

Matematická analýza 1

2026/2027

2. Číselné postupnosti Riešené príklady

Pre správne zobrazenie, fungovanie tooltipov, 2D a 3D animácií je nevyhnutné súbor otvoriť pomocou programu Adobe Reader (zásuvný modul Adobe PDF Plug-In webového prehliadača nestačí).

Kliknutím na text pred ikonou  získate nápomoc.

Kliknutím na skratku v modrej lište vpravo hore sa dostanete na príslušný slajd, druhým kliknutím sa dostanete na koniec tohto slajdu.

Obsah

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Zoznam limit – postupnosti P1

1. $\lim_{n \rightarrow \infty} n^q$. 2. $\lim_{n \rightarrow \infty} \frac{1}{n^q}$. 3. $\lim_{n \rightarrow \infty} (n^2 - n)$. 4. $\lim_{n \rightarrow \infty} a^n$. 5. $\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$. 6. $\lim_{n \rightarrow \infty} \frac{n^3-2}{n^3+n}$. 7. $\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-n}$. 8. $\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2+n}$. 9. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^3+5}{n^3-n^2-n^2+2}$. 10. $\lim_{n \rightarrow \infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^2+5}$. 11. $\lim_{n \rightarrow \infty} \frac{4n^3+5n^4+6n^2+3}{4n^2-2n^3+3}$.
12. $\lim_{n \rightarrow \infty} \frac{4n^2-2n^3+3}{4n^3+5n^4+6n^2+3}$. 13. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$. 14. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4+5}{n^3-n^5-n^2+n^2+2}$. 15. $\lim_{n \rightarrow \infty} \left[\frac{n^2}{n+2} - \frac{n^2}{n+3} \right]$. 16. $\lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^4+1}$. 17. $\lim_{n \rightarrow \infty} \frac{(n+4)!(n+2)!}{(n+3)!}$. 18. $\lim_{n \rightarrow \infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$. 19. $\lim_{n \rightarrow \infty} \frac{n+3^n}{n-3^n}$. 20. $\lim_{n \rightarrow \infty} \frac{n+3^{-n}}{n-3^{-n}}$.
21. $\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[3]{n}+3}$. 22. $\lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n}+4\sqrt[4]{n}-2}{3\sqrt[3]{n+4}\sqrt[4]{n+5}\sqrt[5]{n+6}}$. 23. $\lim_{n \rightarrow \infty} \frac{3\sqrt[5]{n}+4\sqrt[6]{n}+5\sqrt[7]{n}+6}{3\sqrt[3]{n+4}\sqrt[4]{n+5}}$. 24. $\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{n+1}$. 25. $\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n}}$. 26. $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}}$. 27. $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^2-2}+3\sqrt[7]{n+1}}{2\sqrt[3]{n^3+1}+3\sqrt[5]{n^6-1}-\sqrt[7]{n-1}}$.
28. $\lim_{n \rightarrow \infty} \left[\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right]$. 29. $\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$. 30. $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^2+1}}$. 31. $\lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-1)}{\sqrt{9n^2+1}}$. 32. $\lim_{n \rightarrow \infty} \frac{1^2+2^2+3^2+\dots+n^2}{n^3}$. 33. $\lim_{n \rightarrow \infty} \frac{1^2+3^2+5^2+\dots+(2n-1)^2}{n^3}$. 34. $\lim_{n \rightarrow \infty} \frac{1^2+3^2+5^2+\dots+(2n-1)^2}{1^2+2^2+3^2+\dots+n^2}$.
35. $\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4+5^{-n}}{3n+2-n-2^{-n}}$. 36. $\lim_{n \rightarrow \infty} \frac{3n+2-n-2^{-n}}{8+3^{-n}+4+5^{-n}}$. 37. $\lim_{n \rightarrow \infty} \frac{8-3^{-n}+4+5^{-n}}{3n+2-n-2^{-n}}$. 38. $\lim_{n \rightarrow \infty} \frac{3n+2-n-2^{-n}}{8-3^{-n}+4+5^{-n}}$. 39. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4-n^2+2+5^n}{2n^2-n^3+3+5^{n+1}}$. 40. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4-n^2}{2n^2-n^3+3+5^{n+1}}$. 41. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2+2+5^n}{2n^2-n^3}$. 42. $\lim_{n \rightarrow \infty} \frac{2n^3+3n^4-n^2-2+5^n}{2n^2-n^3}$.
43. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4-n^2+2+5^n}{2n^2-n^3+6+7^n}$. 44. $\lim_{n \rightarrow \infty} \frac{2n^2+3n^4-n^2+2+5^n}{2n^2-n^3+6+4^n}$. 45. $\lim_{n \rightarrow \infty} \frac{3n^2+4n^3+5n+6+3^n}{3n+4n^3-2+3^n}$. 46. $\lim_{n \rightarrow \infty} \frac{3n^2+4n^3+5n+6+3^n}{3n+4n^3-2+3^{n+1}}$. 47. $\lim_{n \rightarrow \infty} \frac{-3n^2+4n^3+5n-6-3^n}{3n+4n^3+2+3^{n+1}}$. 48. $\lim_{n \rightarrow \infty} \frac{-3n^2+4n^3+5n-6-e^n}{3n+4n^3+2+e^{n+1}}$. 49. $\lim_{n \rightarrow \infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$.
50. $\lim_{n \rightarrow \infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$. 51. $\lim_{n \rightarrow \infty} \frac{3^n-2^n}{3^{n+1}-2^{n+1}}$. 52. $\lim_{n \rightarrow \infty} \frac{(-2)^n+3^n}{(-2)^{n+1}+3^{n+1}}$. 53. $\lim_{n \rightarrow \infty} \frac{1+(-2)^n+3^{n+1}+(-4)^n+5^{-n-1}}{2^{n+1}-3^{n+1}+4^{n+1}-5^{n+1}}$. 54. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n}-\frac{1}{3^n}}$. 55. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n}-\frac{1}{3^n}}$. 56. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n}-\frac{1}{3^n}}$. 57. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n}-\frac{1}{3^n}}$. 58. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n}-\frac{1}{3^n}}$.
59. $\lim_{n \rightarrow \infty} \frac{\frac{1}{2^n}-\frac{1}{3^n}}{\frac{1}{2^n}}$. 60. $\lim_{n \rightarrow \infty} \sqrt[4]{4^n-3^n}$. 61. $\lim_{n \rightarrow \infty} \sqrt[4]{4^n+3^n}$. 62. $\lim_{n \rightarrow \infty} \sqrt[3]{3^n-2^n}$. 63. $\lim_{n \rightarrow \infty} \sqrt[3]{2^n+1}$. 64. $\lim_{n \rightarrow \infty} \sqrt[3]{3^n-1}$. 65. $\lim_{n \rightarrow \infty} \sqrt[3]{3^n+1}$. 66. $\lim_{n \rightarrow \infty} \frac{a^n n!}{n!}$. 67. $\lim_{n \rightarrow \infty} \frac{a^n}{n!}$. 68. $\lim_{n \rightarrow \infty} \frac{a^n n!}{n!}$. 69. $\lim_{n \rightarrow \infty} \frac{3^n n!}{n!}$.
70. $\lim_{n \rightarrow \infty} \frac{2^n n!}{n!}$. 71. $\lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)!}$. 72. $\lim_{n \rightarrow \infty} \frac{7^n n!}{(2n)!}$. 73. $\lim_{n \rightarrow \infty} \frac{7^n n!}{(3n)!}$. 74. $\lim_{n \rightarrow \infty} \frac{7^n n!}{(5n)!}$. 75. $\lim_{n \rightarrow \infty} \frac{n^2}{(5n)^2}$. 76. $\lim_{n \rightarrow \infty} \frac{n^3}{2^n}$. 77. $\lim_{n \rightarrow \infty} \frac{a^n}{n^n}$. 78. $\lim_{n \rightarrow \infty} \frac{2^n}{n^n}$. 79. $\lim_{n \rightarrow \infty} \frac{a^n}{0.5^n}$. 80. $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n} \right)^4$. 81. $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$. 82. $\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \right)^4$.
83. $\lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \right)^4$. 84. $\lim_{n \rightarrow \infty} 4^{\frac{n+1}{3}}$. 85. $\lim_{n \rightarrow \infty} \sqrt{\frac{5n+1}{n+5}}$. 86. $\lim_{n \rightarrow \infty} \sqrt[3]{\frac{5n-1}{n-5}}$. 87. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{2n}$. 88. $\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+2} \right)^{n+2}$. 89. $\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$. 90. $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$. 91. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{n^2+1}$. 92. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{\sqrt{n}+1}$.
93. $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n} \right)^{n^2+1}$. 94. $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n} \right)^{\sqrt{n}+1}$. 95. $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^2+6}$. 96. $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^2+6}$. 97. $\lim_{n \rightarrow \infty} \left(\frac{2n^2-1}{2n^2+3} \right)^{n+6}$. 98. $\lim_{n \rightarrow \infty} \left(\frac{2n^2-1}{2n^2+3} \right)^{n^2+6}$. 99. $\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[3]{n}-1}{2\sqrt[3]{n}+3} \right)^{\sqrt[5]{n}+6}$. 100. $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[5]{n}+6}$.
101. $\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[5]{n}-1}{2\sqrt[5]{n}+3} \right)^{n+6}$. 102. $\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[5]{n}-1}{2\sqrt[5]{n}+3} \right)^{n+6}$. 103. $\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n}+2}{\sqrt[3]{n}+1} \right)^{\sqrt[3]{n}+1}$. 104. $\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n}+2}{\sqrt[3]{n}+1} \right)^{\sqrt[3]{n}}$. 105. $\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^{\sqrt{n}}$. 106. $\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$. 107. $\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+4}$. 108. $\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+6}$.
109. $\lim_{n \rightarrow \infty} \left(\frac{n+5}{n+3} \right)^{n^4+1}$. 110. $\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{2}+\sqrt[3]{3}}{\sqrt[3]{2}+\sqrt[3]{3}} \right)^n$. 111. $\lim_{n \rightarrow \infty} n \left(\sqrt[3]{3} - \sqrt[3]{2} \right)$. 112. $\lim_{n \rightarrow \infty} n \left(\sqrt[3]{2} - \sqrt[3]{2} \right)$. 113. $\lim_{n \rightarrow \infty} n^2 \left(\sqrt[3]{2} - \sqrt[3]{2} \right)$. 114. $\lim_{n \rightarrow \infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[2^n]{2^n} \right]$. 115. $\lim_{n \rightarrow \infty} \frac{\sin n}{n}$.
116. $\lim_{n \rightarrow \infty} n \sin \frac{1}{n}$. 117. $\lim_{n \rightarrow \infty} \sin \frac{1}{n}$. 118. $\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right]$. 119. $\lim_{n \rightarrow \infty} \left[\sqrt{n^3+1} - n^2 \right]$. 120. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n+1} - n+1 \right]$. 121. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n} - n+1 \right]$. 122. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2-n} - n-1 \right]$.
123. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+2n-1} - n+2 \right]$. 124. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+2n+4} - n-3 \right]$. 125. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+4n+1} - n+1 \right]$. 126. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2-3n+1} - n+3 \right]$. 127. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n+1} - n \right]$.
128. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n} - n \right]$. 129. $\lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right]$. 130. $\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right]$. 131. $\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right]$. 132. $\lim_{n \rightarrow \infty} \left[\sqrt{an+p} - \sqrt{an+q} \right]$. 133. $\lim_{n \rightarrow \infty} \left[\sqrt{2n+24} - \sqrt{2n+33} \right]$.
134. $\lim_{n \rightarrow \infty} \left[\sqrt{3n} - \sqrt{2n} \right]$. 135. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+p} - \sqrt{n^2+q} \right]$. 136. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+2} - \sqrt{n^2+1} \right]$. 137. $\lim_{n \rightarrow \infty} \left[\sqrt{n+2} - \sqrt{n+1} \right]$. 138. $\lim_{n \rightarrow \infty} \left[\sqrt{n^4+n^3-n-2} - \sqrt{n^4+n^3-n^2-1} \right]$.
139. $\lim_{n \rightarrow \infty} \left[\sqrt{n^3-n-2} - \sqrt{n^3-n^2-1} \right]$. 140. $\lim_{n \rightarrow \infty} \left[\sqrt{n^3-2} - \sqrt{n^3-n^2-1} \right]$. 141. $\lim_{n \rightarrow \infty} \left[\sqrt{n^3-2} - \sqrt{n^3-n-1} \right]$. 142. $\lim_{n \rightarrow \infty} \left[\sqrt{n^5+2} - \sqrt{n^5+1} \right]$. 143. $\lim_{n \rightarrow \infty} \left[\sqrt[5]{n+2} - \sqrt[5]{n+1} \right]$.
144. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2} \right]$. 145. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2+n+1} - \sqrt{n-1} \right]$. 146. $\lim_{n \rightarrow \infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-n-1} \right]$.

