 + 0}{\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} \cdot (1-0) + (1-0)^2} = \frac{3}{1+1 \cdot 1+1} = 1.$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right)$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right)$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right)$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n^3\sqrt[3]{n^3+1}+n^2}} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n\sqrt[3]{n^3} = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right] \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n^3}\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2+n^3}\sqrt[3]{n^3+1}+n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n^3}\sqrt[3]{n^3+1}+n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n^3}\sqrt[3]{n^3+1}+n^2} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n\sqrt[3]{n^3} = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1}+n^2} = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ n^2 = n \sqrt[3]{n^3} = \sqrt[3]{n^3 \cdot n^3}. \end{array} \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1+\frac{1}{n^3}\right)^2 + \sqrt[3]{1+\frac{1}{n^3}+1}} \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1+\frac{1}{\infty}\right)^2 + \sqrt[3]{1+\frac{1}{\infty}+1}} \right)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1) - n^3}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \right. \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2 + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2 + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)} \\ &= 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{(1+0)^2 + \sqrt[3]{1+0} + 1 \right)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n}\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2+n}\sqrt[3]{n^3+1}+n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2+n}\sqrt[3]{n^3+1}+n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n}\sqrt[3]{n^3+1}+n^2} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \right. \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1+\frac{1}{n^3}\right)^2 + \sqrt[3]{1+\frac{1}{n^3}+1}} \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1+\frac{1}{\infty}\right)^2 + \sqrt[3]{1+\frac{1}{\infty}+1}} \right)} \\ &= 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{(1+0)^2 + \sqrt[3]{1+0+1}} \right)} = 1 + \frac{1}{\infty \cdot (1+1+1)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1) - n^3}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \right. \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2 + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2 + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)} \\ &= 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{(1+0)^2 + \sqrt[3]{1+0} + 1} \right)} = 1 + \frac{1}{\infty \cdot (1+1+1)} = 1 + \frac{1}{\infty} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right)$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1) - n^3}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \right. \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2 + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2 + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)} \\ &= 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{(1+0)^2 + \sqrt[3]{1+0} + 1 \right)} = 1 + \frac{1}{\infty \cdot (1+1+1)} = 1 + \frac{1}{\infty} = 1 + 0 \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 43, 44

$$\lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n + 1 \right) = 1$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) = 1 + \lim_{n \rightarrow \infty} \left(\sqrt[3]{n^3 + 1} - n \right) \cdot \frac{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{(n^3+1) - n^3}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2+n} \sqrt[3]{n^3+1} + n^2} = \left[\text{Pre všetky } n \in \mathbb{N} \text{ platí } \right. \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left(\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2 + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right)} = 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2 + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right)} \\ &= 1 + \frac{1}{\infty \cdot \left(\sqrt[3]{(1+0)^2 + \sqrt[3]{1+0} + 1 \right)} = 1 + \frac{1}{\infty \cdot (1+1+1)} = 1 + \frac{1}{\infty} = 1 + 0 = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right) = \lim_{n \rightarrow \infty} \left(\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right) \\ &= \lim_{n \rightarrow \infty} \left((n+4) - \frac{1}{n+3} \right) = (\infty + 4) - \frac{1}{\infty+3} = \infty - 0 = \infty. \end{aligned}$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}}$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6}$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}}$$

- $= \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n}$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5}$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n+6}{2n+3}}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5}$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n+6}{2n+3}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2 \cdot 5^n}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 3 \cdot 5}$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n+6}{2n+3}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} = \frac{2}{15}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{2}{15}.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{\left(1 + \frac{1}{n} \right)^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 45, 46

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6} = e^{-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n+6}{2n+3}} = (e^{-4})^{\frac{1}{2}} = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} = \frac{2}{15}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{2}{15}.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

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Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}}$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6}$

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- $= \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n}$

Riešené limity – 47, 48

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$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5}$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

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Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

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$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5}$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

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$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15}$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^{\infty}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst.} \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst.} \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst.} \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst.} \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 47, 48

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{n^6+6}{2n+3}} = (e^{-4})^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst.} \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 + \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 + \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{\infty + 0}{2 + 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + \frac{3n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

Označme $a_n = \frac{n^k}{5^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n+5 \leq 5n+1.$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n+5 \leq 5n+1. \Rightarrow \bullet 5n+1 < 5n+25 = 5(n+5).$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right)^{\frac{n^6+6}{2n-3}}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n+5 \leq 5n+1. \Rightarrow \bullet 5n+1 < 5n+25 = 5(n+5).$$

$$\Rightarrow \bullet 1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right)^{\frac{n^6+6}{2n-3}} = (e^2)^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{2n-3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n-3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí} \right] = e^2.$$

$$\left[\lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n+5 \leq 5n+1. \Rightarrow \bullet 5n+1 < 5n+25 = 5(n+5).$$

$$\Rightarrow \bullet 1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow \bullet 1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right)^{\frac{n^6+6}{2n-3}} = (e^2)^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{2n-3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n-3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 - \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 - \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 - \frac{3}{\infty}} = \frac{\infty + 0}{2 - 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n + 5 \leq 5n + 1. \Rightarrow \bullet 5n + 1 < 5n + 25 = 5(n + 5).$$

$$\Rightarrow \bullet 1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow \bullet 1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} 1 \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{5} = 1.$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right)^{\frac{n^6+6}{2n-3}} = (e^2)^\infty = e^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{2n-3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n-3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 - \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 - \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 - \frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty.$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n+5 \leq 5n+1. \Rightarrow \bullet 5n+1 < 5n+25 = 5(n+5).$$

$$\Rightarrow \bullet 1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow \bullet 1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} 1 \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{5} = 1. \Rightarrow \bullet \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1.$$

Riešené limity – 49, 50

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right)^{\frac{n^6+6}{2n-3}} = (e^2)^\infty = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{2n-3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2n-3 \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3} = \lim_{n \rightarrow \infty} \frac{n \cdot (n^5 + \frac{6}{n})}{n \cdot (2 - \frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5 + \frac{6}{n}}{2 - \frac{3}{n}} = \frac{\infty + \frac{6}{\infty}}{2 - \frac{3}{\infty}} = \frac{\infty + 0}{2 - 0} = \infty.$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1$$

Pre všetky $n \in \mathbb{N}$ platí:

$$\bullet 4 \leq 4n. \Rightarrow \bullet n + 5 \leq 5n + 1. \Rightarrow \bullet 5n + 1 < 5n + 25 = 5(n + 5).$$

$$\Rightarrow \bullet 1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5. \Rightarrow \bullet 1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$$

$$\Rightarrow \bullet 1 = \lim_{n \rightarrow \infty} 1 \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{5} = 1. \Rightarrow \bullet \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1.$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6}$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

- $= \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$

- $= \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)}$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)}$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n+6}{2n^6 + 3}}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot \left(\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty} \right)}$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n+6}{2n^6 + 3}} = (e^{-4})^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst.} \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí} \right. \\ \left. \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{1 - \infty - 0 + 0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot \left(\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty} \right)} \\ = \frac{0 + 3 + 0}{\infty \cdot (0 - 1 - 0 + 0)}$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n+6}{2n^6 + 3}} = (e^{-4})^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{n \cdot \left(2n^5 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n}}{2n^5 + \frac{3}{n}} = \frac{1 + \frac{6}{\infty}}{2 \cdot \infty + \frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{1 - \infty - 0 + 0} = \frac{3}{-\infty}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot \left(\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty} \right)} = \frac{0 + 3 + 0}{\infty \cdot (0 - 1 - 0 + 0)} = \frac{3}{\infty \cdot (-1)} = \frac{3}{-\infty}$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n+6}{2n^6 + 3}} = (e^{-4})^0 = e^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{n \cdot \left(2n^5 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n}}{2n^5 + \frac{3}{n}} = \frac{1 + \frac{6}{\infty}}{2 \cdot \infty + \frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{1 - \infty - 0 + 0} = \frac{3}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot \left(\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty} \right)} = \frac{0 + 3 + 0}{\infty \cdot (0 - 1 - 0 + 0)} = \frac{3}{\infty \cdot (-1)} = \frac{3}{-\infty} = 0.$$

Riešené limity – 51, 52

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n+6}{2n^6 + 3}} = (e^{-4})^0 = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{n \cdot \left(2n^5 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n}}{2n^5 + \frac{3}{n}} = \frac{1 + \frac{6}{\infty}}{2 \cdot \infty + \frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{1 - \infty - 0 + 0} = \frac{3}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot \left(\frac{2}{n} + 3 + \frac{5}{n^3} \right)}{n^4 \cdot \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot \left(\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty} \right)} = \frac{0 + 3 + 0}{\infty \cdot (0 - 1 - 0 + 0)} = \frac{3}{\infty \cdot (-1)} = \frac{3}{-\infty} = 0.$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n^6 + 6}{2n^6 + 3}}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{\infty \cdot \left(\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty} \right)}{\frac{2}{\infty} - 3 + \frac{5}{\infty}}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n^6 + 6}{2n^6 + 3}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí} \right. \\ \left. \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{1 - \infty + 0 + 0}{0 - 3 + 0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{\infty \cdot \left(\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty} \right)}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{\infty \cdot (0 - 1 + 0 + 0)}{0 - 3 + 0}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n^6 + 6}{2n^6 + 3}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6 + 6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n^6 \cdot \left(1 + \frac{6}{n^6} \right)}{n^6 \cdot \left(2 + \frac{3}{n^6} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n^6}}{2 + \frac{3}{n^6}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1 + 0}{2 + 0} = \frac{1 + 0}{2 + 0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{1 - \infty + 0 + 0}{0 - 3 + 0} = \frac{-\infty}{-3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{\infty \cdot \left(\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty} \right)}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{\infty \cdot (0 - 1 + 0 + 0)}{0 - 3 + 0} = \frac{\infty \cdot (-1)}{-3} = \frac{\infty}{3}$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n^6 + 6}{2n^6 + 3}} = (e^{-4})^{\frac{1}{2}} = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6 + 6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n^6 \cdot \left(1 + \frac{6}{n^6} \right)}{n^6 \cdot \left(2 + \frac{3}{n^6} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n^6}}{2 + \frac{3}{n^6}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1 + 0}{2 + 0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{1 - \infty + 0 + 0}{0 - 3 + 0} = \frac{-\infty}{-3} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{\infty \cdot \left(\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty} \right)}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{\infty \cdot (0 - 1 + 0 + 0)}{0 - 3 + 0} = \frac{\infty \cdot (-1)}{-3} = \frac{\infty}{3} = \infty.$$