Zoznam limit – postupnosti P2

147. $\lim_{n \rightarrow \infty} [\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}]$. 148. $\lim_{n \rightarrow \infty} [\sqrt{n^4 + 1} - \sqrt{n + 1}]$. 149. $\lim_{n \rightarrow \infty} [\sqrt{n^2 + 1} - \sqrt{n - 3}]$. 150. $\lim_{n \rightarrow \infty} [\sqrt{\sqrt{n} + 1} - \sqrt{n - 3}]$. 151. $\lim_{n \rightarrow \infty} [\sqrt{n^2 + 1} - \sqrt{n^3 - 3}]$.
 152. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + 1} - n + 1]$. 153. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + 1} - n]$. 154. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^4 + 1} - n]$. 155. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + n + 1} - n + 1]$. 156. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + n^2 + 1} - n + 1]$. 157. $\lim_{n \rightarrow \infty} [\sqrt[3]{n + p} - \sqrt[3]{n + q}]$.
 158. $\lim_{n \rightarrow \infty} [\sqrt[3]{n + 1} - \sqrt[3]{n + 2}]$. 159. $\lim_{n \rightarrow \infty} [\sqrt[3]{n - 1} - \sqrt[3]{n - 2}]$. 160. $\lim_{n \rightarrow \infty} [\sqrt[3]{n - 1} - \sqrt[3]{n + 3}]$. 161. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + p} - \sqrt[3]{n^3 + q}]$. 162. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q}]$.
 163. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1}]$. 164. $\lim_{n \rightarrow \infty} [\sqrt[3]{\sqrt[3]{n} + p} - \sqrt[3]{\sqrt[3]{n} + q}]$. 165. $\lim_{n \rightarrow \infty} [\sqrt[3]{\sqrt[3]{n} + 2} - \sqrt[3]{\sqrt[3]{n} + 1}]$. 166. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2}]$. 167. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1}]$.
 168. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1}]$. 169. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1}]$. 170. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^4 + 1} - \sqrt[3]{n + 1}]$. 171. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^5 + 1} - \sqrt[3]{n^4 + 1}]$. 172. $\lim_{n \rightarrow \infty} [\sqrt[3]{n^4 + 1} - \sqrt[3]{n^4 - 1}]$.
 173. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^3 + p} - \sqrt[4]{n^3 + q}]$. 174. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q}]$. 175. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1}]$. 176. $\lim_{n \rightarrow \infty} [\sqrt[4]{\sqrt[4]{n} + p} - \sqrt[4]{\sqrt[4]{n} + q}]$. 177. $\lim_{n \rightarrow \infty} [\sqrt[4]{\sqrt[4]{n} + 2} - \sqrt[4]{\sqrt[4]{n} + 1}]$.
 178. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^4 + n + 1} - n + 1]$. 179. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^4 + n^2 + 1} - n + 1]$. 180. $\lim_{n \rightarrow \infty} [\sqrt[4]{n^4 + n^3 + 1} - n + 1]$. 181. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n}$. 182. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-n^2}$. 183. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n+1}$. 184. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n+1}$.
 185. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n}-n-1}$. 186. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n-1}-n+2}$. 187. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n+4}-n-3}$. 188. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n+1}-n+1}$. 189. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-3n+1}-n+3}$. 190. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n}$. 191. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n}$. 192. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p}-\sqrt{n+q}}$.
 193. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}$. 194. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n-1}}$. 195. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p}-\sqrt{an+q}}$. 196. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24}-\sqrt{2n+33}}$. 197. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{3n}-\sqrt{2n}}$. 198. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+p}-\sqrt{n^2+q}}$. 199. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}}$. 200. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+2}-\sqrt{n+1}}$.
 201. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4+n^3-n-2}-\sqrt{n^4+n^3-n^2-1}}$. 202. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-n-2}-\sqrt{n^3-n^2-1}}$. 203. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}}$. 204. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}}$. 205. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+2}-\sqrt{n^3+1}}$. 206. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n+2}-\sqrt[5]{n+1}}$.
 207. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$. 208. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n-1}}$. 209. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$. 210. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}}$. 211. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4+1}-\sqrt{n+1}}$. 212. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n-3}}$. 213. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n-3}}$.
 214. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n^3-3}}$. 215. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1}-n+1}$. 216. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1}-n}$. 217. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^4+1}-n}$. 218. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-n+1}$. 219. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n^2+1}-n+1}$. 220. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p}-\sqrt[3]{n+q}}$. 221. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1}-\sqrt[3]{n+2}}$.
 222. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$. 223. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-3}}$. 224. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+p}-\sqrt[3]{n^2+q}}$. 225. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p}-\sqrt[3]{n^3+q}}$. 226. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+2}-\sqrt[3]{n^3+1}}$. 227. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[3]{n}+p}-\sqrt[3]{\sqrt[3]{n}+q}}$. 228. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[3]{n}+2}-\sqrt[3]{\sqrt[3]{n}+1}}$.
 229. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}}$. 230. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+2n+1}}$. 231. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+2n+1}}$. 232. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+n+1}-\sqrt[3]{n-1}}$. 233. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+1}-\sqrt[3]{n+1}}$. 234. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+1}-\sqrt[3]{n^2+1}}$.
 235. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1}-\sqrt[4]{n^4-1}}$. 236. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^2+p}-\sqrt[4]{n^2+q}}$. 237. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p}-\sqrt[4]{n^3+q}}$. 238. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^2+2}-\sqrt[4]{n^2+1}}$. 239. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[4]{n}+p}-\sqrt[4]{\sqrt[4]{n}+q}}$. 240. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[4]{n}+2}-\sqrt[4]{\sqrt[4]{n}+1}}$. 241. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n+1}-n+1}$.
 242. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n^2+1}-n+1}$. 243. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n^2+1}-n+1}$. 244. $\lim_{n \rightarrow \infty} n [\ln n - \ln (n + 2)]$. 245. $\lim_{n \rightarrow \infty} n [\ln (n + 3) - \ln n]$. 246. $\lim_{n \rightarrow \infty} n^2 [\ln (n + 3) - \ln n]$. 247. $\lim_{n \rightarrow \infty} \sqrt{n} [\ln (n + 3) - \ln n]$.
 248. $\lim_{n \rightarrow \infty} a_n$, ak $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots\}$. 249. $\lim_{n \rightarrow \infty} a_n$, ak $a_1 = 1$, $a_{n+1} = \sqrt{a_n + 6}$, $n \in \mathbb{N}$. 250. $\lim_{n \rightarrow \infty} a_n$, ak $a_1 = 2$, $a_{n+1} = \sqrt{2a_n + 3}$, $n \in \mathbb{N}$.
 251. $\lim_{n \rightarrow \infty} a_n$, ak $\{a_n\}_{n=1}^{\infty} = \{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\}$. 252. Vyjadriť ako zlomok 0,23. 253. Vyjadriť ako zlomok 32,1771. 254. Obvod Kochovej vložky.

Dôležité limity – ViL

Postupnosti

[$\boxtimes \in \mathbb{N}$ reprezentuje limitnú premennú.]

$$\lim_{\boxtimes \rightarrow \infty} \sqrt[\boxtimes]{\boxtimes} = 1$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\boxtimes!}{\boxtimes^{\boxtimes}} = 0$$

$$\lim_{\boxtimes \rightarrow \infty} \sqrt[\boxtimes]{\boxtimes!} = \infty$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\boxtimes^{\boxtimes}}{\boxtimes!} = \infty$$

$$\lim_{\boxtimes \rightarrow \infty} \left(1 + \frac{1}{\boxtimes!} + \frac{1}{2!} + \dots + \frac{1}{\boxtimes!}\right) = e$$

$$\lim_{\boxtimes \rightarrow \infty} \left(1 + \frac{\alpha}{\boxtimes}\right)^{\boxtimes} = e^{\alpha} \text{ pre } \alpha \in \mathbb{R}$$

$$\lim_{\boxtimes \rightarrow \infty} \sqrt[\boxtimes]{\alpha} = 1 \text{ pre } \alpha > 0$$

$$\lim_{\boxtimes \rightarrow \infty} \left(1 + \frac{1}{\boxtimes}\right)^{\boxtimes} = e$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\boxtimes^{\vartheta}}{\alpha^{\boxtimes}} = 0 \text{ pre } \vartheta \in \mathbb{N}, \alpha \in (1; \infty)$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\alpha^{\boxtimes}}{\boxtimes^{\vartheta}} = \infty \text{ pre } \vartheta \in \mathbb{N}, \alpha \in (1; \infty)$$

$$\lim_{\boxtimes \rightarrow \infty} \boxtimes(\sqrt[\boxtimes]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(\sqrt[\boxtimes]{\alpha} - 1)} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{\boxtimes \rightarrow \infty} \boxtimes(\sqrt[\boxtimes]{e} - 1) = \ln e = 1$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(\sqrt[\boxtimes]{e} - 1)} = \frac{1}{\ln e} = 1$$

Funkcie

[$\boxtimes \in \mathbb{R}$ reprezentuje limitnú premennú.]

$$\lim_{\boxtimes \rightarrow 0} \frac{\sin \boxtimes}{\boxtimes} = \lim_{\boxtimes \rightarrow 0} \frac{\boxtimes}{\sin \boxtimes} = 1$$

$$\lim_{\boxtimes \rightarrow 0} \frac{\arcsin \boxtimes}{\boxtimes} = \lim_{\boxtimes \rightarrow 0} \frac{\boxtimes}{\arcsin \boxtimes} = 1$$

$$\lim_{\boxtimes \rightarrow \infty} \sqrt[\boxtimes]{\alpha} = \lim_{\boxtimes \rightarrow \infty} \alpha^{\frac{1}{\boxtimes}} = 1 \text{ pre } \alpha > 0$$

$$\lim_{\boxtimes \rightarrow 0} \frac{\alpha^{\boxtimes} - 1}{\boxtimes} = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{\boxtimes \rightarrow 0} \frac{1}{\alpha^{\boxtimes} - 1} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{\boxtimes \rightarrow \infty} \sqrt[\boxtimes]{\boxtimes} = \lim_{\boxtimes \rightarrow \infty} \boxtimes^{\frac{1}{\boxtimes}} = 1$$

$$\lim_{\boxtimes \rightarrow 0} \frac{e^{\boxtimes} - 1}{\boxtimes} = \ln e = 1$$

$$\lim_{\boxtimes \rightarrow 0} \frac{1}{e^{\boxtimes} - 1} = \frac{1}{\ln e} = 1$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\boxtimes^{\delta}}{\alpha^{\boxtimes}} = \begin{cases} \infty & \text{pre } \delta \in \mathbb{R}, \alpha \in (0; 1) \\ 0 & \text{pre } \delta \in \mathbb{R}, \alpha \in (1; \infty) \end{cases}$$

$$\lim_{\boxtimes \rightarrow \infty} \left(1 + \frac{\alpha}{\boxtimes}\right)^{\boxtimes} = \lim_{\boxtimes \rightarrow \infty} \left(1 + \alpha \boxtimes\right)^{\frac{1}{\boxtimes}} = e^{\alpha} \text{ pre } \alpha \in \mathbb{R}$$

$$\lim_{\boxtimes \rightarrow \infty} \boxtimes(\alpha^{\frac{1}{\boxtimes}} - 1) = \lim_{\boxtimes \rightarrow \infty} \boxtimes(\sqrt[\boxtimes]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(\alpha^{\frac{1}{\boxtimes}} - 1)} = \lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(\sqrt[\boxtimes]{\alpha} - 1)} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{\alpha^{\boxtimes}}{\boxtimes^{\delta}} = \begin{cases} 0 & \text{pre } \delta \in \mathbb{R}, \alpha \in (0; 1) \\ \infty & \text{pre } \delta \in \mathbb{R}, \alpha \in (1; \infty) \end{cases}$$

$$\lim_{\boxtimes \rightarrow \infty} \left(1 + \frac{1}{\boxtimes}\right)^{\boxtimes} = \lim_{\boxtimes \rightarrow \infty} \left(1 + \boxtimes\right)^{\frac{1}{\boxtimes}} = e$$

$$\lim_{\boxtimes \rightarrow \infty} \boxtimes(e^{\frac{1}{\boxtimes}} - 1) = \lim_{\boxtimes \rightarrow \infty} \boxtimes(\sqrt[\boxtimes]{e} - 1) = \ln e = 1$$

$$\lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(e^{\frac{1}{\boxtimes}} - 1)} = \lim_{\boxtimes \rightarrow \infty} \frac{1}{\boxtimes(\sqrt[\boxtimes]{e} - 1)} = \frac{1}{\ln e} = 1$$

Dôležité limity – ViL

Postupnosti

[$n \in \mathbb{N}$ reprezentuje limitnú premennú.]

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n!} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{n^n}{n!} = \infty$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}\right) = e$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n}\right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\alpha} = 1 \text{ pre } \alpha > 0$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0 \text{ pre } k \in \mathbb{N}, \alpha \in (1; \infty)$$

$$\lim_{n \rightarrow \infty} \frac{\alpha^n}{n^k} = \infty \text{ pre } k \in \mathbb{N}, \alpha \in (1; \infty)$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n(\sqrt[n]{\alpha} - 1)} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{n \rightarrow \infty} n(\sqrt[n]{e} - 1) = \ln e = 1$$

$$\lim_{n \rightarrow \infty} \frac{1}{n(\sqrt[n]{e} - 1)} = \frac{1}{\ln e} = 1$$

Funkcie

[$x \in \mathbb{R}$ reprezentuje limitnú premennú.]

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\arcsin x} = 1$$

$$\lim_{x \rightarrow \infty} \sqrt[x]{\alpha} = \lim_{x \rightarrow \infty} \alpha^{\frac{1}{x}} = 1 \text{ pre } \alpha > 0$$

$$\lim_{x \rightarrow 0} \frac{\alpha^x - 1}{x} = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{x \rightarrow 0} \frac{x}{\alpha^x - 1} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{x \rightarrow \infty} \sqrt[x]{x} = \lim_{x \rightarrow \infty} x^{\frac{1}{x}} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \ln e = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{e^x - 1} = \frac{1}{\ln e} = 1$$

$$\lim_{x \rightarrow \infty} \frac{x^\delta}{\alpha^x} = \begin{cases} \infty & \text{pre } \delta \in \mathbb{R}, \alpha \in (0; 1) \\ 0 & \text{pre } \delta \in \mathbb{R}, \alpha \in (1; \infty) \end{cases}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{\alpha}{x}\right)^x = \lim_{x \rightarrow \infty} (1 + \alpha x)^{\frac{1}{x}} = e^\alpha \text{ pre } \alpha \in \mathbb{R}$$

$$\lim_{x \rightarrow \infty} x(\alpha^{\frac{1}{x}} - 1) = \lim_{x \rightarrow \infty} x(\sqrt[x]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0$$

$$\lim_{x \rightarrow \infty} \frac{1}{x(\alpha^{\frac{1}{x}} - 1)} = \lim_{x \rightarrow \infty} \frac{1}{x(\sqrt[x]{\alpha} - 1)} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1$$

$$\lim_{x \rightarrow \infty} \frac{\alpha^x}{x^\delta} = \begin{cases} 0 & \text{pre } \delta \in \mathbb{R}, \alpha \in (0; 1) \\ \infty & \text{pre } \delta \in \mathbb{R}, \alpha \in (1; \infty) \end{cases}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow \infty} x(e^{\frac{1}{x}} - 1) = \lim_{x \rightarrow \infty} x(\sqrt[x]{e} - 1) = \ln e = 1$$

$$\lim_{x \rightarrow \infty} \frac{1}{x(e^{\frac{1}{x}} - 1)} = \lim_{x \rightarrow \infty} \frac{1}{x(\sqrt[x]{e} - 1)} = \frac{1}{\ln e} = 1$$

Dôležité limity – ViL

Postupnosti

$$\bullet \lim_{n \rightarrow \infty} \sqrt[n]{n} = \lim_{n \rightarrow \infty} \sqrt[n]{\alpha} = 1 \text{ pre } \alpha > 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{n!}{n^n} = \lim_{n \rightarrow \infty} \frac{n^k}{\alpha^n} = 0 \quad \text{a} \quad \lim_{n \rightarrow \infty} \sqrt[n]{n!} = \lim_{n \rightarrow \infty} \frac{n^n}{n!} = \lim_{n \rightarrow \infty} \frac{\alpha^n}{n^k} = \infty \text{ pre } k \in \mathbb{N}, \alpha \in (1; \infty).$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}\right) = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

$$\bullet \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n}\right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}.$$

$$\bullet \lim_{n \rightarrow \infty} n(\sqrt[n]{e} - 1) = \lim_{n \rightarrow \infty} \frac{1}{n(\sqrt[n]{e}-1)} = 1.$$

$$\bullet \lim_{n \rightarrow \infty} n(\sqrt[n]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0.$$

$$\bullet \lim_{n \rightarrow \infty} \frac{1}{n(\sqrt[n]{\alpha}-1)} = \frac{1}{\ln \alpha} \text{ pre } \alpha > 0, \alpha \neq 1.$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} n(\sqrt[n]{e} - 1) &= \left[\text{Subst. } \sqrt[n]{e} - 1 = x, n \rightarrow \infty (n \in \mathbb{N}). \Rightarrow x \rightarrow 0 (x \in \mathbb{R}). \right. \\ &\quad \left. \sqrt[n]{e} = e^{\frac{1}{n}} = x + 1. \Rightarrow \frac{1}{n} = \ln e^{\frac{1}{n}} = \ln(x + 1). \Rightarrow n = \frac{1}{\ln(x+1)}. \right] \\ &= \lim_{x \rightarrow 0} \frac{x}{\ln(x+1)} = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{x} \ln(x+1)} = \lim_{x \rightarrow 0} \frac{1}{\ln(x+1)^{\frac{1}{x}}} = \left[\text{ViL: } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e. \right] = \frac{1}{\ln e} = \frac{1}{1} = 1. \end{aligned}$$

$$\begin{aligned} \bullet \lim_{n \rightarrow \infty} n(\sqrt[n]{\alpha} - 1) &= \left[\text{Subst. } \sqrt[n]{\alpha} - 1 = x, n \rightarrow \infty (n \in \mathbb{N}). \Rightarrow x \rightarrow 0 (x \in \mathbb{R}). \right. \\ &\quad \left. \sqrt[n]{\alpha} = \alpha^{\frac{1}{n}} = x + 1. \Rightarrow \frac{1}{n} = \log_\alpha \alpha^{\frac{1}{n}} = \log_\alpha (x + 1) = \frac{\ln(x+1)}{\ln \alpha}. \Rightarrow n = \frac{\ln \alpha}{\ln(x+1)}. \right] \\ &= \lim_{x \rightarrow 0} \frac{x \ln \alpha}{\ln(x+1)} = \lim_{x \rightarrow 0} \frac{\ln \alpha}{\frac{1}{x} \ln(x+1)} = \lim_{x \rightarrow 0} \frac{\ln \alpha}{\ln(x+1)^{\frac{1}{x}}} = \left[\text{ViL: } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e. \right] = \frac{\ln \alpha}{\ln e} = \frac{\ln \alpha}{1} = \ln \alpha \text{ pre } \alpha > 0. \end{aligned}$$

Riešené příklady – 01, 02, 03

$$L = \lim_{n \rightarrow \infty} n^q \quad \text{pre } q \in \mathbb{R}. \quad [01]$$

$$L = \lim_{n \rightarrow \infty} \frac{1}{n^q} \quad \text{pre } q \in \mathbb{R}. \quad [02]$$

$$\lim_{n \rightarrow \infty} (n^2 - n) \quad [03]$$

Riešené príklady – 01, 02, 03

$$L = \lim_{n \rightarrow \infty} n^q \quad \text{pre } q \in \mathbb{R}. \quad [01]$$

Pre $q = 0$ platí:

Pre $q > 0$, $n \in \mathbb{N}$ platí $n^q < (n+1)^q$:

Pre $q < 0$, $n \in \mathbb{N}$ platí $p = -q > 0$, $n^q = \frac{1}{n^p} = \frac{1}{n^p}$:

$$L = \lim_{n \rightarrow \infty} \frac{1}{n^q} \quad \text{pre } q \in \mathbb{R}. \quad [02]$$

Pre $q = 0$ platí:

Pre $q > 0$, $n \in \mathbb{N}$ platí $n^q < (n+1)^q$:

Pre $q < 0$, $n \in \mathbb{N}$ platí $p = -q > 0$, $\frac{1}{n^q} = n^{-q} = n^p$:

$$\lim_{n \rightarrow \infty} (n^2 - n) \quad [03]$$

$$\bullet = \lim_{n \rightarrow \infty} n(n-1)$$

$$\bullet = \lim_{n \rightarrow \infty} n^2 \left(1 - \frac{1}{n}\right)$$

Riešené príklady – 01, 02, 03

$$L = \lim_{n \rightarrow \infty} n^q = \begin{cases} 0 & \text{pre } q < 0, \\ 1 & \text{pre } q = 0, \\ \infty & \text{pre } q > 0. \end{cases} \quad [01]$$

Pre $q = 0$ platí:

$$\bullet L = \lim_{n \rightarrow \infty} n^0 = \lim_{n \rightarrow \infty} 1 = 1.$$

Pre $q > 0$, $n \in \mathbb{N}$ platí $n^q < (n+1)^q$:

$$\bullet L = \infty.$$

[Postupnosť $\{n^q\}_{n=1}^{\infty}$ je rastúca, neohraničená zhora. $\Rightarrow \lim_{n \rightarrow \infty} n^q = \infty$.]

Pre $q < 0$, $n \in \mathbb{N}$ platí $p = -q > 0$, $n^q = \frac{1}{n^p} = \frac{1}{n^p}$:

$$\bullet L = \lim_{n \rightarrow \infty} \frac{1}{n^p} = \frac{1}{\infty} = 0.$$

$$L = \lim_{n \rightarrow \infty} \frac{1}{n^q} = \begin{cases} \infty & \text{pre } q < 0, \\ 1 & \text{pre } q = 0, \\ 0 & \text{pre } q > 0. \end{cases} \quad [02]$$

Pre $q = 0$ platí:

$$\bullet L = \lim_{n \rightarrow \infty} \frac{1}{n^0} = \lim_{n \rightarrow \infty} 1 = 1.$$

Pre $q > 0$, $n \in \mathbb{N}$ platí $n^q < (n+1)^q$:

$$\bullet L = \frac{1}{\lim_{n \rightarrow \infty} n^q} = \frac{1}{\infty} = 0.$$

[Postupnosť $\{n^q\}_{n=1}^{\infty}$ je rastúca, neohraničená zhora. $\Rightarrow \lim_{n \rightarrow \infty} n^q = \infty$.]

Pre $q < 0$, $n \in \mathbb{N}$ platí $p = -q > 0$, $\frac{1}{n^q} = n^{-q} = n^p$:

$$\bullet L = \lim_{n \rightarrow \infty} n^p = \infty.$$

$$\lim_{n \rightarrow \infty} (n^2 - n) = \infty \quad [03]$$

$$\bullet = \lim_{n \rightarrow \infty} n(n-1) = \infty \cdot (\infty - 1) = \infty \cdot \infty = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} n^2 \left(1 - \frac{1}{n}\right) = \infty \cdot \left(1 - \frac{1}{\infty}\right) = \infty \cdot (1 - 0) = \infty \cdot 1 = \infty.$$

Riešené príklady – 04

$$L = \lim_{n \rightarrow \infty} a^n \quad \text{pre } a \in \mathbb{R}.$$

[04]

Riešené príklady – 04

$$L = \lim_{n \rightarrow \infty} a^n \quad \text{pre } a \in \mathbb{R}.$$

[04]

Pre $a \in (1; \infty)$, $n \in \mathbb{N}$ platí $a^n < a^{n+1}$:

Pre $a = 1$, $n \in \mathbb{N}$ platí $a^n = 1^n = 1$:

Pre $a \in (-1; 1)$, $n \in \mathbb{N}$ platí $|a^n| > |a^{n+1}|$:

Pre $a = -1$, $n = 2k$ ($n \in \mathbb{N}$ párne) platí $a^{2k} = (-1)^{2k} = 1$ pre $n = 2k$ (párne), $a^{2k+1} = (-1)^{2k+1} = -1$ pre $n = 2k + 1$ (nepárne), pre všetky $k \in \mathbb{N}$:

Pre $a \in (-\infty; -1)$ platí $1 < a^{2k} < a^{2(k+1)}$ (hodnoty rastú a sú neohraničené zhora) pre $n = 2k$ (párne),
 $-1 > a^{2k-1} > a^{2(k+1)-1}$ (hodnoty klesajú a sú neohraničené zdola) pre $n = 2k - 1$ (nepárne), pre všetky $k \in \mathbb{N}$:

Riešené príklady – 04

$$L = \lim_{n \rightarrow \infty} a^n = \begin{cases} \infty & \text{pre } a \in (1; \infty), \\ 1 & \text{pre } a = 1, \\ 0 & \text{pre } a \in (-1; 1), \\ \# & \text{pre } a \in (-\infty; -1). \end{cases} \quad [04]$$

Pre $a \in (1; \infty)$, $n \in \mathbb{N}$ platí $a^n < a^{n+1}$:

- $L = \infty$.

[Postupnosť $\{a^n\}_{n=1}^{\infty}$ je rastúca, neohraničená zhora. $\Rightarrow \lim_{n \rightarrow \infty} a^n = \infty$.]

Pre $a = 1$, $n \in \mathbb{N}$ platí $a^n = 1^n = 1$:

- $L = \lim_{n \rightarrow \infty} 1^n = \lim_{n \rightarrow \infty} 1 = 1$.

[Postupnosť $\{a^n\}_{n=1}^{\infty} = \{1\}_{n=1}^{\infty}$ je konštantná.]

Pre $a \in (-1; 1)$, $n \in \mathbb{N}$ platí $|a^n| > |a^{n+1}|$:

- $L = 0$.

[Postupnosť $\{|a^n|\}_{n=1}^{\infty} \rightarrow 0$ je klesajúca, ohraničená zdola.] $\left[\lim_{n \rightarrow \infty} |a^n| = 0 \Rightarrow \lim_{n \rightarrow \infty} a^n = 0 \right]$

Pre $a = -1$, $n = 2k$ ($n \in \mathbb{N}$ párne) platí $a^{2k} = (-1)^{2k} = 1$ pre $n = 2k$ (párne), $a^{2k+1} = (-1)^{2k+1} = -1$ pre $n = 2k+1$ (nepárne), pre všetky $k \in \mathbb{N}$:

- L **neexistuje**.

[Postupnosť $\{a^n\}_{n=1}^{\infty}$ má dve hromadné hodnoty ± 1 , $\lim_{n \rightarrow \infty} a^n$ neexistuje, postupnosť $\{a^n\}_{n=1}^{\infty}$ osciluje.]

Pre $a \in (-\infty; -1)$ platí $1 < a^{2k} < a^{2(k+1)}$ (hodnoty rastú a sú neohraničené zhora) pre $n = 2k$ (párne),

$-1 > a^{2k+1} > a^{2(k+1)+1}$ (hodnoty klesajú a sú neohraničené zdola) pre $n = 2k+1$ (nepárne), pre všetky $k \in \mathbb{N}$:

- L **neexistuje**.

[Postupnosť $\{a^n\}_{n=1}^{\infty}$ má dve hromadné hodnoty $\pm \infty$, $\lim_{n \rightarrow \infty} a^n$ neexistuje, postupnosť $\{a^n\}_{n=1}^{\infty}$ osciluje.]