Riešené limity – 53, 54

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6} = e^{-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n^6 + 3 - 4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{n^6 + 6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} \right)^{\frac{n^6 + 6}{2n^6 + 3}} = (e^{-4})^{\frac{1}{2}} = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6 + 3} \right)^{2n^6 + 3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n^6 + 3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n^6 + 6}{2n^6 + 3} = \lim_{n \rightarrow \infty} \frac{n^6 \cdot \left(1 + \frac{6}{n^6} \right)}{n^6 \cdot \left(2 + \frac{3}{n^6} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{n^6}}{2 + \frac{3}{n^6}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1 + 0}{2 + 0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{1 - \infty + 0 + 0}{0 - 3 + 0} = \frac{-\infty}{-3} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{n^3 \left(\frac{2}{n} - 3 + \frac{5}{n^3} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4} \right)}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{\infty \cdot \left(\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty} \right)}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{\infty \cdot (0 - 1 + 0 + 0)}{0 - 3 + 0} = \frac{\infty \cdot (-1)}{-3} = \frac{\infty}{3} = \infty.$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3} \right)^{\sqrt[6]{n}+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6}$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

- $= \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{1}{n^4}$

- $= \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)}$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{1}{n^4} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}}$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n+3}}}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{1}{n^4} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}}$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}-1}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}+3-4}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} \right)^{\frac{\sqrt[6]{n}+6}{2^{\sqrt[6]{n}+3}}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} = \left[\text{Subst. } \begin{array}{l} m = 2^{\sqrt[6]{n}+3} \\ n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0}$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}-1}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}+3-4}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} \right)^{\frac{\sqrt[6]{n}+6}{2^{\sqrt[6]{n}+3}}} = (e^{-4})^{\frac{1}{2}}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} = \left[\begin{array}{l} \text{Subst.} \\ m = 2^{\sqrt[6]{n}+3} \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2^{\sqrt[6]{n}+3}} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1}$$

Riešené limity – 55, 56

$$\lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}-1}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = e^{-2}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[6]{n}+3-4}}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} \right)^{\frac{\sqrt[6]{n}+6}{2^{\sqrt[6]{n}+3}}} \\ = (e^{-4})^{\frac{1}{2}} = e^{-2}.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2^{\sqrt[6]{n}+3}} \right)^{2^{\sqrt[6]{n}+3}} = \left[\begin{array}{l} \text{Subst.} \\ m = 2^{\sqrt[6]{n}+3} \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2^{\sqrt[6]{n}+3}} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{1 + \frac{6}{\infty}}{2 + \frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} = -3$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1} = -3.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n^2} + 3 + \frac{5}{n^4} \right)}{n^4 \left(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4} \right)} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0+3+0}{0-1-0+0} = \frac{3}{-1} = -3.$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[n]{n+6}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

- $$= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

- $$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{2^n n!}}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{(n+1)^{n+1}} \frac{n^n}{2^n n!} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0 = e^0$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{2^n n!}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 2 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0 = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 2 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 2e^{-1} = \frac{2}{e}$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0 = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{2^n n!}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 2 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 2e^{-1} = \frac{2}{e} < 1. \quad [2 < e \approx 2,718.]$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0 = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{matrix} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{matrix} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{matrix} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{matrix} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 2 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 2e^{-1} = \frac{2}{e} < 1. \quad [2 < e \approx 2,718.]$$

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} = 0.$$

Riešené limity – 57, 58

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n+6}} = 1$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n+6}} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right)^{\frac{\sqrt[6]{n+6}}{2n+3}} = (e^{-4})^0 = e^0 = 1.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } \begin{matrix} n \rightarrow \infty \\ m = 2n+3 \\ m \rightarrow \infty \end{matrix} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n+6}}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot \left(1 + \frac{6}{\sqrt[6]{n}} \right)}{n \cdot \left(2 + \frac{3}{n} \right)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5} \cdot \left(2 + \frac{3}{n} \right)} = \frac{1 + \frac{6}{\infty}}{\infty \cdot \left(2 + \frac{3}{\infty} \right)} = \frac{1+0}{\infty \cdot (2+0)} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} = 0$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{2^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{2^n n!}} = \lim_{n \rightarrow \infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2n^n}{(n+1)^n} = 2 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= 2 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 2 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 2e^{-1} = \frac{2}{e} < 1. \quad [2 < e \approx 2,718.]$$

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} = 0.$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}+3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

- $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}}$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ = (e^{-4})^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\text{Subst. } \begin{array}{l} m = 2\sqrt[6]{n+3} \\ n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^{-4}.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ = (e^{-4})^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n+3} \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{3^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{3^n n!}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ = 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= (e^{-4})^\infty = e^{-\infty} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\begin{array}{l} \text{Subst.} \\ m = 2\sqrt[6]{n+3} \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{3^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{3^n n!}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ &= 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 3 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} \end{aligned}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= (e^{-4})^\infty = e^{-\infty} = \frac{1}{\infty} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\begin{array}{l} \text{Subst.} \\ m = 2\sqrt[6]{n+3} \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^{n!}}{n^n}$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{3^{n+1} (n+1)!}{\frac{(n+1)^{n+1}}{3^{n!}}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ &= 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 3 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 3e^{-1} = \frac{3}{e} \end{aligned}$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6} = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= (e^{-4})^\infty = e^{-\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n+3} \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^{n n!}}{n^n}$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^{n n!}}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ &= 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 3 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 3e^{-1} = \frac{3}{e} > 1. \end{aligned}$$

[3 > e ≈ 2,718.]

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6} = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= (e^{-4})^\infty = e^{-\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n+3} \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ &= 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 3 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 3e^{-1} = \frac{3}{e} > 1. \end{aligned}$$

[3 > e ≈ 2,718.]

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} = \infty.$$

Riešené limity – 59, 60

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6} = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right)^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= (e^{-4})^\infty = e^{-\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\begin{array}{l} \text{Subst.} \\ m = 2\sqrt[6]{n+3} \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{-4}{m} \right)^m = \left[\begin{array}{l} \text{Pre všetky } a \in \mathbb{R} \text{ platí} \\ \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^{-4}.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[6]{n} \cdot \left(2 + \frac{3}{\sqrt[6]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 + \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 + \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2+0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} = \infty$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3n^n}{(n+1)^n} = 3 \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n \\ &= 3 \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = 3 \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = 3e^{-1} = \frac{3}{e} > 1. \end{aligned}$$

[3 > e ≈ 2,718.]

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} = \infty.$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6}$$

- $= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

- $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}}$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{n+6}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[n]{n}-1}}{2^{\sqrt[n]{n}-3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[n]{n}-3+2}}{2^{\sqrt[n]{n}-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2^{\sqrt[n]{n}-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2^{\sqrt[n]{n}-3}} \right)^{2^{\sqrt[n]{n}-3}} \right)^{\frac{n+6}{2^{\sqrt[n]{n}-3}}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[n]{n}-1}}{2^{\sqrt[n]{n}-3}} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2^{\sqrt[n]{n}-3+2}}{2^{\sqrt[n]{n}-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2^{\sqrt[n]{n}-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2^{\sqrt[n]{n}-3}} \right)^{2^{\sqrt[n]{n}-3}} \right)^{\frac{n+6}{2^{\sqrt[n]{n}-3}}} = (e^2)^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2^{\sqrt[n]{n}-3}} \right)^{2^{\sqrt[n]{n}-3}} = \left[\text{Subst. } \left. \begin{array}{l} m = 2^{\sqrt[n]{n}-3} \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^2.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} \right)^{\frac{n+6}{2\sqrt[6]{n}-3}} = (e^2)^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n}-3 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot (1 + \frac{6}{n})}{\sqrt[6]{n} \cdot (2 - \frac{3}{\sqrt[6]{n}})} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot (1 + \frac{6}{n})}{2 - \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot (1 + \frac{6}{\infty})}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}} \cdot \frac{3^n n^n}{8^n n!} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} \right)^{\frac{n+6}{2\sqrt[6]{n}-3}} = (e^2)^\infty = e^\infty$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n}-3 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left[\lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{\sqrt[6]{n} \cdot \left(2 - \frac{3}{\sqrt[6]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n} \right)}{2 - \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty} \right)}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}} \cdot \frac{3^n n^n}{8^n n!} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}-3} \right)^{n+6}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[n]{n}-3+2}{2\sqrt[n]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{2\sqrt[n]{n}-3} \right)^{\frac{n+6}{2\sqrt[n]{n}-3}}$$

$$= (e^2)^\infty = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{2\sqrt[n]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} n \rightarrow \infty \\ m = 2\sqrt[n]{n}-3 \end{array} \right| \begin{array}{l} m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[n]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{\sqrt[n]{n} \cdot \left(2 - \frac{3}{\sqrt[n]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^5} \cdot \left(1 + \frac{6}{n} \right)}{2 - \frac{3}{\sqrt[n]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty} \right)}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}} \cdot \frac{3^n n^n}{8^n n!} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = \frac{8}{3} e^{-1} = \frac{8}{3e}$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[n]{n}-1}{2\sqrt[n]{n}-3} \right)^{n+6} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[n]{n}-3+2}{2\sqrt[n]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{2\sqrt[n]{n}-3} \right)^{\frac{n+6}{2\sqrt[n]{n}-3}} = (e^2)^\infty = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[n]{n}-3} \right)^{2\sqrt[n]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[n]{n}-3 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[n]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n}\right)}{\sqrt[n]{n} \cdot \left(2 - \frac{3}{\sqrt[n]{n}}\right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^5} \cdot \left(1 + \frac{6}{n}\right)}{2 - \frac{3}{\sqrt[n]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty}\right)}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = \frac{8}{3} e^{-1} = \frac{8}{3e} < 1. \quad [8 < 3e \approx 3 \cdot 2,718 = 8,154.]$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} \right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$= (e^2)^\infty = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n}-3 \\ n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right| \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{\sqrt[6]{n} \cdot \left(2 - \frac{3}{\sqrt[6]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n} \right)}{2 - \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty} \right)}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n}$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1}(n+1)!}{3^{n+1}(n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = \frac{8}{3} e^{-1} = \frac{8}{3e} < 1. \quad [8 < 3e \approx 3 \cdot 2,718 = 8,154.]$$

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n} = 0.$$

Riešené limity – 61, 62

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3} \right)^{n+6} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(\left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} \right)^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$= (e^2)^\infty = e^\infty = \infty.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n}-3} \right)^{2\sqrt[6]{n}-3} = \left[\text{Subst. } \left. \begin{array}{l} m = 2\sqrt[6]{n}-3 \\ m \rightarrow \infty \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{2}{m} \right)^m = \left[\text{Pre všetky } a \in \mathbb{R} \text{ platí } \left. \begin{array}{l} \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^n = e^a. \end{array} \right] = e^2.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n}-3} = \lim_{n \rightarrow \infty} \frac{n \cdot \left(1 + \frac{6}{n} \right)}{\sqrt[6]{n} \cdot \left(2 - \frac{3}{\sqrt[6]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5} \cdot \left(1 + \frac{6}{n} \right)}{2 - \frac{3}{\sqrt[6]{n}}} = \frac{\infty \cdot \left(1 + \frac{6}{\infty} \right)}{2 - \frac{3}{\infty}} = \frac{\infty \cdot (1+0)}{2-0} = \frac{\infty}{2} = \infty.$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n} = 0$$

$$\bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}} \cdot \frac{3^n n^n}{8^n n!} = \lim_{n \rightarrow \infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n$$

$$= \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^n \right)^{-1} = \frac{8}{3} \lim_{n \rightarrow \infty} \left(\left(1 + \frac{1}{n} \right)^n \right)^{-1} = \frac{8}{3} e^{-1} = \frac{8}{3e} < 1. \quad [8 < 3e \approx 3 \cdot 2,718 = 8,154.]$$

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{3^n n^n} = 0.$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty \right)}{\frac{2}{\infty} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty \right)}{\frac{2}{\infty} - 1} = \frac{\infty(0+3-0-\infty)}{0-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2+3n-\frac{1}{n}-2 \cdot \frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3 \cdot \infty - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2+\infty-0-\infty}{0-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0+3-0-\infty}{0-0}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty \right)}{\frac{2}{\infty} - 1} = \frac{\infty(0+3-0-\infty)}{0-1} = \frac{\infty(0+3-0-\infty)}{0-1} = \frac{\infty \cdot (-\infty)}{-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2+3n-\frac{1}{n}-2 \cdot \frac{5^n}{n^3}}{\frac{2}{n}-1} = \frac{2+3 \cdot \infty - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2+\infty-0-\infty}{0-1} = \frac{2+\infty-0-\infty}{0-1} ?$$

[Problém nekonečno mínus nekonečno.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0+3-0-\infty}{0-0} = \frac{-\infty}{0}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1+\frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 63

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} = \infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty \right)}{\frac{2}{\infty} - 1} = \frac{\infty(0 + 3 - 0 - \infty)}{0 - 1} = \frac{\infty \cdot (-\infty)}{-1} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 - \infty}{0 - 1} = \frac{?}{-1}.$$

[Problém nekonečno mínus nekonečno.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 - \infty}{0 - 0} = \frac{-\infty}{0} ?.$$

[Problém s delením nulou.]

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \left. \begin{array}{l} \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = \frac{5}{1^k} = 5 > 1. \\ \text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{\frac{1}{7^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{\frac{1}{5^n}}$$

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{7^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}}$$

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{7^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}}$$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0.$$

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{\frac{1}{7^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}}$$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{5^n}{7^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n = 0.$$

[Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

$$\bullet \lim_{n \rightarrow \infty} \frac{7^n}{5^n} = \lim_{n \rightarrow \infty} \left(\frac{7}{5}\right)^n = \infty.$$

[Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot 0}{2 \cdot 0 - 0 + 6}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 6 \cdot \infty}$$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{5^n}{7^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n = 0.$$

[Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

$$\bullet \lim_{n \rightarrow \infty} \frac{7^n}{5^n} = \lim_{n \rightarrow \infty} \left(\frac{7}{5}\right)^n = \infty.$$

[Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot 0}{2 \cdot 0 - 0 + 6} = \frac{0}{6}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 6 \cdot \infty} = \frac{2}{\infty}$$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{5^n}{7^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n = 0.$$

[Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

$$\bullet \lim_{n \rightarrow \infty} \frac{7^n}{5^n} = \lim_{n \rightarrow \infty} \left(\frac{7}{5}\right)^n = \infty.$$

[Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 64

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{7^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot 0}{2 \cdot 0 - 0 + 6} = \frac{0}{6} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 6 \cdot \infty} = \frac{2}{\infty} = 0.$$

Označme $a_n = \frac{n^k}{\alpha^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $\alpha > 1$ (špeciálne $\alpha = 5$ a $\alpha = 7$).

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{\alpha} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{\alpha} = \frac{1^k}{\alpha} = \frac{1}{\alpha} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{\alpha^{n+1}}}{\frac{n^k}{\alpha^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{\alpha n^k} = \lim_{n \rightarrow \infty} \frac{(1 + \frac{1}{n})^k}{\alpha} = \frac{(1+0)^k}{\alpha} = \frac{1}{\alpha} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{5^n}{7^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n = 0.$$

[Limita geometrickej postupnosti pre $-1 < q = \frac{5}{7} < 1$.]

$$\bullet \lim_{n \rightarrow \infty} \frac{7^n}{5^n} = \lim_{n \rightarrow \infty} \left(\frac{7}{5}\right)^n = \infty.$$

[Limita geometrickej postupnosti pre $q = \frac{7}{5} > 1$.]

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{c} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right]$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[15]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[15]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[5]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(1 + \frac{1}{n}\right)}}{2 \cdot \sqrt[3]{1 + \frac{1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[15]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(1 + \frac{1}{n}\right)}}{2 \cdot \sqrt[3]{1 + \frac{1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n}\right)}} = \frac{\sqrt[3]{0-0} - \sqrt[15]{0 \cdot (1-0)^3} + 3 \cdot \sqrt[6]{0 \cdot (1+0)}}{2 \cdot \sqrt[3]{1+0} + 3 \cdot \sqrt[12]{0 \cdot (1-0)^3} - \sqrt[6]{0 \cdot (1-0)}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[5]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[5]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(1 + \frac{1}{n}\right)}}{2 \cdot \sqrt[3]{1 + \frac{1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n}\right)}} = \frac{\sqrt[3]{0-0} - \sqrt[5]{0 \cdot (1-0)^3} + 3 \cdot \sqrt[6]{0 \cdot (1+0)}}{2 \cdot \sqrt[3]{1+0} + 3 \cdot \sqrt[12]{0 \cdot (1-0)^3} - \sqrt[6]{0 \cdot (1-0)}} \\ = \frac{0-0+3 \cdot 0}{2 \cdot 1 + 3 \cdot 0 - 0}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[15]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(1 + \frac{1}{n}\right)}}{2 \cdot \sqrt[3]{1 + \frac{1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n}\right)}} = \frac{\sqrt[3]{0-0} - \sqrt[15]{0 \cdot (1-0)^3} + 3 \cdot \sqrt[6]{0 \cdot (1+0)}}{2 \cdot \sqrt[3]{1+0} + 3 \cdot \sqrt[12]{0 \cdot (1-0)^3} - \sqrt[6]{0 \cdot (1-0)}}$$

$$= \frac{0-0+3 \cdot 0}{2 \cdot 1 + 3 \cdot 0 - 0} = \frac{0}{2}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = 0$$

$$\bullet = \left[\begin{array}{l} \text{Pre všetky } n \in \mathbb{N} \text{ platí} \\ \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[15]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} - \sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{(n^6-2)^3}{n^7 \cdot (n^6)^3}} + 3 \cdot \sqrt[6]{\frac{(n+1)^3}{n^7 \cdot n^3}}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{(n^6-1)^3}{n^2 \cdot (n^6)^3}} - \sqrt[6]{\frac{(n-1)^3}{n^7 \cdot n^3}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(\frac{n^6-2}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n+1}{n}\right)^3}}{2 \cdot \sqrt[3]{\frac{n^5+1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(\frac{n^6-1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n^6}\right)^3} + 3 \cdot \sqrt[6]{\frac{1}{n^7} \cdot \left(1 + \frac{1}{n}\right)}}{2 \cdot \sqrt[3]{1 + \frac{1}{n^5}} + 3 \cdot \sqrt[12]{\frac{1}{n^2} \cdot \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \cdot \left(1 - \frac{1}{n}\right)}} = \frac{\sqrt[3]{0-0} - \sqrt[15]{0 \cdot (1-0)^3} + 3 \cdot \sqrt[6]{0 \cdot (1+0)}}{2 \cdot \sqrt[3]{1+0} + 3 \cdot \sqrt[12]{0 \cdot (1-0)^3} - \sqrt[6]{0 \cdot (1-0)}}$$