Riešené príklady – 05, 06

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^2 - 2}$$

[05]

$$\lim_{n \rightarrow \infty} \frac{n^3 - 2}{n^2 + n}$$

[06]

Riešené príklady – 05, 06

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2}$$

[05]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$$\lim_{n \rightarrow \infty} \frac{n^3-2}{n^2+n}$$

[06]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3(2-\frac{2}{n^3})}{n^2(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{n(2-\frac{2}{n^3})}{1+\frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(2n-\frac{2}{n})}{n^2(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2n-\frac{2}{n}}{1+\frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}}$$

Riešené príklady – 05, 06

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} = 1$$

[05]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{2}{\infty}} = \frac{1+0}{1-0} = 1.$$

$$\lim_{n \rightarrow \infty} \frac{n^3-2}{n^2+n} = \infty$$

[06]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3(2-\frac{2}{n^3})}{n^2(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{n(2-\frac{2}{n^3})}{1+\frac{1}{n}} = \frac{\infty(2-\frac{2}{\infty})}{1+\frac{1}{\infty}} = \frac{\infty(2-0)}{1+0} = \frac{\infty \cdot 2}{1} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(2n-\frac{2}{n^2})}{n^2(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{2\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{2\infty-0}{1+0} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$$

Riešené príklady – 07, 08

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

[07]

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 + n^3}$$

[08]

Riešené príklady – 07, 08

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3}$$

[07]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n})}{n^4(1 - \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{n^2(1 - \frac{1}{n})}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n})}{n^2(n^2 - n)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{n^2 - n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 - \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 + n^3}$$

[08]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n})}{n^4(1 + \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{n^2(1 + \frac{1}{n})}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n})}{n^2(n^2 + n)} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{n^2 + n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2 + n}{n^4 + n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{1 + \frac{1}{n}}$$

Riešené príklady – 07, 08

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^4-n^3} = 0$$

[07]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^4(1-\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{n^2(1-\frac{1}{n})} = \frac{1+\frac{1}{\infty}}{\infty(1-\frac{1}{\infty})} = \frac{1+0}{\infty(1-0)} = \frac{1}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(n^2-n)} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{n^2-n} = \frac{1+\frac{1}{\infty}}{\infty-\infty} = \frac{1+0}{??}.$$

[Postup nemôžeme použiť.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{1}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n^2+n}{n^4+n^3} = 0$$

[08]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^4(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{n^2(1+\frac{1}{n})} = \frac{1+\frac{1}{\infty}}{\infty(1+\frac{1}{\infty})} = \frac{1+0}{\infty(1+0)} = \frac{1}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{n^2(n^2+n)} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{n^2+n} = \frac{1+\frac{1}{\infty}}{\infty+\infty} = \frac{1+0}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2+n}{n^4+n^3} \cdot \frac{1}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1+\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1+\frac{1}{\infty}} = \frac{0+0}{1+0} = 0.$$

Riešené príklady – 09, 10

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

[09]

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

[10]

Riešené príklady – 09, 10

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2}$$

[09]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{2}{n} + 3 + \frac{5}{n^3})}{n^4 \cdot (\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5}$$

[10]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4})}{n^3(\frac{2}{n} - 3 + \frac{5}{n^3})} = \lim_{n \rightarrow \infty} \frac{n(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4})}{\frac{2}{n} - 3 + \frac{5}{n^3}}$$

Riešené príklady – 09, 10

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} = 0$$

[09]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{1 - n - \frac{1}{n} + \frac{2}{n^3}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{1 - \infty - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{1 - \infty - 0 + 0} = \frac{3}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{2}{n} + 3 + \frac{5}{n^3})}{n^4 \cdot (\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 + \frac{5}{n^3}}{n(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})} \\ = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\infty \cdot (\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty})} = \frac{0 + 3 + 0}{\infty \cdot (0 - 1 - 0 + 0)} = \frac{3}{\infty \cdot (-1)} = \frac{3}{-\infty} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} = \infty$$

[10]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 - n^4 + 2n^2 + 2}{2n^2 - 3n^3 + 5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{1 - n + \frac{2}{n} + \frac{2}{n^3}}{\frac{2}{n} - 3 + \frac{5}{n^3}} = \frac{1 - \infty + \frac{2}{\infty} + \frac{2}{\infty}}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{1 - \infty + 0 + 0}{0 - 3 + 0} = \frac{-\infty}{-3} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4})}{n^3(\frac{2}{n} - 3 + \frac{5}{n^3})} = \lim_{n \rightarrow \infty} \frac{n(\frac{1}{n} - 1 + \frac{2}{n^2} + \frac{2}{n^4})}{\frac{2}{n} - 3 + \frac{5}{n^3}} \\ = \frac{\infty \cdot (\frac{1}{\infty} - 1 + \frac{2}{\infty} + \frac{2}{\infty})}{\frac{2}{\infty} - 3 + \frac{5}{\infty}} = \frac{\infty \cdot (0 - 1 + 0 + 0)}{0 - 3 + 0} = \frac{\infty \cdot (-1)}{-3} = \frac{\infty}{-3} = \infty.$$

Riešené príklady – 11

$$\lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}$$

[11]

Riešené príklady – 11

$$\lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}$$

[11]

$$\bullet = \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}{\frac{4}{n} - 2 + \frac{3}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4}}{\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{n^3 \cdot (\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{n \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{\frac{4}{n} - 2 + \frac{3}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{n^3 \cdot (\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}{\frac{4}{n} - 2 + \frac{3}{n^3}}$$

Riešené príklady – 11

$$\lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} = -\infty$$

[11]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}{\frac{4}{n} - 2 + \frac{3}{n^3}} = \frac{4 + 5 \cdot \infty + \frac{6}{\infty} + \frac{3}{\infty}}{\frac{4}{\infty} - 2 + \frac{3}{\infty}} \\ &= \frac{4 + \infty + 0 + 0}{0 - 2 + 0} = \frac{\infty}{-2} = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4}}{\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4}} = \frac{\frac{4}{\infty} + 5 + \frac{6}{\infty} + \frac{3}{\infty}}{\frac{4}{\infty} - \frac{2}{\infty} + \frac{3}{\infty}} \\ &= \frac{0 + 5 + 0 + 0}{0 - 0 + 0} = \frac{5}{0} = ?? \quad \left[\text{Postup nemôžeme použiť.} \right] \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^4 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})} = \lim_{n \rightarrow \infty} \frac{n \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{\frac{4}{n} - 2 + \frac{3}{n^3}} = \frac{\infty \cdot (\frac{4}{\infty} + 5 + \frac{6}{\infty} + \frac{3}{\infty})}{\frac{4}{\infty} - 2 + \frac{3}{\infty}} \\ &= \frac{\infty(0 + 5 + 0 + 0)}{0 - 2 + 0} = \frac{\infty \cdot 5}{-2} = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})} = \lim_{n \rightarrow \infty} \frac{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}{\frac{4}{n} - 2 + \frac{3}{n^3}} = \frac{4 + 5 \cdot \infty + \frac{6}{\infty} + \frac{3}{\infty}}{\frac{4}{\infty} - 2 + \frac{3}{\infty}} \\ &= \frac{4 + \infty + 0 + 0}{0 - 2 + 0} = \frac{\infty}{-2} = -\infty. \end{aligned}$$

Riešené príklady – 12

$$\lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3}$$

[12]

Riešené príklady – 12

$$\lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3}$$

[12]

$$\bullet = \lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4}}{\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})}{n^4 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{n \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})}{n^3 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{4 + 5n + \frac{6}{n} + \frac{3}{n^3}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}} = \frac{1}{\lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}}$$

Riešené příklady – 12

$$\lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3} = 0$$

[12]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{4 + 5n + \frac{6}{n} + \frac{3}{n^3}} \\ &= \frac{\frac{4}{\infty} - 2 + \frac{3}{\infty}}{4 + 5 \cdot \infty + \frac{6}{\infty} + \frac{3}{\infty}} = \frac{0 - 2 + 0}{4 + \infty + 0 + 0} = \frac{-2}{\infty} = 0. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{4n^2 - 2n^3 + 3}{4n^3 + 5n^4 + 6n^2 + 3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n^2} - \frac{2}{n} + \frac{3}{n^4}}{\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4}} = \frac{\frac{4}{\infty} - \frac{2}{\infty} + \frac{3}{\infty}}{\frac{4}{\infty} + 5 + \frac{6}{\infty} + \frac{3}{\infty}} = \frac{0 - 0 + 0}{0 + 5 + 0 + 0} = \frac{0}{5} = 0.$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})}{n^4 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{n \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})} \\ &= \frac{\frac{4}{\infty} - 2 + \frac{3}{\infty}}{\infty \cdot (\frac{4}{\infty} + 5 + \frac{6}{\infty} + \frac{3}{\infty})} = \frac{0 - 2 + 0}{\infty \cdot (0 + 5 + 0 + 0)} = \frac{-2}{\infty \cdot 5} = 0. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3 \cdot (\frac{4}{n} - 2 + \frac{3}{n^3})}{n^3 \cdot (\frac{4}{n} + 5 + \frac{6}{n^2} + \frac{3}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{4}{n} - 2 + \frac{3}{n^3}}{4 + 5n + \frac{6}{n} + \frac{3}{n^3}} = \frac{\frac{4}{\infty} - 2 + \frac{3}{\infty}}{4 + 5 \cdot \infty + \frac{6}{\infty} + \frac{3}{\infty}} = \frac{0 - 2 + 0}{4 + \infty + 0 + 0} = \frac{-2}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}} = \frac{1}{\lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3}} = \left[\text{Pr. 11: } \lim_{n \rightarrow \infty} \frac{4n^3 + 5n^4 + 6n^2 + 3}{4n^2 - 2n^3 + 3} = -\infty. \right] = \frac{1}{-\infty} = 0.$$

Riešené príklady – 13, 14

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^4 - n^2 + 2}$$

[13]

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^5 - n^2 + n + 2}$$

[14]

Riešené príklady – 13, 14

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^4 - n^2 + 2}$$

[13]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4(\frac{2}{n^2} + 3 + \frac{5}{n^4})}{n^4(\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^5 - n^2 + n + 2}$$

[14]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^5 - n^2 + n + 2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4(\frac{2}{n^2} + 3 + \frac{5}{n^4})}{n^4(\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4}}$$

Riešené príklady – 13, 14

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^4 - n^2 + 2} = -3$$

[13]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^4 - n^2 + 2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{0 - 1 - 0 + 0} = \frac{3}{-1} = -3.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 (\frac{2}{n^2} + 3 + \frac{5}{n^4})}{n^4 (\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - 1 - \frac{1}{n^2} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - 1 - \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{0 - 1 - 0 + 0} = \frac{3}{-1} = -3.$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^5 - n^2 + n + 2} = 0$$

[14]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^4 + 5}{n^3 - n^5 - n^2 + n + 2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - \infty - \frac{1}{\infty} + \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{0 - \infty - 0 + 0 + 0} = \frac{3}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 (\frac{2}{n^2} + 3 + \frac{5}{n^4})}{n^4 (\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4})} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 + \frac{5}{n^4}}{\frac{1}{n} - n - \frac{1}{n^2} + \frac{1}{n^3} + \frac{2}{n^4}} = \frac{\frac{2}{\infty} + 3 + \frac{5}{\infty}}{\frac{1}{\infty} - \infty - \frac{1}{\infty} + \frac{1}{\infty} + \frac{2}{\infty}} = \frac{0 + 3 + 0}{0 - \infty - 0 + 0 + 0} = \frac{3}{-\infty} = 0.$$

Riešené príklady – 15, 16, 17

$$\lim_{n \rightarrow \infty} \left[\frac{n^2}{n+2} - \frac{n^2}{n+3} \right]$$

[15]

$$\lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^5+1}$$

[16]

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

[17]

Riešené příklady – 15, 16, 17

$$\lim_{n \rightarrow \infty} \left[\frac{n^2}{n+2} - \frac{n^2}{n+3} \right]$$

[15]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2(1+\frac{2}{n})(1+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{1}{(1+\frac{2}{n})(1+\frac{3}{n})}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^5+1}$$

[16]

$$\bullet = \lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^5+1} \cdot \frac{\frac{1}{n} \cdot \frac{1}{n} \cdot \frac{1}{n} \cdot \frac{1}{n}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})(1+\frac{2}{n})(1+\frac{3}{n})(1+\frac{4}{n})}{n+\frac{1}{n^4}}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!}$$

[17]

$$\bullet = \lim_{n \rightarrow \infty} \left[\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right] = \lim_{n \rightarrow \infty} \left[\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right]$$

$$= \lim_{n \rightarrow \infty} \left[(n+4) - \frac{1}{n+3} \right]$$

Riešené příklady – 15, 16, 17

$$\lim_{n \rightarrow \infty} \left[\frac{n^2}{n+2} - \frac{n^2}{n+3} \right] = 1 \quad [15]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{n^2(n+3) - n^2(n+2)}{(n+2)(n+3)} = \lim_{n \rightarrow \infty} \frac{n^2(n+3-n-2)}{n^2(1+\frac{2}{n})(1+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{1}{(1+\frac{2}{n})(1+\frac{3}{n})} = \frac{1}{(1+\frac{2}{\infty})(1+\frac{3}{\infty})} \\ &= \frac{1}{(1+0)(1+0)} = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^5+1} = 0 \quad [16]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{(n+1)(n+2)(n+3)(n+4)}{n^5+1} \cdot \frac{\frac{1}{n} \cdot \frac{1}{n} \cdot \frac{1}{n} \cdot \frac{1}{n}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})(1+\frac{2}{n})(1+\frac{3}{n})(1+\frac{4}{n})}{n+\frac{1}{n^4}} \\ &= \frac{(1+\frac{1}{\infty})(1+\frac{2}{\infty})(1+\frac{3}{\infty})(1+\frac{4}{\infty})}{\infty+\frac{1}{\infty}} = \frac{(1+0)(1+0)(1+0)(1+0)}{\infty+0} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{(n+4)! - (n+2)!}{(n+3)!} = \infty \quad [17]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\frac{(n+4)!}{(n+3)!} - \frac{(n+2)!}{(n+3)!} \right] = \lim_{n \rightarrow \infty} \left[\frac{(n+4)(n+3)!}{(n+3)!} - \frac{(n+2)!}{(n+3)(n+2)!} \right] \\ &= \lim_{n \rightarrow \infty} \left[(n+4) - \frac{1}{n+3} \right] = \infty + 4 - \frac{1}{\infty} = \infty - 0 = \infty. \end{aligned}$$

Riešené príklady – 18, 19, 20

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

[18]

$$\lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n}$$

[19]

$$\lim_{n \rightarrow \infty} \frac{n + 3^{-n}}{n - 3^{-n}}$$

[20]

Riešené príklady – 18, 19, 20

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}}$$

[18]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1}$$

$$\lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n}$$

[19]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \left[\begin{array}{l} a_n = \frac{n}{3^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0. \\ a_n = \frac{n}{3^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0. \end{array} \right] \cdot \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1}$$

$$\lim_{n \rightarrow \infty} \frac{n + 3^{-n}}{n - 3^{-n}}$$

[20]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}}$$

Riešené příklady – 18, 19, 20

$$\lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} = -1$$

[18]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n}} - 1}{\frac{1}{\sqrt{n}} + 1} = \frac{\frac{1}{\sqrt{\infty}} - 1}{\frac{1}{\sqrt{\infty}} + 1} = \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1} = \frac{0 - 1}{0 + 1} = -1.$$

$$\lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n} = -1$$

[19]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \left[\begin{array}{l} a_n = \frac{n}{3^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{3n} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0. \\ a_n = \frac{n}{3^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{3^n} = 0. \end{array} \right] \cdot \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1} = \frac{0 + 1}{0 - 1} = -1.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + 3^n}{n - 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{n}{3^n} + 1}{\frac{n}{3^n} - 1} = \left[\text{ViL: } \lim_{n \rightarrow \infty} \frac{\alpha^{\vartheta}}{\alpha^{\vartheta}} = 0 \text{ pre } \vartheta \in \mathbb{N}, \alpha \in (1; \infty). \right] = \frac{0 + 1}{0 - 1} = -1.$$

$$\lim_{n \rightarrow \infty} \frac{n + 3^{-n}}{n - 3^{-n}} = 1$$

[20]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{n + \frac{1}{3^n}}{n - \frac{1}{3^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n \cdot 3^n}}{1 - \frac{1}{n \cdot 3^n}} = \frac{1 + \frac{1}{\infty \cdot \infty}}{1 - \frac{1}{\infty \cdot \infty}} = \frac{1 + \frac{1}{\infty}}{1 - \frac{1}{\infty}} = \frac{1 + 0}{1 - 0} = 1.$$