$$= \frac{0-0+3 \cdot 0}{2 \cdot 1 + 3 \cdot 0 - 0} = \frac{0}{2} = 0.$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ \left. \sqrt{n} = \sqrt[3]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right]$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right]$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[3]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15}{n^2} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{1}}{n^5} \cdot \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12}{n^2} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{6}{n^7} \cdot \sqrt{1-\frac{1}{n}} \right)} \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15}{n^2} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{1}}{n^5} \cdot \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12}{n^2} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{6}{n^7} \cdot \sqrt{1-\frac{1}{n}} \right)} \\ &= \frac{\sqrt[3]{1-0} - \frac{15}{\sqrt[3]{0}} \cdot \sqrt[5]{1-0} + 3 \cdot \frac{\sqrt[6]{0}}{\sqrt[3]{0}} \cdot \sqrt{1+0}}{\sqrt[3]{\infty} \cdot \left(2 \sqrt[3]{1+0} + 3 \cdot \frac{12}{\sqrt[3]{0}} \cdot \sqrt[4]{1-0} - \frac{6}{\sqrt[3]{0}} \cdot \sqrt{1-0} \right)} \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{\frac{1}{n^2}}}{15\sqrt[15]{\frac{1}{n^2}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{\frac{1}{n^3}}}{\sqrt[6]{\frac{1}{n^5}}} \cdot \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{\frac{1}{n^2}}}{12\sqrt[12]{\frac{1}{n^2}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{\frac{1}{n^3}}}{\sqrt[6]{\frac{1}{n^7}}} \cdot \sqrt{1-\frac{1}{n}} \right)} \\ &= \frac{\sqrt[3]{1-0} - \frac{15\sqrt[15]{0}}{15\sqrt[15]{0}} \cdot \sqrt[5]{1-0} + 3 \cdot \frac{\sqrt[6]{0} \cdot \sqrt{1+0}}{\sqrt[6]{0} \cdot \sqrt{1-0}}}{\sqrt[3]{\infty} \cdot (2\sqrt[3]{1+0} + 3 \cdot \frac{12\sqrt[12]{0}}{12\sqrt[12]{0}} \cdot \sqrt[4]{1-0} - \frac{\sqrt[6]{0} \cdot \sqrt{1-0}}{\sqrt[6]{0} \cdot \sqrt{1-0}})} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty(2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{\frac{1}{n^2}}}{15\sqrt[15]{\frac{1}{n^2}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{\frac{1}{n^5}}}{\sqrt[6]{\frac{1}{n^5}}} \cdot \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{\frac{1}{n^2}}}{12\sqrt[12]{\frac{1}{n^2}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{\frac{1}{n^7}}}{\sqrt[6]{\frac{1}{n^7}}} \cdot \sqrt{1-\frac{1}{n}} \right)} \\ &= \frac{\sqrt[3]{1-0} - \frac{15\sqrt[15]{0}}{15\sqrt[15]{0}} \cdot \sqrt[5]{1-0} + 3 \cdot \frac{\sqrt[6]{0} \cdot \sqrt{1+0}}{\sqrt[6]{0} \cdot \sqrt{1+0}}}{\sqrt[3]{\infty} \cdot (2\sqrt[3]{1+0} + 3 \cdot \frac{12\sqrt[12]{0}}{12\sqrt[12]{0}} \cdot \sqrt[4]{1-0} - \frac{\sqrt[6]{0} \cdot \sqrt{1-0}}{\sqrt[6]{0} \cdot \sqrt{1-0}})} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty(2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = 0$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)} - \sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)} + 3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)} + 3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)} - \sqrt{n\left(1-\frac{1}{n}\right)}} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \cdot \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \cdot \sqrt{1-\frac{1}{n}}} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \cdot \left(\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \cdot \sqrt{1+\frac{1}{n}} \right)}{\sqrt[3]{n^5} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \cdot \sqrt{1-\frac{1}{n}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15}{n^2} \cdot \sqrt[5]{1-\frac{2}{n^6}} + 3 \cdot \frac{\sqrt[6]{1}}{n^5} \cdot \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \cdot \left(2 \cdot \sqrt[3]{1+\frac{1}{n^5}} + 3 \cdot \frac{12}{n^2} \cdot \sqrt[4]{1-\frac{1}{n^6}} - \frac{6}{n^7} \cdot \sqrt{1-\frac{1}{n}} \right)} \\ &= \frac{\sqrt[3]{1-0} - \frac{15}{\sqrt[3]{0}} \cdot \sqrt[5]{1-0} + 3 \cdot \frac{6}{\sqrt[3]{0}} \cdot \sqrt{1+0}}{\sqrt[3]{\infty} \cdot (2\sqrt[3]{1+0} + 3 \cdot \frac{12}{\sqrt[3]{0}} \cdot \sqrt[4]{1-0} - \frac{6}{\sqrt[3]{0}} \cdot \sqrt{1-0})} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty(2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} = 0. \end{aligned}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}-\frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}-\frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}-\frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}-\frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}-\frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}-\frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}-\frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}-\frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3}-\frac{4}{3}} = n^{-\frac{0}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}} = n^{\frac{4}{3} - \frac{5}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[m]{n^k}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[m]{n^k}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

$$= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 3 \cdot 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot \left(2 \cdot (1+0)^{\frac{1}{3}} + 3 \cdot 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}} \right)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[m]{n^k}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

$$= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 3 \cdot 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot \left(2 \cdot (1+0)^{\frac{1}{3}} + 3 \cdot 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}} \right)} = \frac{1-0+3 \cdot 0}{\infty \cdot (2 \cdot 1 + 3 \cdot 0 - 0 \cdot 1)}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[m]{n^k}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

$$= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 3 \cdot 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot \left(2 \cdot (1+0)^{\frac{1}{3}} + 3 \cdot 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}} \right)} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty \cdot (2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^{\frac{0}{3}}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{m}{m} \cdot \frac{k}{m}}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

$$= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 3 \cdot 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot \left(2 \cdot (1+0)^{\frac{1}{3}} + 3 \cdot 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}} \right)} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty \cdot (2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} = \frac{1}{\infty}$$

Riešené limity – 65

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \cdot \left((1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5} - \frac{4}{3}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2} - \frac{4}{3}} \cdot (1+\frac{1}{n})^{\frac{1}{2}} \right)}{n^{\frac{5}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4} - \frac{5}{3}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2} - \frac{5}{3}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)} = \left[\text{Pre } n \in \mathbb{N} \text{ platí } \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3} - \frac{4}{3}} = n^{-\frac{0}{3}} = \frac{1}{n^0} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{-\frac{2}{15}} \cdot (1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{-\frac{5}{6}} \cdot (1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \cdot \left(2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{-\frac{2}{12}} \cdot (1-\frac{1}{n^6})^{\frac{1}{4}} - n^{-\frac{7}{6}} \cdot (1-\frac{1}{n})^{\frac{1}{2}} \right)}$$

$$= \left[\lim_{n \rightarrow \infty} n^{\frac{1}{3}} = \lim_{n \rightarrow \infty} \sqrt[3]{n} = \infty. \text{ Pre všetky } k, m \in \mathbb{N} \text{ platí } \lim_{n \rightarrow \infty} n^{-\frac{k}{m}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{k}{m}}} = \lim_{n \rightarrow \infty} \frac{1}{n^{\frac{m}{m} \cdot \frac{k}{m}}} = \frac{1}{\sqrt[m]{\infty^k}} = \frac{1}{\sqrt[m]{\infty}} = \frac{1}{\infty} = 0. \right]$$

$$= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 3 \cdot 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot \left(2 \cdot (1+0)^{\frac{1}{3}} + 3 \cdot 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}} \right)} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty \cdot (2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} = \frac{1}{\infty} = 0.$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}}$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n}$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{\left(\frac{4}{5}\right)^n - 1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n - 1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{1 - \left(\frac{4}{3}\right)^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n - 1}$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \quad \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \quad \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \quad \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{1 - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \quad \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \quad \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right]$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{\infty}{0-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{1}{0-\infty}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{1 - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{0}{1-\infty}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1}$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{\infty}{0-1} = \frac{\infty}{-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{1}{0-\infty} = \frac{1}{-\infty}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{1 - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{0}{1-\infty} = \frac{0}{-\infty}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1}$$

Riešené limity – 66, 67, 68

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} = -\infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{\infty}{0-1} = \frac{\infty}{-1} = -\infty.$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{1}{0-\infty} = \frac{1}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} = 0$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{4}{5}\right)^n}{1 - \left(\frac{4}{3}\right)^n} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty. \end{array} \right] = \frac{0}{1-\infty} = \frac{0}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n - 1} = \left[\begin{array}{l} \text{Geometrické postupnosti:} \\ \lim_{n \rightarrow \infty} \left(\frac{3}{4}\right)^n = 0, \lim_{n \rightarrow \infty} \left(\frac{3}{5}\right)^n = 0. \end{array} \right] = \frac{0}{0-1} = \frac{0}{-1} = 0.$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{1}{\frac{1}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{1}{\frac{1}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)}$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n}{n}\right)^k} = \frac{5}{1^k} = 5 > 1. \\ \text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \cdot \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty \right)}{\frac{1}{\infty} - 1}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{1}{n}\right)^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{5^n}{n^k}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 + \infty}{-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 + \infty}{0 - 0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \cdot \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty \right)}{\frac{1}{\infty} - 1} = \frac{\infty \cdot (0 + 3 - 0 + \infty)}{-1}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in N$ pre $k \in N$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n}{n}\right)^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 + \infty}{-1} = \frac{\infty}{-1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 + \infty}{0 - 0} = \frac{\infty}{0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \cdot \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty \right)}{\frac{1}{\infty} - 1} = \frac{\infty \cdot (0 + 3 - 0 + \infty)}{-1} = \frac{\infty}{-1}$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in N$ pre $k \in N$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n}{n}\right)^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1.$$

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 5 > 1. \\ \text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 5 > 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 69

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} = -\infty$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 + \infty}{-1} = \frac{\infty}{-1} = -\infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 + \infty}{0 - 0} = \frac{\infty}{0} ?.$$

[Problém s delením nulou.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \cdot \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty \right)}{\frac{1}{\infty} - 1} = \frac{\infty \cdot (0 + 3 - 0 + \infty)}{-1} = \frac{\infty}{-1} = -\infty.$$

Označme $a_n = \frac{5^n}{n^k}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\sqrt[n]{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{1}{n}\right)^k} = \frac{5}{1^k} = 5 > 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5n^k}{(n+1)^k} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = \frac{5}{(1+0)^k} = 5 > 1.$$

$$\left. \begin{array}{l} \bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = 5 > 1. \\ \text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 5 > 1. \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{n + n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{n + n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{n + n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + n^{\frac{1}{2} - 2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{\left(n + \left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot \left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{\left(n + n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{n + n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + n^{\frac{1}{2} - 2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \left(\frac{1}{n} + n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{\infty} + \frac{1}{\infty})^{\frac{1}{2}})^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} \\ &= \frac{1}{\sqrt{1 + 0}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{\infty} + \frac{1}{\infty})^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + (0 + 0)^{\frac{1}{2}})^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} = 1$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} \\ &= \frac{1}{\sqrt{1 + 0}} = 1. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{\infty} + \frac{1}{\infty})^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + (0 + 0)^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + 0)^{\frac{1}{2}}} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} = 1$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} \\ &= \frac{1}{\sqrt{1 + 0}} = 1. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{\infty} + \frac{1}{\infty})^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + (0 + 0)^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + 0)^{\frac{1}{2}}} = \frac{1}{1} \end{aligned}$$

Riešené limity – 70

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{(n + (n + n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} = 1$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{\sqrt{n^4}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} \\ &= \frac{1}{\sqrt{1 + 0}} = 1. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \cdot (1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{n + n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{\frac{1}{2} - 2})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + n^{-\frac{3}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}})^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{1}{(1 + (\frac{1}{\infty} + \frac{1}{\infty})^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + (0 + 0)^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{1}{(1 + 0)^{\frac{1}{2}}} = \frac{1}{1} = 1. \end{aligned}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} \end{aligned}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt[4]{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n}{n^2}} - 1}{2 - \sqrt[10]{\frac{n^2}{n^5}} + \frac{3}{\sqrt{n}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{n^{\frac{3}{10}}} + \frac{3}{n^{\frac{1}{2}}}} \end{aligned}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n}{n^2}} - 1}{2 - \sqrt[10]{\frac{n^2}{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{n}} + 3 \cdot \sqrt[4]{\frac{1}{n}} - 1}{2 - \sqrt[10]{\frac{1}{n^3}} + 3 \cdot \sqrt{\frac{1}{n}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{\sqrt[6]{n}} + \frac{3}{\sqrt[4]{n}} - 1}{2 - \frac{1}{\sqrt[10]{n^3}} + \frac{3}{\sqrt{n}}} \end{aligned}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n}{n^2}} - 1}{2 - \sqrt[10]{\frac{n^2}{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{n}} + 3 \cdot \sqrt[4]{\frac{1}{n}} - 1}{2 - \sqrt[10]{\frac{1}{n^3}} + 3 \cdot \sqrt{\frac{1}{n}}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{\infty}} + 3 \cdot \sqrt[4]{\frac{1}{\infty}} - 1}{2 - \sqrt[10]{\frac{1}{\infty}} + 3 \cdot \sqrt{\frac{1}{\infty}}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{n^{\frac{3}{10}}} + \frac{3}{n^{\frac{1}{2}}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{\sqrt[6]{\infty}} + \frac{3}{\sqrt[4]{\infty}} - 1}{2 - \frac{1}{\sqrt[10]{\infty}} + \frac{3}{\sqrt{\infty}}}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n^2}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n^2}{n^2}} - 1}{2 - \frac{\sqrt[10]{\frac{n^2}{n^5}}}{\sqrt{\frac{n^2}{n^5}}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{n}} + 3 \cdot \sqrt[4]{\frac{1}{n}} - 1}{2 - \frac{\sqrt[10]{\frac{1}{n^3}}}{\sqrt{\frac{1}{n^3}}} + 3 \cdot \sqrt{\frac{1}{n}}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{\infty}} + 3 \cdot \sqrt[4]{\frac{1}{\infty}} - 1}{2 - \frac{\sqrt[10]{\frac{1}{\infty}}}{\sqrt{\frac{1}{\infty}}} + 3 \cdot \sqrt{\frac{1}{\infty}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{0} + 3 \cdot \sqrt[4]{0} - 1}{2 - \frac{\sqrt[10]{0}}{\sqrt{0}} + 3 \cdot \sqrt{0}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{n^{\frac{3}{10}}} + \frac{3}{n^{\frac{1}{2}}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{\sqrt[6]{\infty}} + \frac{3}{\sqrt[4]{\infty}} - 1}{2 - \frac{1}{\sqrt[10]{\infty}} + \frac{3}{\sqrt{\infty}}}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{2}{\infty} + \frac{3}{\infty} - 1}{2 - \frac{1}{\infty} + \frac{3}{\infty}}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n^2}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n^2}{n^2}} - 1}{2 - \frac{\sqrt[10]{\frac{n^2}{n^5}}}{\sqrt{\frac{n^2}{n^5}}} + \frac{3}{\sqrt{\frac{n^2}{n^5}}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{n}} + 3 \cdot \sqrt[4]{\frac{1}{n}} - 1}{2 - \frac{\sqrt[10]{\frac{1}{n^3}}}{\sqrt{\frac{1}{n^3}}} + 3 \cdot \sqrt{\frac{1}{n}}} \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{\infty}} + 3 \cdot \sqrt[4]{\frac{1}{\infty}} - 1}{2 - \frac{\sqrt[10]{\frac{1}{\infty}}}{\sqrt{\frac{1}{\infty}}} + 3 \cdot \sqrt{\frac{1}{\infty}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{0} + 3 \cdot \sqrt[4]{0} - 1}{2 - \frac{\sqrt[10]{0}}{\sqrt{0}} + 3 \cdot \sqrt{0}} = \frac{0+0-1}{2-0+0} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{\sqrt[10]{n^3}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{\sqrt[6]{\infty}} + \frac{3}{\sqrt[4]{\infty}} - 1}{2 - \frac{1}{\sqrt[10]{\infty}} + \frac{3}{\sqrt{\infty}}} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{2}{\infty} + \frac{3}{\infty} - 1}{2 - \frac{1}{\infty} + \frac{3}{\infty}} = \frac{0+0-1}{2-0+0} \end{aligned}$$