Riešené príklady – 21

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

[21]

Riešené príklady – 21

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3}$$

[21]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\frac{\sqrt[3]{n}}{\sqrt{n}} + 3\frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[3]{n} = \sqrt[6]{n^2}, \quad \sqrt[4]{n} = \sqrt[10]{n^2}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[4]{n^2} = \sqrt[6]{n^3} = \sqrt[10]{n^5}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}} + 3\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}} + 3\sqrt[4]{\frac{n}{n^2}} - 1}{2 - \sqrt[10]{\frac{n^2}{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\sqrt[6]{\frac{1}{n}} + 3\sqrt[4]{\frac{1}{n}} - 1}{2 - \sqrt[10]{\frac{1}{n^3}} + \frac{3}{\sqrt{n}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{1/6}} + \frac{3}{n^{1/4}} - 1}{2 - \frac{1}{n^{3/10}} + \frac{3}{n^{1/2}}} \end{aligned}$$

Riešené príklady – 21

$$\lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} = -\frac{1}{2}$$

[21]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2\sqrt[3]{n} + 3\sqrt[4]{n} - \sqrt{n}}{2\sqrt{n} - \sqrt[5]{n} + 3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\frac{\sqrt[3]{n}}{\sqrt{n}} + 3\frac{\sqrt[4]{n}}{\sqrt{n}} - 1}{2 - \frac{\sqrt[5]{n}}{\sqrt{n}} + \frac{3}{\sqrt{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[3]{n} = \sqrt[6]{n^2}, \quad \sqrt[4]{n} = \sqrt[10]{n^2}, \right. \\ &\quad \left. \sqrt{n} = \sqrt[4]{n^2} = \sqrt[6]{n^3} = \sqrt[10]{n^5} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}} + 3\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}} - 1}{2 - \frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}} + 3\sqrt[4]{\frac{n}{n^2}} - 1}{2 - \sqrt[10]{\frac{n^2}{n^5}} + \frac{3}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{2\sqrt[6]{\frac{1}{n}} + 3\sqrt[4]{\frac{1}{n}} - 1}{2 - \sqrt[10]{\frac{1}{n^3}} + \frac{3}{\sqrt{n}}} \\ &= \frac{2\sqrt[6]{\frac{1}{\infty}} + 3\sqrt[4]{\frac{1}{\infty}} - 1}{2 - \sqrt[10]{\frac{1}{\infty}} + \frac{3}{\sqrt{\infty}}} = \frac{2\sqrt[6]{0} + 3\sqrt[4]{0} - 1}{2 - \sqrt[10]{0} + 0} = \frac{0 + 0 - 1}{2 - 0 + 0} = -\frac{1}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3}} + 3n^{\frac{1}{4}} - n^{\frac{1}{2}}}{2n^{\frac{1}{2}} - n^{\frac{1}{5}} + 3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{1}{3} - \frac{1}{2}} + 3n^{\frac{1}{4} - \frac{1}{2}} - 1}{2 - n^{\frac{1}{5} - \frac{1}{2}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{2n^{\frac{2-3}{6}} + 3n^{\frac{1-2}{4}} - 1}{2 - n^{\frac{2-5}{10}} + 3n^{-\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{2n^{-\frac{1}{6}} + 3n^{-\frac{1}{4}} - 1}{2 - n^{-\frac{3}{10}} + 3n^{-\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^{\frac{1}{6}}} + \frac{3}{n^{\frac{1}{4}}} - 1}{2 - \frac{1}{n^{\frac{3}{10}}} + \frac{3}{n^{\frac{1}{2}}}} \\ &= \frac{\frac{2}{\infty^{\frac{1}{6}}} + \frac{3}{\infty^{\frac{1}{4}}} - 1}{2 - \frac{1}{\infty^{\frac{3}{10}}} + \frac{3}{\infty^{\frac{1}{2}}}} = \frac{\frac{2}{\infty} + \frac{3}{\infty} - 1}{2 - \frac{1}{\infty} + \frac{3}{\infty}} = \frac{0 + 0 - 1}{2 - 0 + 0} = -\frac{1}{2}. \end{aligned}$$

Riešené príklady – 22

$$\lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n} + 4\sqrt[4]{n} - 2}{3\sqrt[4]{n} + 4\sqrt[5]{n} + 5\sqrt[3]{n} + 6}$$

[22]

Riešené príklady – 22

$$\lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n} + 4\sqrt[4]{n} - 2}{3\sqrt[4]{n} + 4\sqrt[5]{n} + 5\sqrt[3]{n} + 6}$$

[22]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n} + 4\sqrt[4]{n} - 2}{3\sqrt[4]{n} + 4\sqrt[5]{n} + 5\sqrt[3]{n} + 6} \cdot \frac{\frac{1}{\sqrt[3]{n}}}{\frac{1}{\sqrt[3]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[4]{n}}{\sqrt[3]{n}} - \frac{2}{\sqrt[3]{n}}}{3\frac{\sqrt[4]{n}}{\sqrt[3]{n}} + 4\frac{\sqrt[5]{n}}{\sqrt[3]{n}} + 5 + \frac{6}{\sqrt[3]{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[4]{n} = \sqrt[12]{n^3}, \sqrt[5]{n} = \sqrt[15]{n^3}, \right. \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4\frac{12\sqrt[12]{n^3}}{\sqrt[12]{n^4}} - \frac{2}{\sqrt[3]{n}}}{3\frac{12\sqrt[12]{n^3}}{\sqrt[12]{n^4}} + 4\frac{15\sqrt[15]{n^3}}{\sqrt[15]{n^5}} + 5 + \frac{6}{\sqrt[3]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[12]{\frac{1}{n}} - \frac{2}{\sqrt[3]{n}}}{3\sqrt[12]{\frac{1}{n}} + 4\sqrt[15]{\frac{1}{n^2}} + 5 + \frac{6}{\sqrt[3]{n}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3n^{\frac{1}{3}} + 4n^{\frac{1}{4}} - 2}{3n^{\frac{1}{4}} + 4n^{\frac{1}{5}} + 5n^{\frac{1}{3}} + 6} \cdot \frac{n^{-\frac{1}{3}}}{n^{-\frac{1}{3}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{1}{4} - \frac{1}{3}} - 2n^{-\frac{1}{3}}}{3n^{\frac{1}{4} - \frac{1}{3}} + 4n^{\frac{1}{5} - \frac{1}{3}} + 5 + 6n^{-\frac{1}{3}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{3-4}{12}} - 2n^{-\frac{1}{3}}}{3n^{\frac{3-4}{12}} + 4n^{\frac{3-5}{15}} + 5 + 6n^{-\frac{1}{3}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{-\frac{1}{12}} - 2n^{-\frac{1}{3}}}{3n^{-\frac{1}{12}} + 4n^{-\frac{2}{15}} + 5 + 6n^{-\frac{1}{3}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + \frac{4}{n^{1/12}} - \frac{2}{n^{1/3}}}{\frac{3}{n^{1/12}} + \frac{4}{n^{2/15}} + 5 + \frac{6}{n^{1/3}}} \end{aligned}$$

Riešené příklady – 22

$$\lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n} + 4\sqrt[4]{n} - 2}{3\sqrt[4]{n} + 4\sqrt[5]{n} + 5\sqrt[3]{n} + 6} = \frac{3}{5}$$

[22]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3\sqrt[3]{n} + 4\sqrt[4]{n} - 2}{3\sqrt[4]{n} + 4\sqrt[5]{n} + 5\sqrt[3]{n} + 6} \cdot \frac{\frac{1}{\sqrt[3]{n}}}{\frac{1}{\sqrt[3]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[4]{n}}{\sqrt[3]{n}} - \frac{2}{\sqrt[3]{n}}}{3\frac{\sqrt[4]{n}}{\sqrt[3]{n}} + 4\frac{\sqrt[5]{n}}{\sqrt[3]{n}} + 5 + \frac{6}{\sqrt[3]{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[4]{n} = \sqrt[12]{n^3}, \sqrt[5]{n} = \sqrt[15]{n^3}, \sqrt[3]{n} = \sqrt[15]{n^4} = \sqrt[15]{n^5} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4\frac{12\sqrt[12]{n^3}}{\sqrt[15]{n^4}} - \frac{2}{\sqrt[15]{n^5}}}{3\frac{12\sqrt[12]{n^3}}{\sqrt[15]{n^4}} + 4\frac{15\sqrt[15]{n^3}}{\sqrt[15]{n^5}} + 5 + \frac{6}{\sqrt[15]{n^5}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[12]{\frac{n^3}{n^4}} - \frac{2}{\sqrt[15]{n}}}{3\sqrt[12]{\frac{n^3}{n^4}} + 4\sqrt[15]{\frac{n^3}{n^5}} + 5 + \frac{6}{\sqrt[15]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[12]{\frac{1}{n}} - \frac{2}{\sqrt[15]{n}}}{3\sqrt[12]{\frac{1}{n}} + 4\sqrt[15]{\frac{1}{n^2}} + 5 + \frac{6}{\sqrt[15]{n}}} \\ &= \frac{3 + 4\sqrt[12]{\frac{1}{\infty}} - \frac{2}{\sqrt[15]{\infty}}}{3\sqrt[12]{\frac{1}{\infty}} + 4\sqrt[15]{\frac{1}{\infty}} + 5 + \frac{6}{\sqrt[15]{\infty}}} = \frac{3 + 4\sqrt[12]{0} - 0}{3\sqrt[12]{0} + 4\sqrt[15]{0} + 5 + 0} = \frac{3 + 0 - 0}{0 + 0 + 5 + 0} = \frac{3}{5}. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3n^{\frac{1}{3}} + 4n^{\frac{1}{4}} - 2}{3n^{\frac{1}{4}} + 4n^{\frac{1}{5}} + 5n^{\frac{1}{3}} + 6} \cdot \frac{n^{-\frac{1}{3}}}{n^{-\frac{1}{3}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{1}{4} - \frac{1}{3}} - 2n^{-\frac{1}{3}}}{3n^{\frac{1}{4} - \frac{1}{3}} + 4n^{\frac{1}{5} - \frac{1}{3}} + 5 + 6n^{-\frac{1}{3}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4n^{-\frac{1}{12}} - 2n^{-\frac{1}{3}}}{3n^{-\frac{1}{12}} + 4n^{-\frac{2}{15}} + 5 + 6n^{-\frac{1}{3}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + \frac{4}{\infty^{\frac{1}{12}}} - \frac{2}{\infty^{\frac{1}{3}}}}{\frac{3}{\infty^{\frac{1}{12}}} + \frac{4}{\infty^{\frac{2}{15}}} + 5 + \frac{6}{\infty^{\frac{1}{3}}}} = \frac{3 + \frac{4}{\infty} - \frac{2}{\infty}}{\frac{3}{\infty} + \frac{4}{\infty} + 5 + \frac{6}{\infty}} = \frac{3 + 0 - 0}{0 + 0 + 5 + 0} = \frac{3}{5}. \end{aligned}$$

Riešené příklady – 23

$$\lim_{x \rightarrow \infty} \frac{3\sqrt[5]{n} + 4\sqrt[6]{n} + 5\sqrt[4]{n} + 6}{3\sqrt[6]{n} + 4\sqrt[5]{n} - 2}$$

[23]

Riešené príklady – 23

$$\lim_{x \rightarrow \infty} \frac{3\sqrt[5]{n} + 4\sqrt[6]{n} + 5\sqrt[4]{n} + 6}{3\sqrt[6]{n} + 4\sqrt[5]{n} - 2}$$

[23]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3\sqrt[5]{n} + 4\sqrt[6]{n} + 5\sqrt[4]{n} + 6}{3\sqrt[6]{n} + 4\sqrt[5]{n} - 2} \cdot \frac{\frac{1}{\sqrt[5]{n}}}{\frac{1}{\sqrt[5]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[6]{n}}{\sqrt[5]{n}} + 5\frac{\sqrt[4]{n}}{\sqrt[5]{n}} + \frac{6}{\sqrt[5]{n}}}{3\frac{\sqrt[6]{n}}{\sqrt[5]{n}} + 4 - \frac{2}{\sqrt[5]{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[5]{n} = \sqrt[30]{n^6}, \sqrt[4]{n} = \sqrt[20]{n^5}, \sqrt[6]{n} = \sqrt[30]{n^5} = \sqrt[20]{n^4} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[30]{n^5}}{\sqrt[30]{n^6}} + 5\frac{\sqrt[20]{n^5}}{\sqrt[20]{n^4}} + \frac{6}{\sqrt[5]{n^6}}}{3\frac{\sqrt[30]{n^5}}{\sqrt[30]{n^6}} + 4 - \frac{2}{\sqrt[5]{n^6}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[30]{\frac{n^5}{n^6}} + 5\sqrt[20]{\frac{n^5}{n^4}} + \frac{6}{\sqrt[5]{n^6}}}{3\sqrt[30]{\frac{n^5}{n^6}} + 4 - \frac{2}{\sqrt[5]{n^6}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[30]{\frac{1}{n}} + 5\sqrt[20]{n} + \frac{6}{\sqrt[5]{n^6}}}{3\sqrt[30]{\frac{1}{n}} + 4 - \frac{2}{\sqrt[5]{n^6}}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3n^{\frac{1}{5}} + 4n^{\frac{1}{6}} + 5n^{\frac{1}{4}} + 6}{3n^{\frac{1}{6}} + 4n^{\frac{1}{5}} - 2} \cdot \frac{n^{-\frac{1}{5}}}{n^{-\frac{1}{5}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{1}{6} - \frac{1}{5}} + 5n^{\frac{1}{4} - \frac{1}{5}} + 6n^{-\frac{1}{5}}}{3n^{\frac{1}{6} - \frac{1}{5}} + 4 - 2n^{-\frac{1}{5}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{5-6}{30}} + 5n^{\frac{5-4}{20}} + 6n^{-\frac{1}{5}}}{3n^{\frac{5-6}{30}} + 4 - 2n^{-\frac{1}{5}}} = \lim_{n \rightarrow \infty} \frac{3 + \frac{4}{n^{1/30}} + 5n^{1/20} + \frac{6}{n^{1/5}}}{\frac{3}{n^{1/30}} + 4 - \frac{2}{n^{1/5}}} \end{aligned}$$

Riešené príklady – 23

$$\lim_{x \rightarrow \infty} \frac{3\sqrt[5]{n} + 4\sqrt[6]{n} + 5\sqrt[4]{n} + 6}{3\sqrt[6]{n} + 4\sqrt[5]{n} - 2} = \infty$$

[23]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{3\sqrt[5]{n} + 4\sqrt[6]{n} + 5\sqrt[4]{n} + 6}{3\sqrt[6]{n} + 4\sqrt[5]{n} - 2} \cdot \frac{\frac{1}{\sqrt[5]{n}}}{\frac{1}{\sqrt[5]{n}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[6]{n}}{\sqrt[5]{n}} + 5\frac{\sqrt[4]{n}}{\sqrt[5]{n}} + \frac{6}{\sqrt[5]{n}}}{3\frac{\sqrt[6]{n}}{\sqrt[5]{n}} + 4 - \frac{2}{\sqrt[5]{n}}} = \left[n \in \mathbb{N} \Rightarrow \sqrt[5]{n} = \sqrt[30]{n^6}, \sqrt[4]{n} = \sqrt[20]{n^5}, \right. \\ &\quad \left. \sqrt[6]{n} = \sqrt[30]{n^5}, \sqrt[5]{n} = \sqrt[30]{n^6} = \sqrt[20]{n^4} \right] \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4\frac{\sqrt[30]{n^5}}{\sqrt[30]{n^6}} + 5\frac{\sqrt[20]{n^5}}{\sqrt[20]{n^4}} + \frac{6}{\sqrt[30]{n^6}}}{3\frac{\sqrt[30]{n^5}}{\sqrt[30]{n^6}} + 4 - \frac{2}{\sqrt[30]{n^6}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[30]{\frac{n^5}{n^6}} + 5\sqrt[20]{\frac{n^5}{n^4}} + \frac{6}{\sqrt[30]{n^6}}}{3\sqrt[30]{\frac{n^5}{n^6}} + 4 - \frac{2}{\sqrt[30]{n^6}}} = \lim_{n \rightarrow \infty} \frac{3 + 4\sqrt[30]{\frac{1}{n}} + 5\sqrt[20]{\frac{1}{n}} + \frac{6}{\sqrt[30]{n^6}}}{3\sqrt[30]{\frac{1}{n}} + 4 - \frac{2}{\sqrt[30]{n^6}}} \\ &= \frac{3 + 4\sqrt[30]{\frac{1}{\infty}} + 5\sqrt[20]{\frac{1}{\infty}} + \frac{6}{\sqrt[30]{\infty}}}{3\sqrt[30]{\frac{1}{\infty}} + 4 - \frac{2}{\sqrt[30]{\infty}}} = \frac{3 + 4\sqrt[30]{0} + 5 \cdot \infty + 0}{3\sqrt[30]{0} + 4 - 0} = \frac{3 + 0 + \infty + 0}{0 + 4 - 0} = \frac{\infty}{4} = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{3n^{\frac{1}{5}} + 4n^{\frac{1}{6}} + 5n^{\frac{1}{4}} + 6}{3n^{\frac{1}{6}} + 4n^{\frac{1}{5}} - 2} \cdot \frac{n^{-\frac{1}{5}}}{n^{-\frac{1}{5}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{1}{6} - \frac{1}{5}} + 5n^{\frac{1}{4} - \frac{1}{5}} + 6n^{-\frac{1}{5}}}{3n^{\frac{1}{6} - \frac{1}{5}} + 4 - 2n^{-\frac{1}{5}}} \\ &= \lim_{n \rightarrow \infty} \frac{3 + 4n^{\frac{5-6}{30}} + 5n^{\frac{5-4}{20}} + 6n^{-\frac{1}{5}}}{3n^{\frac{5-6}{30}} + 4 - 2n^{-\frac{1}{5}}} = \lim_{n \rightarrow \infty} \frac{3 + 4n^{-\frac{1}{30}} + 5n^{\frac{1}{20}} + 6n^{-\frac{1}{5}}}{3n^{-\frac{1}{30}} + 4 - 2n^{-\frac{1}{5}}} = \lim_{n \rightarrow \infty} \frac{3 + \frac{4}{n^{1/30}} + 5n^{\frac{1}{20}} + \frac{6}{n^{1/5}}}{\frac{3}{n^{1/30}} + 4 - \frac{2}{n^{1/5}}} \\ &= \frac{3 + \frac{4}{\infty^{1/30}} + 5 \cdot \infty^{\frac{1}{20}} + \frac{6}{\infty^{1/5}}}{\frac{3}{\infty^{1/30}} + 4 - \frac{2}{\infty^{1/5}}} = \frac{3 + \frac{4}{\infty} + 5 \cdot \infty + \frac{6}{\infty}}{\frac{3}{\infty} + 4 - \frac{2}{\infty}} = \frac{3 + 0 + \infty + 0}{0 + 4 - 0} = \frac{\infty}{4} = \infty. \end{aligned}$$

Riešené příklady – 24

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}}{n+1}$$

[24]

Riešené príklady – 24

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}}{n+1}$$

[24]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n^3+1}}{\sqrt{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n^3+1}{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} \right] = \lim_{n \rightarrow \infty} \frac{n \sqrt{\frac{n^3+1}{n^2}}}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \cdot \sqrt{\frac{n^3+1}{n^3}}}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \cdot \sqrt{1+\frac{1}{n^3}}}{\sqrt{n^2} \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n} \cdot \sqrt{1+\frac{1}{n^3}}}{1+\frac{1}{n}}$$

Riešené príklady – 24

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}}{n+1} = \infty$$

[24]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}}{n+1} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n^3+1}}{\sqrt{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n^3+1}{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}} = \frac{\sqrt{\infty+\frac{1}{\infty}}}{1+\frac{1}{\infty}} \\ &= \frac{\sqrt{\infty+0}}{1+0} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2}. \right] = \lim_{n \rightarrow \infty} \frac{n \sqrt{\frac{n^3+1}{n^2}}}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\frac{1}{n^2}}}{1+\frac{1}{n}} = \frac{\sqrt{\infty+\frac{1}{\infty}}}{1+\frac{1}{\infty}} \\ &= \frac{\sqrt{\infty+0}}{1+0} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \cdot \sqrt{\frac{n^3+1}{n^3}}}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \cdot \sqrt{1+\frac{1}{n^3}}}{\sqrt{n^2} \cdot (1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{\sqrt{n} \cdot \sqrt{1+\frac{1}{n^3}}}{1+\frac{1}{n}} \\ &= \frac{\sqrt{\infty} \cdot \sqrt{1+\frac{1}{\infty}}}{1+\frac{1}{\infty}} = \frac{\infty \cdot \sqrt{1+0}}{1+0} = \frac{\infty \cdot 1}{1} = \infty. \end{aligned}$$

Riešené príklady – 25

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}}$$

[25]

Riešené príklady – 25

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}}$$

[25]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{n+\sqrt{n+1}}} = \left[\frac{n+1}{n+\sqrt{n+1}} = \frac{n+1}{n+\sqrt{n+1}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1+\frac{1}{n}}{1+\frac{\sqrt{n+1}}{n}} = \frac{1+\frac{1}{n}}{1+\frac{\sqrt{n+1}}{\sqrt{n^2}}} = \frac{1+\frac{1}{n}}{1+\sqrt{\frac{n+1}{n^2}}} = \frac{1+\frac{1}{n}}{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}} \right] \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{1+\frac{1}{n}}{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} = \frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{\frac{\sqrt{n+1}}{\sqrt{n}}}{\frac{\sqrt{n+\sqrt{n+1}}}{\sqrt{n}}} = \frac{\sqrt{\frac{n+1}{n}}}{\sqrt{\frac{n+\sqrt{n+1}}{n}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{\sqrt{n+1}}{\sqrt{n^2}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{\sqrt{n+1}}{\sqrt{n}}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\sqrt{\frac{n+1}{n}}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} = \frac{(n+1)^{\frac{1}{2}}}{[n+(n+1)^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{(n+1)^{\frac{1}{2}}}{[n+(n+1)^{\frac{1}{2}}]^{\frac{1}{2}}} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \frac{[(n+1) \cdot n^{-1}]^{\frac{1}{2}}}{[[n+(n+1)^{\frac{1}{2}}] \cdot n^{-1}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n+1)^{\frac{1}{2}} \cdot (n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+((n+1) \cdot n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n^{-1}+n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n^{-1}+n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{\left[1+\frac{1}{n}\right]^{\frac{1}{2}}}{\left[1+\left(\frac{1}{n}+\frac{1}{n^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}} \end{aligned}$$

Riešené príklady – 25

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} = 1$$

[25]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{n+\sqrt{n+1}}} = \left[\frac{n+1}{n+\sqrt{n+1}} = \frac{n+1}{n+\sqrt{n+1}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1+\frac{1}{n}}{1+\frac{\sqrt{n+1}}{n}} = \frac{1+\frac{1}{n}}{1+\frac{\sqrt{n+1}}{\sqrt{n^2}}} = \frac{1+\frac{1}{n}}{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}} = \frac{1+\frac{1}{n}}{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}} \right] \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{1+\frac{1}{n}}{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} = \sqrt{\frac{1+\frac{1}{\infty}}{1+\sqrt{\frac{1}{\infty}+\frac{1}{\infty}}}} = \sqrt{\frac{1+0}{1+\sqrt{0+0}}} = \sqrt{\frac{1}{1+0}} = 1. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} = \frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{\frac{\sqrt{n+1}}{\sqrt{n}}}{\frac{n+\sqrt{n+1}}{\sqrt{n}}} = \frac{\sqrt{\frac{n+1}{n}}}{\frac{n+\sqrt{n+1}}{\sqrt{n}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{\sqrt{n+1}}{\sqrt{n^2}}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{\sqrt{n+1}}{\sqrt{n}}}} = \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{1}{n}}}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{n^2}}}} = \frac{\sqrt{1+\frac{1}{\infty}}}{\sqrt{1+\sqrt{\frac{1}{\infty}+\frac{1}{\infty}}}} = \frac{\sqrt{1+0}}{\sqrt{1+\sqrt{0+0}}} = \frac{1}{1} = 1. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n+1}}{\sqrt{n+\sqrt{n+1}}} = \frac{(n+1)^{\frac{1}{2}}}{[n+(n+1)^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{(n+1)^{\frac{1}{2}}}{[n+(n+1)^{\frac{1}{2}}]^{\frac{1}{2}}} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \frac{[(n+1) \cdot n^{-1}]^{\frac{1}{2}}}{[[n+(n+1)^{\frac{1}{2}}] \cdot n^{-1}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n+1)^{\frac{1}{2}} \cdot (n^{-1})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+((n+1) \cdot n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n^{-1}+n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{[1+n^{-1}]^{\frac{1}{2}}}{[1+(n^{-1}+n^{-2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{[1+\frac{1}{n}]^{\frac{1}{2}}}{[1+(\frac{1}{n}+\frac{1}{n^2})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+\frac{1}{\infty}]^{\frac{1}{2}}}{[1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{[1+0]^{\frac{1}{2}}}{[1+(0+0)^{\frac{1}{2}}]^{\frac{1}{2}}} \\ &= \frac{1}{1} = 1. \end{aligned}$$

Riešené príklady – 26

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}}$$

[26]

Riešené príklady – 26

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}}$$

[26]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{\frac{n}{n + \sqrt{n + \sqrt{n}}}} = \left[\frac{n}{n + \sqrt{n + \sqrt{n}}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1}{1 + \frac{\sqrt{n + \sqrt{n}}}{n}} = \frac{1}{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}} = \frac{1}{1 + \sqrt{\frac{n + \sqrt{n}}{n^2}}} = \frac{1}{1 + \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{n^2}}} = \frac{1}{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{1}{\sqrt{\frac{n + \sqrt{n + \sqrt{n}}}{n}}} = \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{n}}} = \frac{1}{\sqrt{1 + \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}}} \right] \left(\ast \right) = \frac{\sqrt{n + \sqrt{n}}}{n} = \frac{\sqrt{n + \sqrt{n}}}{\sqrt{n^2}} = \sqrt{\frac{n + \sqrt{n}}{n^2}} = \sqrt{\frac{1}{n} + \frac{\sqrt{n}}{n^2}} = \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}} = \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{n} + \sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \sqrt{\frac{1}{\infty} + \sqrt{\frac{1}{\infty}}}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n}}{\sqrt{n + \sqrt{n + \sqrt{n}}}} = \frac{n^{\frac{1}{2}}}{[n + (n + n^{\frac{1}{2}})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}} \left[\frac{n + (n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n} \right]^{\frac{1}{2}}} = \frac{1}{\left[1 + \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n} \right]^{\frac{1}{2}}} = \frac{1}{(1 + \ast)^{\frac{1}{2}}} \right] \left(\ast \right) = \frac{(n + n^{\frac{1}{2}})^{\frac{1}{2}}}{n} = \left(\frac{n + n^{\frac{1}{2}}}{n^2} \right)^{\frac{1}{2}} = \left(\frac{1}{n} + \frac{n^{\frac{1}{2}}}{n^2} \right)^{\frac{1}{2}} = \left(\frac{1}{n} + n^{\frac{1}{2} - 2} \right)^{\frac{1}{2}} \\ &= \left(\frac{1}{n} + n^{-\frac{3}{2}} \right)^{\frac{1}{2}} = \left(\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}} \right)^{\frac{1}{2}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\left[1 + \left(\frac{1}{n} + \frac{1}{n^{\frac{3}{2}}} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}} \end{aligned}$$