Riešené limity – 71

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} = -\frac{1}{2}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[3]{n}}{\sqrt{n}} + 3 \cdot \frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[\begin{array}{l} \text{Pre } n \in \mathbb{N} \text{ platí } \sqrt[3]{n} = \sqrt[6]{n^2}, \sqrt[5]{n} = \sqrt[10]{n^2}, \\ \sqrt{n} = \sqrt[4]{n^2}, \sqrt{n} = \sqrt[6]{n^3}, \sqrt{n} = \sqrt[10]{n^5}. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{\sqrt[6]{n^2}}{\sqrt{n^3}} + 3 \cdot \frac{\sqrt[4]{n^2}}{\sqrt{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{n^2}{n^3}} + 3 \cdot \sqrt[4]{\frac{n^2}{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{n}} + 3 \cdot \sqrt[4]{\frac{1}{n}} - 1}{2 - \frac{\sqrt[10]{1}}{n^{\frac{2}{5}}} + 3 \cdot \sqrt{\frac{1}{n}}} \\ &= \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{\frac{1}{\infty}} + 3 \cdot \sqrt[4]{\frac{1}{\infty}} - 1}{2 - \frac{\sqrt[10]{1}}{\infty} + 3 \cdot \sqrt{\frac{1}{\infty}}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \sqrt[6]{0} + 3 \cdot \sqrt[4]{0} - 1}{2 - \frac{\sqrt[10]{0}}{\infty} + 3 \cdot \sqrt{0}} = \frac{0+0-1}{2-0+0} = -\frac{1}{2}. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2n^{\frac{1}{2} - \frac{1}{2}} - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{n^{\frac{3}{10}}} + \frac{3}{n^{\frac{1}{2}}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{\sqrt[6]{\infty}} + \frac{3}{\sqrt[4]{\infty}} - 1}{2 - \frac{1}{\sqrt[10]{\infty}} + \frac{3}{\sqrt{\infty}}} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{2}{\infty} + \frac{3}{\infty} - 1}{2 - \frac{1}{\infty} + \frac{3}{\infty}} = \frac{0+0-1}{2-0+0} = -\frac{1}{2}. \end{aligned}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\bullet = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\bullet = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1)$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\bullet = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\bullet = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1)$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\bullet = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right]$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\begin{array}{l} \text{Subst. } n \rightarrow \infty \mid \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ m = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \rightarrow \infty \mid \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1-1) \cdot (\sqrt[n+1]{2} - 1) \end{aligned}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \mid \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ m = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \rightarrow \infty \mid \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\bullet \lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2} - 1) + n(\sqrt[n]{3} - 1)}{2} = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1 - 1) \cdot (\sqrt[n+1]{2} - 1) = \ln 2 - \lim_{n \rightarrow \infty} \left((n+1) \cdot (\sqrt[n+1]{2} - 1) - (\sqrt[n+1]{2} - 1) \right) \end{aligned}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n = \sqrt{6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} = \sqrt{6}. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \mid \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ m = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \rightarrow \infty \mid \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\bullet \lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2} - 1) + n(\sqrt[n]{3} - 1)}{2} = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1-1) \cdot (\sqrt[n+1]{2} - 1) = \ln 2 - \lim_{n \rightarrow \infty} \left((n+1) \cdot (\sqrt[n+1]{2} - 1) - (\sqrt[n+1]{2} - 1) \right) \\ &= \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n+1 \mid m \rightarrow \infty \end{array} \right] \end{aligned}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n = \sqrt{6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} = \sqrt{6}. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\begin{array}{l} \text{Subst. } n \rightarrow \infty \left| \begin{array}{l} \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \\ \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \end{array} \right. \left| \begin{array}{l} \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right. \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\bullet \lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2} - 1) + n(\sqrt[n]{3} - 1)}{2} = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1-1) \cdot (\sqrt[n+1]{2} - 1) = \ln 2 - \lim_{n \rightarrow \infty} \left((n+1) \cdot (\sqrt[n+1]{2} - 1) - (\sqrt[n+1]{2} - 1) \right) \\ &= \left[\begin{array}{l} \text{Subst. } \left| \begin{array}{l} n \rightarrow \infty \\ m = n+1 \\ m \rightarrow \infty \end{array} \right. \right] = \ln 2 - \lim_{m \rightarrow \infty} m(\sqrt[m]{2} - 1) + \lim_{m \rightarrow \infty} (\sqrt[m]{2} - 1) \end{array}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n = \sqrt{6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} = \sqrt{6}. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \mid \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ m = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \rightarrow \infty \mid \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\bullet \lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2} - 1) + n(\sqrt[n]{3} - 1)}{2} = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2})$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1-1) \cdot (\sqrt[n+1]{2} - 1) = \ln 2 - \lim_{n \rightarrow \infty} \left((n+1) \cdot (\sqrt[n+1]{2} - 1) - (\sqrt[n+1]{2} - 1) \right) \\ &= \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n+1 \mid m \rightarrow \infty \end{array} \right] = \ln 2 - \lim_{m \rightarrow \infty} m(\sqrt[m]{2} - 1) + \lim_{m \rightarrow \infty} (\sqrt[m]{2} - 1) = \ln 2 - \ln 2 + (1-1) \end{aligned}$$

Riešené limity – 72, 73

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n = \sqrt{6}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right)^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} = e^{\ln \sqrt{6}} = \sqrt{6}. \end{aligned}$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \mid \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \\ m = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \rightarrow \infty \mid \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \mid \sqrt[n]{2} + \sqrt[n]{3} - 2 \rightarrow 0^+ \end{array} \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m} \right)^m = e.$$

$$\bullet \lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2} - 1) + n(\sqrt[n]{3} - 1)}{2} = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}.$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{2} - \sqrt[n+1]{2}) = 0$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1 - \sqrt[n+1]{2} + 1) = \lim_{n \rightarrow \infty} n(\sqrt[n]{2} - 1) - \lim_{n \rightarrow \infty} n(\sqrt[n+1]{2} - 1) = \left[\begin{array}{l} \text{Pre } a > 0 \text{ platí} \\ \lim_{n \rightarrow \infty} n(\sqrt[n]{a} - 1) = \ln a \end{array} \right] \\ &= \ln 2 - \lim_{n \rightarrow \infty} (n+1-1) \cdot (\sqrt[n+1]{2} - 1) = \ln 2 - \lim_{n \rightarrow \infty} \left((n+1) \cdot (\sqrt[n+1]{2} - 1) - (\sqrt[n+1]{2} - 1) \right) \\ &= \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n+1 \mid m \rightarrow \infty \end{array} \right] = \ln 2 - \lim_{m \rightarrow \infty} m(\sqrt[m]{2} - 1) + \lim_{m \rightarrow \infty} (\sqrt[m]{2} - 1) = \ln 2 - \ln 2 + (1-1) = 0. \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

- $= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right)$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right)$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right)
 \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n+1]{2}} \cdot \left(\sqrt[n+1]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n+1]{2}} \cdot \left(\sqrt[n+1]{2} - 1 \right) \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}}
 \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\ &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}} \\ &= \left[\begin{array}{l} \text{Subst.} \\ m = n \cdot (n+1) \end{array} \middle| \begin{array}{l} n \rightarrow \infty \\ m \rightarrow \infty \end{array} \right] \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n - (n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}} \\
 &= \left[\text{Subst.} \right. \\
 &\quad \left. m = n \cdot (n+1) \left| \begin{matrix} n \rightarrow \infty \\ m \rightarrow \infty \end{matrix} \right. \right] = \lim_{m \rightarrow \infty} m \left(\sqrt[m]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{m \rightarrow \infty} \frac{1}{\sqrt{2}}
 \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\ &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\ &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}} \\ &= \left[\text{Subst.} \right. \\ &\quad \left. m = n \cdot (n+1) \right]_{m \rightarrow \infty}^{n \rightarrow \infty} = \lim_{m \rightarrow \infty} m \left(\sqrt[m]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{m \rightarrow \infty} \frac{1}{\sqrt[m]{2}} \\ &= \ln 2 \cdot \frac{1}{1 + \frac{1}{\infty}} \cdot 1 \cdot 1 \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}} \\
 &= \left[\text{Subst.} \right. \\
 &\quad \left. m = n \cdot (n+1) \left| \begin{matrix} n \rightarrow \infty \\ m \rightarrow \infty \end{matrix} \right. \right] = \lim_{m \rightarrow \infty} m \left(\sqrt[m]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{m \rightarrow \infty} \frac{1}{\sqrt[m]{2}} \\
 &\quad = \ln 2 \cdot \frac{1}{1 + \frac{1}{\infty}} \cdot 1 \cdot 1 = \ln 2 \cdot \frac{1}{1+0} \cdot 1 \cdot 1
 \end{aligned}$$

Riešené limity – 74

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \ln 2$$

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}} - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = \lim_{n \rightarrow \infty} n^2 \cdot 2^{\frac{1}{n}} \cdot \frac{1}{2^{\frac{1}{n(n+1)}}} \cdot \left(2^{\frac{1}{n(n+1)}} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n^2 \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) = \lim_{n \rightarrow \infty} \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n]{2} \cdot \frac{1}{\sqrt[n(n+1)]{2}} \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \\
 &= \lim_{n \rightarrow \infty} n \cdot (n+1) \cdot \left(\sqrt[n(n+1)]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n(n+1)]{2}} \\
 &= \left[\text{Subst.} \right. \\
 &\quad \left. m = n \cdot (n+1) \left| \begin{matrix} n \rightarrow \infty \\ m \rightarrow \infty \end{matrix} \right. \right] = \lim_{m \rightarrow \infty} m \left(\sqrt[m]{2} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{m \rightarrow \infty} \frac{1}{\sqrt[m]{2}} \\
 &\quad = \ln 2 \cdot \frac{1}{1 + \frac{1}{\infty}} \cdot 1 \cdot 1 = \ln 2 \cdot \frac{1}{1+0} \cdot 1 \cdot 1 = \ln 2.
 \end{aligned}$$

Riešené limity – 75

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} \right)\end{aligned}$$

Riešené limity – 75

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right)\end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}}$

Riešené limity – 75

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} \right)\end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$

Riešené limity – 75

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} \right)\end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n.$
- \Rightarrow • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n.$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet \quad 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet \quad 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\Rightarrow \bullet \quad \begin{aligned} \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln (n \cdot 2^n) \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet \quad 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet \quad 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet \quad n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln (n \cdot 2^n) = \ln n + \ln 2^n \end{aligned}$$

Riešené limity – 75

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right)\end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned}\Rightarrow \bullet n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.\end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

- $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$
- \Rightarrow • $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$
- \Rightarrow • $n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right)$
 $< \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$
- \Rightarrow • $\ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right)$
 $< \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet \quad 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet \quad 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet \quad n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \quad \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \quad \lim_{n \rightarrow \infty} \ln 2 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ \leq \lim_{n \rightarrow \infty} \left(\ln \sqrt[n]{n} + \ln 2 \right) \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 = \lim_{n \rightarrow \infty} \ln 2 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ \leq \lim_{n \rightarrow \infty} \left(\ln \sqrt[n]{n} + \ln 2 \right) = \ln 1 + \ln 2 \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet \quad 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet \quad 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet \quad n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \quad \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \quad \ln 2 = \lim_{n \rightarrow \infty} \ln 2 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ \leq \lim_{n \rightarrow \infty} \left(\ln \sqrt[n]{n} + \ln 2 \right) = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2. \end{aligned}$$

Riešené limity – 75

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \right) = \ln 2 \end{aligned}$$

Pre všetky $n \in \mathbb{N}$, $n > 1$ platí:

$$\bullet 2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n.$$

$$\Rightarrow \bullet 2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} < n \cdot 2^n.$$

$$\begin{aligned} \Rightarrow \bullet n \cdot \ln 2 = \ln 2^n < \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \ln \left(n \cdot 2^n \right) = \ln n + \ln 2^n = \ln n + n \cdot \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 < \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \end{aligned}$$

$$\begin{aligned} \Rightarrow \bullet \ln 2 = \lim_{n \rightarrow \infty} \ln 2 \leq \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) \\ \leq \lim_{n \rightarrow \infty} \left(\ln \sqrt[n]{n} + \ln 2 \right) = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2. \end{aligned}$$

$$\Rightarrow \bullet \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right) = \ln 2.$$

Riešené limity – 76, 77

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

Riešené limity – 76, 77

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

$\{a_n\}_{n=1}^{\infty}$

má tvar:

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

$\{a_n\}_{n=1}^{\infty}$

má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne).

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{1}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right]$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

$\{a_n\}_{n=1}^{\infty}$

má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2},$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right]$$

$$= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1.$$

$$\text{Resp. } \bullet \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

$\{a_n\}_{n=1}^{\infty}$ má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2},$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{1}{4^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right] \\ &= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} \end{aligned}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\begin{aligned} \bullet & \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \\ \text{Resp. } \bullet & \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \end{aligned} \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n$$

$\{a_n\}_{n=1}^{\infty}$

má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}, a_3 = \sqrt{\sqrt{\sqrt{2}}} = \sqrt[8]{2} = \sqrt[2^3]{2}, \dots,$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{1}{4^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right] \\ &= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} = \infty. \end{aligned}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\begin{aligned} \bullet & \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \\ \text{Resp. } \bullet & \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \end{aligned} \quad \left. \vphantom{\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} 2^{\frac{n}{2^n}}$$

$\{a_n\}_{n=1}^{\infty}$

má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}, a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}, \dots, a_n = \sqrt[2^n]{2}, n \in \mathbb{N} \text{ (explicitne).}$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right] \\ &= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} = \infty. \end{aligned}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\begin{aligned} \bullet &\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \\ \text{Resp. } \bullet &\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \end{aligned} \left. \vphantom{\lim_{n \rightarrow \infty} \sqrt[n]{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \sqrt[2^n]{2} = \left[\begin{array}{l} \text{Subst. } \left| \begin{array}{l} n \rightarrow \infty \\ m = n^2 \\ m \rightarrow \infty \end{array} \right. \end{array} \right]$$

$\{a_n\}_{n=1}^{\infty}$

má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}, a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}, \dots, a_n = \sqrt[2^n]{2}, n \in \mathbb{N} \text{ (explicitne).}$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right] \\ &= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} = \infty. \end{aligned}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\begin{aligned} \bullet &\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \\ \text{Resp. } \bullet &\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \end{aligned} \quad \left. \vphantom{\lim_{n \rightarrow \infty} \sqrt[n]{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \sqrt[n]{2} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n^2 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \sqrt[m]{2}$$

$\{a_n\}_{n=1}^{\infty}$ je vybraná z postupnosti $\{\sqrt[n]{2}\}_{n=1}^{\infty}$ a má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}, a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}, \dots, a_n = \sqrt[2^n]{2}, n \in \mathbb{N} \text{ (explicitne).}$$

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$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} = \infty$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n \rightarrow \infty} \frac{5^n}{4^n} = \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right] \\ &= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} = \infty. \end{aligned}$$

Označme $a_n = \frac{n^k}{4^n}$, $n \in \mathbb{N}$ pre $k \in \mathbb{N}$.

$$\begin{aligned} \bullet &\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \\ \text{Resp. } \bullet &\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1. \end{aligned} \quad \left. \vphantom{\lim_{n \rightarrow \infty} \sqrt[n]{a_n}} \right\} \Rightarrow \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$.

$$\bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \sqrt[n]{2} = \left[\text{Subst. } \begin{array}{l} n \rightarrow \infty \\ m = n^2 \\ m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \sqrt[m]{2} = 1.$$

$\{a_n\}_{n=1}^{\infty}$ je vybraná z postupnosti $\{\sqrt[n]{2}\}_{n=1}^{\infty}$ a má tvar: $\bullet a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{a_n}$, $n \in \mathbb{N}$ (rekurentne),

$$\bullet a_1 = \sqrt{2}, a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}, a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}, \dots, a_n = \sqrt[2^n]{2}, n \in \mathbb{N} \text{ (explicitne).}$$

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Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots\}$.



Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots\}$.

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Ešte musíme ukázať, že existuje $\lim_{n \rightarrow \infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \rightarrow 2$.

$\bullet \{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 2, a_n - 2 < 0$ pre všetky $n \in \mathbb{N}$. [Dôkaz matematickou indukciou.]

$$\boxed{\text{Krok 1.}} \quad a_1 = \sqrt{2} < 2, a_2 = \sqrt{2 + \sqrt{2}} < \sqrt{2 + 2} = \sqrt{4} = 2.$$

$$\boxed{\text{Krok 2.}} \quad \text{Pre } k \in \mathbb{N} \text{ platí } a_k < 2. \Rightarrow \text{Pre } k+1 \text{ platí } a_{k+1} = \sqrt{2 + a_k} < \sqrt{2 + 2} = \sqrt{4} = 2.$$

$\bullet \{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in \mathbb{N}$.

$$a_n^2 - a_{n+1}^2 = a_n^2 - (a_n + 2) = a_n^2 - a_n - 2 = (a_n - 2)(a_n + 1) < 0. \Rightarrow a_n^2 < a_{n+1}^2. \Rightarrow a_n < a_{n+1}.$$

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots\}$.

Postupnosť $\{a_n\}_{n=1}^{\infty}$ je rekurentne zadaná: $\bullet a_1 = \sqrt{2}, a_{n+1} = \sqrt{2 + a_n}, n \in \mathbb{N}$.

Označme $\lim_{n \rightarrow \infty} a_n = a$. Platí $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in \mathbb{N}$.

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (2 + a_n) = 2 + \lim_{n \rightarrow \infty} a_n = 2 + a.$$

$$\Rightarrow \bullet a \text{ je riešením rovnice } a^2 = 2 + a. \Rightarrow \bullet a^2 - a - 2 = (a - 2) \cdot (a + 1) = 0.$$

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Krok 1. $a_1 = \sqrt{2} < 2, a_2 = \sqrt{2 + \sqrt{2}} < \sqrt{2 + 2} = \sqrt{4} = 2.$

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Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2, a_{n+1} = \sqrt{2a_n + 3}, n \in \mathbb{N}$.



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Riešené limity – 79

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Riešené limity – 79

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• $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora,

• $\{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 79

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$$\bullet \{a_n\}_{n=1}^{\infty} \text{ je ohraničená zhora, t. j. } a_n < 3, a_n - 3 < 0, \text{ pre všetky } n \in \mathbb{N}. \quad [\text{Dôkaz matematickou indukciou.}]$$

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Riešené limity – 79

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Krok 1.

 $a_1 = 2 < 3, a_2 = \sqrt{2 \cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$

$\bullet \{a_n\}_{n=1}^{\infty}$ je rastúca,

Riešené limity – 79

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[Dôkaz matematickou indukciou.]

Krok 1. $a_1 = 2 < 3, a_2 = \sqrt{2 \cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$

Krok 2. Pre $k \in \mathbb{N}$ platí $a_k < 3$.

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Riešené limity – 79

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Riešené limity – 79

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Riešené limity – 79

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Riešené limity – 79

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$$\boxed{\text{Krok 1.}} \quad a_1 = 2 < 3, a_2 = \sqrt{2 \cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$$

$$\boxed{\text{Krok 2.}} \quad \text{Pre } k \in \mathbb{N} \text{ platí } a_k < 3. \Rightarrow \text{Pre } k+1 \text{ platí } a_{k+1} = \sqrt{2a_k + 3} < \sqrt{2 \cdot 3 + 3} = \sqrt{9} = 3.$$

- $\bullet \{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in \mathbb{N}$.

$$a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3 = (a_n - 3)(a_n + 1) < 0. \Rightarrow a_n^2 < a_{n+1}^2.$$

Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2$, $a_{n+1} = \sqrt{2a_n + 3}$, $n \in \mathbb{N}$.

Označme $\lim_{n \rightarrow \infty} a_n = a$. Platí $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in \mathbb{N}$.

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (2a_n + 3) = 2 \lim_{n \rightarrow \infty} a_n + 3 = 2a + 3.$$

$$\Rightarrow \bullet a \text{ je riešením rovnice } a^2 = 2a + 3. \Rightarrow \bullet a^2 - 2a - 3 = (a - 3) \cdot (a + 1) = 0.$$

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \text{ (pokiaľ limita existuje).} \quad [\text{Koreň } a = -1 < 0 \text{ nevyhovuje, pretože } a_n > 0 \text{ pre všetky } n \in \mathbb{N}.]$$

Ešte musíme ukázať, že existuje $\lim_{n \rightarrow \infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \rightarrow 3$.

- $\bullet \{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 3$, $a_n - 3 < 0$, pre všetky $n \in \mathbb{N}$. [Dôkaz matematickou indukciou.]

$$\boxed{\text{Krok 1.}} \quad a_1 = 2 < 3, \quad a_2 = \sqrt{2 \cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$$

$$\boxed{\text{Krok 2.}} \quad \text{Pre } k \in \mathbb{N} \text{ platí } a_k < 3. \Rightarrow \text{Pre } k+1 \text{ platí } a_{k+1} = \sqrt{2a_k + 3} < \sqrt{2 \cdot 3 + 3} = \sqrt{9} = 3.$$

- $\bullet \{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in \mathbb{N}$.

$$a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3 = (a_n - 3)(a_n + 1) < 0. \Rightarrow a_n^2 < a_{n+1}^2. \Rightarrow a_n < a_{n+1}.$$

Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1 = 2, a_{n+1} = \sqrt{2a_n + 3}, n \in \mathbb{N}$.