Riešené príklady – 26

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = 1$$

[26]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{\frac{n}{n+\sqrt{n+\sqrt{n}}}} = \left[\frac{n}{n+\sqrt{n+\sqrt{n}}} = \frac{n}{n+\sqrt{n+\sqrt{n}}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1}{1+\frac{\sqrt{n+\sqrt{n}}}{n}} = \frac{1}{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}} = \frac{1}{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}} = \frac{1}{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{n^2}}} = \frac{1}{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}} = \frac{1}{\sqrt{1+0}} = 1. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{n}}} = \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}} \right] \left(\ast = \frac{\sqrt{n+\sqrt{n}}}{n} = \frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}} = \sqrt{\frac{n+\sqrt{n}}{n^2}} = \sqrt{\frac{1}{n}+\frac{\sqrt{n}}{n^2}} = \sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}} = \sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}} = \frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}} = \frac{1}{\sqrt{1+0}} = 1. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \frac{n^{\frac{1}{2}}}{[n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{n^{\frac{1}{2}}}{n^{\frac{1}{2}}[\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}]{n^{\frac{1}{2}}}}^{\frac{1}{2}}} = \frac{1}{[1+\frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}]^{\frac{1}{2}}} = \frac{1}{(1+\frac{1}{n})^{\frac{1}{2}}} \right] \left(\ast = \frac{(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n} = (\frac{n+n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}} = (\frac{1}{n}+\frac{n^{\frac{1}{2}}}{n^2})^{\frac{1}{2}} = (\frac{1}{n}+n^{\frac{1}{2}-2})^{\frac{1}{2}} = (\frac{1}{n}+n^{-\frac{3}{2}})^{\frac{1}{2}} = (\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{[1+(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{1}{[1+(\frac{1}{\infty}+\frac{1}{\infty})^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{1}{[1+(0+0)^{\frac{1}{2}}]^{\frac{1}{2}}} = \frac{1}{(1+0)^{\frac{1}{2}}} = \frac{1}{1} = 1. \end{aligned}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[27]

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[27]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}} = \lim_{n \rightarrow \infty} \frac{\frac{\sqrt[3]{n^4-1}}{\sqrt[3]{n^5}} - \frac{\sqrt[5]{(n^6-2)^3}}{\sqrt[15]{n^{25}}} + 3\frac{\sqrt[6]{(n+1)^3}}{\sqrt[6]{n^{10}}}}{2\frac{\sqrt[3]{n^5+1}}{\sqrt[3]{n^5}} + 3\frac{\sqrt[12]{(n^6-1)^3}}{\sqrt[12]{n^{20}}} - \frac{\sqrt[6]{(n-1)^3}}{\sqrt[6]{n^{10}}}}$$

$$= \left[\begin{array}{l} \frac{\sqrt[3]{n^4-1}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^4-1}{n^5}} = \sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} \\ \frac{\sqrt[5]{(n^6-2)^3}}{\sqrt[15]{n^{25}}} = \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} = \sqrt[15]{\frac{(n^6-2)^3}{n^7(n^6)^3}} = \sqrt[15]{\frac{1}{n^7} \left(\frac{n^6-2}{n^6}\right)^3} = \sqrt[15]{\frac{1}{n^7} \left(1 - \frac{2}{n^6}\right)^3} \\ \frac{\sqrt[6]{(n+1)^3}}{\sqrt[6]{n^{10}}} = \sqrt[6]{\frac{(n+1)^3}{n^{10}}} = \sqrt[6]{\frac{(n+1)^3}{n^7 n^3}} = \sqrt[6]{\frac{1}{n^7} \left(\frac{n+1}{n}\right)^3} = \sqrt[6]{\frac{1}{n^7} \left(1 + \frac{1}{n}\right)^3} \end{array} \right] \left[\begin{array}{l} \frac{\sqrt[3]{n^5+1}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^5+1}{n^5}} = \sqrt[3]{1 + \frac{1}{n^5}} \\ \frac{\sqrt[12]{(n^6-1)^3}}{\sqrt[12]{n^{20}}} = \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} = \sqrt[12]{\frac{(n^6-1)^3}{n^2(n^6)^3}} = \sqrt[12]{\frac{1}{n^2} \left(\frac{n^6-1}{n^6}\right)^3} = \sqrt[12]{\frac{1}{n^2} \left(1 - \frac{1}{n^6}\right)^3} \\ \frac{\sqrt[6]{(n-1)^3}}{\sqrt[6]{n^{10}}} = \sqrt[6]{\frac{(n-1)^3}{n^{10}}} = \sqrt[6]{\frac{(n-1)^3}{n^7 n^3}} = \sqrt[6]{\frac{1}{n^7} \left(\frac{n-1}{n}\right)^3} = \sqrt[6]{\frac{1}{n^7} \left(1 - \frac{1}{n}\right)^3} \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \left(1 - \frac{2}{n^6}\right)^3} + 3\sqrt[6]{\frac{1}{n^7} \left(1 + \frac{1}{n}\right)^3}}{2\sqrt[3]{1 + \frac{1}{n^5}} + 3\sqrt[12]{\frac{1}{n^2} \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \left(1 - \frac{1}{n}\right)^3}}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = 0$$

[27]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{(n^6-2)^3} + 3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1} + 3\sqrt[12]{(n^6-1)^3} - \sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}} = \lim_{n \rightarrow \infty} \frac{\frac{\sqrt[3]{n^4-1}}{\sqrt[3]{n^5}} - \frac{\sqrt[5]{(n^6-2)^3}}{\sqrt[15]{n^{25}}} + 3\frac{\sqrt[6]{(n+1)^3}}{\sqrt[6]{n^{10}}}}{2\frac{\sqrt[3]{n^5+1}}{\sqrt[3]{n^5}} + 3\frac{\sqrt[12]{(n^6-1)^3}}{\sqrt[12]{n^{20}}} - \frac{\sqrt[6]{(n-1)^3}}{\sqrt[6]{n^{10}}}} \\ &= \left[\begin{array}{l} \frac{\sqrt[3]{n^4-1}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^4-1}{n^5}} = \sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} \\ \frac{\sqrt[5]{(n^6-2)^3}}{\sqrt[15]{n^{25}}} = \sqrt[15]{\frac{(n^6-2)^3}{n^{25}}} = \sqrt[15]{\frac{(n^6-2)^3}{n^7(n^6)^3}} = \sqrt[15]{\frac{1}{n^7} \left(\frac{n^6-2}{n^6}\right)^3} = \sqrt[15]{\frac{1}{n^7} \left(1 - \frac{2}{n^6}\right)^3} \\ \frac{\sqrt[6]{(n+1)^3}}{\sqrt[6]{n^{10}}} = \sqrt[6]{\frac{(n+1)^3}{n^{10}}} = \sqrt[6]{\frac{(n+1)^3}{n^7 n^3}} = \sqrt[6]{\frac{1}{n^7} \left(\frac{n+1}{n}\right)^3} = \sqrt[6]{\frac{1}{n^7} \left(1 + \frac{1}{n}\right)^3} \end{array} \right] \left[\begin{array}{l} \frac{\sqrt[3]{n^5+1}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^5+1}{n^5}} = \sqrt[3]{1 + \frac{1}{n^5}} \\ \frac{\sqrt[12]{(n^6-1)^3}}{\sqrt[12]{n^{20}}} = \sqrt[12]{\frac{(n^6-1)^3}{n^{20}}} = \sqrt[12]{\frac{(n^6-1)^3}{n^2(n^6)^3}} = \sqrt[12]{\frac{1}{n^2} \left(\frac{n^6-1}{n^6}\right)^3} = \sqrt[12]{\frac{1}{n^2} \left(1 - \frac{1}{n^6}\right)^3} \\ \frac{\sqrt[6]{(n-1)^3}}{\sqrt[6]{n^{10}}} = \sqrt[6]{\frac{(n-1)^3}{n^{10}}} = \sqrt[6]{\frac{(n-1)^3}{n^7 n^3}} = \sqrt[6]{\frac{1}{n^7} \left(\frac{n-1}{n}\right)^3} = \sqrt[6]{\frac{1}{n^7} \left(1 - \frac{1}{n}\right)^3} \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{n} - \frac{1}{n^5}} - \sqrt[15]{\frac{1}{n^7} \left(1 - \frac{2}{n^6}\right)^3} + 3\sqrt[6]{\frac{1}{n^7} \left(1 + \frac{1}{n}\right)^3}}{2\sqrt[3]{1 + \frac{1}{n^5}} + 3\sqrt[12]{\frac{1}{n^2} \left(1 - \frac{1}{n^6}\right)^3} - \sqrt[6]{\frac{1}{n^7} \left(1 - \frac{1}{n}\right)^3}} = \frac{\sqrt[3]{\frac{1}{\infty} - \frac{1}{\infty}} - \sqrt[15]{\frac{1}{\infty} \left(1 - \frac{2}{\infty}\right)^3} + 3\sqrt[6]{\frac{1}{\infty} \left(1 + \frac{1}{\infty}\right)^3}}{2\sqrt[3]{1 + \frac{1}{\infty}} + 3\sqrt[12]{\frac{1}{\infty} \left(1 - \frac{1}{\infty}\right)^3} - \sqrt[6]{\frac{1}{\infty} \left(1 - \frac{1}{\infty}\right)^3}} \\ &= \frac{\sqrt[3]{0-0} - \sqrt[15]{0 \cdot (1-0)^3} + 3\sqrt[6]{0 \cdot (1+0)}}{2\sqrt[3]{1+0} + 3\sqrt[12]{0 \cdot (1-0)^3} - \sqrt[6]{0 \cdot (1-0)}} = \frac{0-0+3 \cdot 0}{2 \cdot 1 + 3 \cdot 0 - 0} = \frac{0}{2} = 0. \end{aligned}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[Iné riešenie 27]

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[Iné riešenie 27]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4(1-\frac{1}{n^4})} - \sqrt[5]{n^6(1-\frac{2}{n^6})} + 3\sqrt{n(1+\frac{1}{n})}}{2\sqrt[3]{n^5(1+\frac{1}{n^5})} + 3\sqrt[4]{n^6(1-\frac{1}{n^6})} - \sqrt{n(1-\frac{1}{n})}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \sqrt{1-\frac{1}{n}}} \\ &= \left[n \in \mathbb{N}. \Rightarrow \begin{array}{l} \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \\ \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \left[\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \sqrt[5]{1-\frac{2}{n^6}} + 3 \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \sqrt{1+\frac{1}{n}} \right]}{\sqrt[3]{n^5} \left[2\sqrt[3]{1+\frac{1}{n^5}} + 3 \frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \sqrt{1-\frac{1}{n}} \right]} \\ &= \left[n \in \mathbb{N}. \Rightarrow \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^4}{n^5}} = \sqrt[3]{\frac{1}{n}} = \frac{1}{\sqrt[3]{n}}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{1-\frac{2}{n^6}}}{15\sqrt[15]{n^2}} \sqrt[5]{1-\frac{2}{n^6}} + 3 \sqrt[6]{\frac{1}{n^5}} \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \left[2\sqrt[3]{1+\frac{1}{n^5}} + 3 \sqrt[12]{\frac{1}{n^2}} \sqrt[4]{1-\frac{1}{n^6}} - \sqrt[6]{\frac{1}{n^7}} \sqrt{1-\frac{1}{n}} \right]} \end{aligned}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = 0$$

[Iné riešenie 27]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4(1-\frac{1}{n^4})} - \sqrt[5]{n^6(1-\frac{2}{n^6})} + 3\sqrt{n(1+\frac{1}{n})}}{2\sqrt[3]{n^5(1+\frac{1}{n^5})} + 3\sqrt[4]{n^6(1-\frac{1}{n^6})} - \sqrt{n(1-\frac{1}{n})}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \sqrt[3]{1-\frac{1}{n^4}} - \sqrt[5]{n^6} \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt{n} \sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5} \sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[4]{n^6} \sqrt[4]{1-\frac{1}{n^6}} - \sqrt{n} \sqrt{1-\frac{1}{n}}} \\ &= \left[n \in \mathbb{N}. \Rightarrow \begin{array}{l} \sqrt[3]{n^4} = \sqrt[6]{n^8} = \sqrt[15]{n^{20}}, \quad \sqrt[5]{n^6} = \sqrt[15]{n^{18}}, \\ \sqrt{n} = \sqrt[6]{n^3}, \quad \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}}, \quad \sqrt[4]{n^6} = \sqrt[12]{n^{18}}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4} \left[\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{n^{18}}}{15\sqrt[15]{n^{20}}} \sqrt[5]{1-\frac{2}{n^6}} + 3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}} \sqrt{1+\frac{1}{n}} \right]}{\sqrt[3]{n^5} \left[2\sqrt[3]{1+\frac{1}{n^5}} + 3\frac{12\sqrt[12]{n^{18}}}{12\sqrt[12]{n^{20}}} \sqrt[4]{1-\frac{1}{n^6}} - \frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}} \sqrt{1-\frac{1}{n}} \right]} \\ &= \left[n \in \mathbb{N}. \Rightarrow \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \sqrt[3]{\frac{n^4}{n^5}} = \sqrt[3]{\frac{1}{n}} = \frac{1}{\sqrt[3]{n}}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{1-\frac{1}{n^4}} - \frac{15\sqrt[15]{1-\frac{2}{n^6}}}{15\sqrt[15]{n^2}} \sqrt[5]{1-\frac{2}{n^6}} + 3\sqrt[6]{\frac{1}{n^5}} \sqrt{1+\frac{1}{n}}}{\sqrt[3]{n} \left[2\sqrt[3]{1+\frac{1}{n^5}} + 3\sqrt[12]{\frac{1}{n^2}} \sqrt[4]{1-\frac{1}{n^6}} - \sqrt[6]{\frac{1}{n^7}} \sqrt{1-\frac{1}{n}} \right]} \\ &= \frac{\sqrt[3]{1-\frac{1}{\infty}} - \frac{15\sqrt[15]{\frac{1}{\infty}}}{15\sqrt[15]{\infty}} \sqrt[5]{1-\frac{2}{\infty}} + 3\sqrt[6]{\frac{1}{\infty}} \sqrt{1+\frac{1}{\infty}}}{\sqrt[3]{\infty} \left[2\sqrt[3]{1+\frac{1}{\infty}} + 3\sqrt[12]{\frac{1}{\infty}} \sqrt[4]{1-\frac{1}{\infty}} - \sqrt[6]{\frac{1}{\infty}} \sqrt{1-\frac{1}{\infty}} \right]} \\ &= \frac{\sqrt[3]{1-0} - \frac{15\sqrt[15]{0}}{15\sqrt[15]{0}} \cdot \sqrt[5]{1-0} + 3\sqrt[6]{0} \cdot \sqrt{1+0}}{\sqrt[3]{\infty} \cdot [2\sqrt[3]{1+0} + 3\sqrt[12]{0} \cdot \sqrt[4]{1-0} - \sqrt[6]{0} \cdot \sqrt{1-0}]} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty(2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[Iné riešenie 27]