Označme $\lim_{n \rightarrow \infty} a_n = a$. Platí $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1}$, pričom $a_n > 0$ pre všetky $n \in \mathbb{N}$.

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (2a_n + 3) = 2 \lim_{n \rightarrow \infty} a_n + 3 = 2a + 3.$$

$$\Rightarrow \bullet a \text{ je riešením rovnice } a^2 = 2a + 3. \Rightarrow \bullet a^2 - 2a - 3 = (a - 3) \cdot (a + 1) = 0.$$

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \text{ (pokiaľ limita existuje).} \quad [\text{Koreň } a = -1 < 0 \text{ nevyhovuje, pretože } a_n > 0 \text{ pre všetky } n \in \mathbb{N}.]$$

Ešte musíme ukázať, že existuje $\lim_{n \rightarrow \infty} a_n$, t. j. $\{a_n\}_{n=1}^{\infty} \rightarrow 3$.

- $\bullet \{a_n\}_{n=1}^{\infty}$ je ohraničená zhora, t. j. $a_n < 3, a_n - 3 < 0$, pre všetky $n \in \mathbb{N}$. [Dôkaz matematickou indukciou.]

$$\boxed{\text{Krok 1.}} \quad a_1 = 2 < 3, a_2 = \sqrt{2 \cdot 2 + 3} = \sqrt{7} < \sqrt{9} = 3.$$

$$\boxed{\text{Krok 2.}} \quad \text{Pre } k \in \mathbb{N} \text{ platí } a_k < 3. \Rightarrow \text{Pre } k+1 \text{ platí } a_{k+1} = \sqrt{2a_k + 3} < \sqrt{2 \cdot 3 + 3} = \sqrt{9} = 3.$$

- $\bullet \{a_n\}_{n=1}^{\infty}$ je rastúca, t. j. $a_n < a_{n+1}$ pre všetky $n \in \mathbb{N}$.

$$a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3 = (a_n - 3)(a_n + 1) < 0. \Rightarrow a_n^2 < a_{n+1}^2. \Rightarrow a_n < a_{n+1}.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots$

1

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots$

Periodické číslo $x = 32,17\overline{71}$

2

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots$

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

3

- $x = 32,17\overline{71} = 32,177171\dots$

- $x = 32,17\overline{71} = 32,177171\dots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots$

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

4

- $x = 32,17\overline{71} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots$

- $x = 32,17\overline{71} = 32,177171\dots$
- $100x = 3217,\overline{71} = 3217,717171\dots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots$

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

5

$$\begin{aligned} \bullet \quad x &= 32,17\overline{71} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\ &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \end{aligned}$$

-
- $x = 32,17\overline{71} = 32,177171\dots$
 - $100x = 3217,\overline{71} = 3217,717171\dots$
 - $10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171\dots$

Periodické číslo $x = 32,1\overline{771}$ môžeme vyjadriť ako súčet členov postupnosti:

6

$$\begin{aligned} \bullet x &= 32,1\overline{771} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\ &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\ &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots\right) \end{aligned}$$

$$\begin{aligned} \bullet x &= 32,1\overline{771} = 32,177171\dots \\ \bullet 100x &= 3217,\overline{71} = 3217,717171\dots \\ \bullet 10\,000x &= 321\,771,\overline{71} = 321\,771,717171\dots \end{aligned}$$

$$\Rightarrow \bullet 10\,000x - 100x$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171\dots$

Periodické číslo $x = 32,1\overline{771}$ môžeme vyjadriť ako súčet členov postupnosti:

7

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots \right) = \left[\begin{array}{l} \text{Geometrický rad s kvocientom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right]
 \end{aligned}$$

$$\bullet \quad x = 32,1\overline{771} = 32,177171\dots$$

$$\bullet \quad 100x = 3217,\overline{71} = 3217,717171\dots$$

$$\bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots$$

$$\begin{aligned}
 \Rightarrow \bullet \quad & 10\,000x - 100x = 321\,771,\overline{71} - 3217,\overline{71} \\
 & = 321\,771,717171\dots - 3217,717171\dots
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171\dots$

Periodické číslo $x = 32,1\overline{771}$ môžeme vyjadriť ako súčet členov postupnosti:

8

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots \right) = \left[\begin{array}{l} \text{Geometrický rad s kvociantom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}}
 \end{aligned}$$

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177171\dots \\
 \bullet \quad 100x &= 3217,\overline{71} = 3217,717171\dots \\
 \bullet \quad 10\,000x &= 321\,771,\overline{71} = 321\,771,717171\dots
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 10\,000x - 100x &= 321\,771,\overline{71} - 3217,\overline{71} \\
 &= 321\,771,717171\dots - 3217,717171\dots = 321\,771 - 3217
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171\dots$

Periodické číslo $x = 32,1\overline{771}$ môžeme vyjadriť ako súčet členov postupnosti:

9

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots \right) = \left[\begin{array}{l} \text{Geometrický rad s kvociantom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{3217}{100} + \frac{71}{100 \cdot (100-1)}
 \end{aligned}$$

$$\left. \begin{array}{l}
 \bullet \quad x = 32,1\overline{771} = 32,177171\dots \\
 \bullet \quad 100x = 3217,\overline{71} = 3217,717171\dots \\
 \bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots
 \end{array} \right\} \Rightarrow 9\,900x = 318\,554$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 9\,900x &= 10\,000x - 100x = 321\,771,\overline{71} - 3217,\overline{71} \\
 &= 321\,771,717171\dots - 3217,717171\dots = 321\,771 - 3217 = 318\,554.
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

10

$$\begin{aligned}
 \bullet \quad x &= 32,17\overline{71} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots\right) = \left[\begin{array}{l} \text{Geometrický rad s kvociantom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100 \cdot (100-1)} \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 99}
 \end{aligned}$$

$$\left. \begin{array}{l}
 \bullet \quad x = 32,17\overline{71} = 32,177171\dots \\
 \bullet \quad 100x = 3\,217,\overline{71} = 3\,217,717171\dots \\
 \bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots
 \end{array} \right\} \Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 9\,900x &= 10\,000x - 100x = 321\,771,\overline{71} - 3\,217,\overline{71} \\
 &= 321\,771,717171\dots - 3\,217,717171\dots = 321\,771 - 3\,217 = 318\,554.
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

11

$$\begin{aligned}
 \bullet \quad x &= 32,17\overline{71} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots\right) = \left[\begin{array}{l} \text{Geometrický rad s kvocientom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100 \cdot (100-1)} \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 99} = \frac{3\,217 \cdot 99 + 71}{100 \cdot 99}
 \end{aligned}$$

$$\left. \begin{array}{l}
 \bullet \quad x = 32,17\overline{71} = 32,177171\dots \\
 \bullet \quad 100x = 3\,217,\overline{71} = 3\,217,717171\dots \\
 \bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots
 \end{array} \right\} \Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 9\,900x &= 10\,000x - 100x = 321\,771,\overline{71} - 3\,217,\overline{71} \\
 &= 321\,771,717171\dots - 3\,217,717171\dots = 321\,771 - 3\,217 = 318\,554.
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,17717171\dots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32,17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

12

$$\begin{aligned}
 \bullet \quad x &= 32,17\overline{71} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots\right) = \left[\begin{array}{l} \text{Geometrický rad s kvociantom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100 \cdot (100-1)} \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 99} = \frac{3\,217 \cdot 99 + 71}{100 \cdot 99} = \frac{318\,483 + 71}{9\,900}
 \end{aligned}$$

$$\left. \begin{array}{l}
 \bullet \quad x = 32,17\overline{71} = 32,177171\dots \\
 \bullet \quad 100x = 3\,217,\overline{71} = 3\,217,717171\dots \\
 \bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots
 \end{array} \right\} \Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 9\,900x &= 10\,000x - 100x = 321\,771,\overline{71} - 3\,217,\overline{71} \\
 &= 321\,771,717171\dots - 3\,217,717171\dots = 321\,771 - 3\,217 = 318\,554.
 \end{aligned}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171\dots = \frac{318\,554}{9\,900}$.

Periodické číslo $x = 32,1\overline{771}$ môžeme vyjadriť ako súčet členov postupnosti:

13

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177171\dots = 32,17 + 0,0071 + 0,000071 + 0,00000071 + \dots \\
 &= 32,17 + \frac{71}{100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100} + \frac{71}{100 \cdot 100 \cdot 100 \cdot 100} + \dots \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \left(1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots\right) = \left[\begin{array}{l} \text{Geometrický rad s kvociantom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100 \cdot 100} + \dots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}} \end{array} \right] \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 100} \cdot \frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100 \cdot (100-1)} \\
 &= \frac{3\,217}{100} + \frac{71}{100 \cdot 99} = \frac{3\,217 \cdot 99 + 71}{100 \cdot 99} = \frac{318\,483 + 71}{9\,900} = \frac{318\,554}{9\,900}.
 \end{aligned}$$

$$\left. \begin{array}{l}
 \bullet \quad x = 32,1\overline{771} = 32,177171\dots \\
 \bullet \quad 100x = 3\,217,\overline{71} = 3\,217,717171\dots \\
 \bullet \quad 10\,000x = 321\,771,\overline{71} = 321\,771,717171\dots
 \end{array} \right\} \Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}.$$

$$\begin{aligned}
 \Rightarrow \bullet \quad 9\,900x &= 10\,000x - 100x = 321\,771,\overline{71} - 3\,217,\overline{71} \\
 &= 321\,771,717171\dots - 3\,217,717171\dots = 321\,771 - 3\,217 = 318\,554.
 \end{aligned}$$

Koniec 2. časti (príklady)

Ďakujem za pozornosť.