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}}$$

[Iné riešenie 27]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}} \\ &= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \left[(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}} \right]}{n^{\frac{5}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3}-\frac{5}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-10}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} = \left[n \in \mathbb{N}. \Rightarrow n^{\frac{18-20}{15}} = n^{-\frac{2}{15}} = \frac{1}{n^{\frac{2}{15}}}, n^{\frac{3-10}{6}} = n^{-\frac{7}{6}} = \frac{1}{n^{\frac{7}{6}}}, \right. \\ &\quad \left. n^{\frac{18-20}{12}} = n^{-\frac{2}{12}} = \frac{1}{n^{\frac{1}{6}}}, n^{\frac{3-10}{6}} = n^{-\frac{7}{6}} = \frac{1}{n^{\frac{7}{6}}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - \frac{1}{n^{2/15}}(1-\frac{2}{n^6})^{\frac{1}{5}} + \frac{3}{n^{5/6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + \frac{3}{n^{2/12}}(1-\frac{1}{n^6})^{\frac{1}{4}} - \frac{1}{n^{7/6}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} \end{aligned}$$

Riešené príklady – 27

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^4-1} - \sqrt[5]{n^6-2} + 3\sqrt{n+1}}{2\sqrt[3]{n^5+1} + 3\sqrt[4]{n^6-1} - \sqrt{n-1}} = 0$$

[Iné riešenie 27]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{(n^4-1)^{\frac{1}{3}} - (n^6-2)^{\frac{1}{5}} + 3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}} + 3(n^6-1)^{\frac{1}{4}} - (n-1)^{\frac{1}{2}}} = \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}} \\
 &= \lim_{n \rightarrow \infty} \frac{n^{\frac{4}{3}} \left[(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}} \right]}{n^{\frac{5}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^{\frac{4}{3}}}{n^{\frac{4}{3}}} = n^{\frac{4}{3}-\frac{5}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}}. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}} + 3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + 3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}} - n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} = \left[n \in \mathbb{N}. \Rightarrow n^{\frac{18-20}{15}} = n^{-\frac{2}{15}} = \frac{1}{n^{\frac{2}{15}}}, n^{\frac{3-8}{6}} = n^{-\frac{5}{6}} = \frac{1}{n^{\frac{5}{6}}}, \right. \\
 &\quad \left. n^{\frac{18-20}{12}} = n^{-\frac{2}{12}} = \frac{1}{n^{\frac{1}{6}}}, n^{\frac{3-10}{6}} = n^{-\frac{7}{6}} = \frac{1}{n^{\frac{7}{6}}}. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{(1-\frac{1}{n^4})^{\frac{1}{3}} - \frac{1}{n^{\frac{2}{15}}}(1-\frac{2}{n^6})^{\frac{1}{5}} + \frac{3}{n^{\frac{5}{6}}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}} \left[2(1+\frac{1}{n^5})^{\frac{1}{3}} + \frac{3}{n^{\frac{2}{12}}}(1-\frac{1}{n^6})^{\frac{1}{4}} - \frac{1}{n^{\frac{7}{6}}}(1-\frac{1}{n})^{\frac{1}{2}} \right]} = \frac{(1-\frac{1}{\infty})^{\frac{1}{3}} - \frac{1}{\infty}(1-\frac{2}{\infty})^{\frac{1}{5}} + \frac{3}{\infty}(1+\frac{1}{\infty})^{\frac{1}{2}}}{\infty \left[2(1+\frac{1}{\infty})^{\frac{1}{3}} + \frac{3}{\infty}(1-\frac{1}{\infty})^{\frac{1}{4}} - \frac{1}{\infty}(1-\frac{1}{\infty})^{\frac{1}{2}} \right]} \\
 &= \frac{(1-0)^{\frac{1}{3}} - 0 \cdot (1-0)^{\frac{1}{5}} + 0 \cdot (1+0)^{\frac{1}{2}}}{\infty \cdot (2 \cdot (1+0)^{\frac{1}{3}} + 0 \cdot (1-0)^{\frac{1}{4}} - 0 \cdot (1-0)^{\frac{1}{2}})} = \frac{1-0 \cdot 1 + 3 \cdot 0 \cdot 1}{\infty \cdot (2 \cdot 1 + 3 \cdot 0 \cdot 1 - 0 \cdot 1)} = \frac{1}{\infty \cdot 2} = \frac{1}{\infty} = 0.
 \end{aligned}$$

Riešené príklady – 28, 29

Aritmetické konečné rady:

$$\bullet 1 + 2 + \dots + n = \frac{(1+n)n}{2}.$$

$$\bullet 1 + 3 + \dots + (2n-1) = \frac{(1+2n-1)n}{2} = \frac{2n \cdot n}{2} = n^2.$$

$$\lim_{n \rightarrow \infty} \left[\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right]$$

[28]

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

[29]

Riešené príklady – 28, 29

Aritmetické konečné rady:

$$\bullet 1 + 2 + \dots + n = \frac{(1+n)n}{2}.$$

$$\bullet 1 + 3 + \dots + (2n-1) = \frac{(1+2n-1)n}{2} = \frac{2n \cdot n}{2} = n^2.$$

$$\lim_{n \rightarrow \infty} \left[\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right]$$

[28]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow \frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} = \frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} = \frac{n}{2} - \frac{n(n+1)}{2(n+2)} = \frac{n(n+2)-n(n+1)}{2(n+2)} = \frac{n^2+2n-n^2-n}{2(n+2)} = \frac{n}{2n+4} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{n(n+\frac{4}{n})} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow \frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} = \frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} = \frac{n}{2} - \frac{n(n+1)}{2(n+2)} = \frac{n(n+2)-n(n+1)}{2(n+2)} = \frac{n^2+2n-n^2-n}{2(n+2)} = \frac{n}{2n+4} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n}$$

[29]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{2n}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}}$$

Riešené príklady – 28, 29

Aritmetické konečné rady:

$$\bullet 1 + 2 + \dots + n = \frac{(1+n)n}{2}.$$

$$\bullet 1 + 3 + \dots + (2n-1) = \frac{(1+2n-1)n}{2} = \frac{2n \cdot n}{2} = n^2.$$

$$\lim_{n \rightarrow \infty} \left[\frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} \right] = \frac{1}{2}$$

[28]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} = \frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} = \frac{n}{2} - \frac{n(n+1)}{2(n+2)} = \frac{n(n+2)-n(n+1)}{2(n+2)} = \frac{n^2+2n-n^2-n}{2(n+2)} = \frac{n}{2n+4} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{n(n+\frac{4}{n})} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} = \frac{1}{2}.$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{2} - \frac{1+2+3+\dots+n}{n+2} = \frac{n}{2} - \frac{\frac{n(n+1)}{2}}{n+2} = \frac{n}{2} - \frac{n(n+1)}{2(n+2)} = \frac{n(n+2)-n(n+1)}{2(n+2)} = \frac{n^2+2n-n^2-n}{2(n+2)} = \frac{n}{2n+4} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{n}{2n+4} = \lim_{n \rightarrow \infty} \frac{n}{2n+4} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{2+\frac{4}{n}} = \frac{1}{2+\frac{4}{\infty}} = \frac{1}{2+0} = \frac{1}{2}.$$

$$\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{1+2+3+\dots+n} = 2$$

[29]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{2n}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^2}{\frac{(1+n)n}{2}} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = \lim_{n \rightarrow \infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Riešené príklady – 30, 31

Aritmetické konečné rady: • $1 + 2 + \dots + n = \frac{(1+n)n}{2}$. • $1 + 4 + 7 + \dots + (3n - 1) = \frac{(1+3n-1)n}{2} = \frac{3n \cdot n}{2} = \frac{3n^2}{2}$.

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

[30]

$$\lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-1)}{\sqrt{9n^4+1}}$$

[31]

Riešené príklady – 30, 31

Aritmetické konečné rady: • $1 + 2 + \dots + n = \frac{(1+n)n}{2}$. • $1 + 4 + 7 + \dots + (3n - 1) = \frac{(1+3n-1)n}{2} = \frac{3n \cdot n}{2} = \frac{3n^2}{2}$.

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}}$$

[30]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n \cdot n(1+\frac{1}{n})}{2\sqrt{n^4}\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{2n^2\sqrt{9+\frac{1}{n^4}}} \\ &= \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-1)}{\sqrt{9n^4+1}}$$

[31]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{3n^2}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2\sqrt{n^4}\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2n^2\sqrt{9+\frac{1}{n^4}}} \\ &= \lim_{n \rightarrow \infty} \frac{3}{2\sqrt{9+\frac{1}{n^4}}} = \frac{3}{2\sqrt{9+\frac{1}{\infty}}} \end{aligned}$$

Riešené príklady – 30, 31

Aritmetické konečné rady: • $1 + 2 + \dots + n = \frac{(1+n)n}{2}$. • $1 + 4 + 7 + \dots + (3n - 1) = \frac{(1+3n-1)n}{2} = \frac{3n \cdot n}{2} = \frac{3n^2}{2}$.

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{\sqrt{9n^4+1}} = \frac{1}{6} \quad [30]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{n \cdot n(1+\frac{1}{n})}{2\sqrt{n^4}\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{2n^2\sqrt{9+\frac{1}{n^4}}} \\ &= \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}} = \frac{1+0}{2\sqrt{9+0}} = \frac{1}{2 \cdot 3} = \frac{1}{6}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1+4+7+\dots+(3n-1)}{\sqrt{9n^4+1}} = \frac{1}{2} \quad [31]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{3n^2}{2}}{\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2\sqrt{9n^4+1}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2\sqrt{n^4}\sqrt{9+\frac{1}{n^4}}} = \lim_{n \rightarrow \infty} \frac{3n^2}{2n^2\sqrt{9+\frac{1}{n^4}}} \\ &= \lim_{n \rightarrow \infty} \frac{3}{2\sqrt{9+\frac{1}{n^4}}} = \frac{3}{2\sqrt{9+\frac{1}{\infty}}} = \frac{3}{2\sqrt{9+0}} = \frac{3}{2 \cdot 3} = \frac{1}{2}. \end{aligned}$$

Riešené příklady – 32, 33, 34

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

[32]

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

[33]

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{1^2 + 2^2 + 3^2 + \dots + n^2}$$

[34]

Riešené príklady – 32, 33, 34

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

[32]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} = \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3}$$

[33]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right)$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{1^2 + 2^2 + 3^2 + \dots + n^2}$$

[34]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{\frac{n(n+1)(2n+1)}{6}} = \lim_{n \rightarrow \infty} \frac{2(4n^2-1)}{(n+1)(2n+1)} = \lim_{n \rightarrow \infty} \frac{2n^2(4 - \frac{1}{n^2})}{n^2(1 + \frac{1}{n})(2 + \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2(4 - \frac{1}{n^2})}{(1 + \frac{1}{n})(2 + \frac{1}{n})}$$

Riešené príklady – 32, 33, 34

$$\bullet 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\bullet 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} = \frac{1}{3}$$

[32]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n \cdot n} = \frac{1}{6} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \\ &= \frac{1}{6} \left(1 + \frac{1}{\infty}\right) \left(2 + \frac{1}{\infty}\right) = \frac{1}{6} (1 + 0) (2 + 0) = \frac{1}{6} \cdot 1 \cdot 2 = \frac{1}{3}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{n^3} = \frac{4}{3}$$

[33]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{3n^2} = \lim_{n \rightarrow \infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2}{1^2 + 2^2 + 3^2 + \dots + n^2} = 4$$

[34]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{n(4n^2-1)}{3}}{\frac{n(n+1)(2n+1)}{6}} = \lim_{n \rightarrow \infty} \frac{2(4n^2-1)}{(n+1)(2n+1)} = \lim_{n \rightarrow \infty} \frac{2n^2(4 - \frac{1}{n^2})}{n^2(1 + \frac{1}{n})(2 + \frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{2(4 - \frac{1}{n^2})}{(1 + \frac{1}{n})(2 + \frac{1}{n})} \\ &= \frac{2(4 - \frac{1}{\infty})}{(1 + \frac{1}{\infty})(2 + \frac{1}{\infty})} = \frac{2(4-0)}{(1+0)(2+0)} = \frac{2 \cdot 4}{2} = 4. \end{aligned}$$

Riešené príklady – 35, 36

- $a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$
- $a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}}$$

[35]

$$\lim_{n \rightarrow \infty} \frac{3n+2-n \cdot 2^{-n}}{8+3^{-n}+4 \cdot 5^{-n}}$$

[36]

Riešené príklady – 35, 36

- $a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$
- $a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}}$$

[35]

$$\bullet = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right] = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{8}{n}+\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}{3+\frac{2}{n}-\frac{1}{2^n}}$$

$$\lim_{n \rightarrow \infty} \frac{3n+2-n \cdot 2^{-n}}{8+3^{-n}+4 \cdot 5^{-n}}$$

[36]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8+\frac{1}{3^n}+\frac{4}{5^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8+\frac{1}{3^n}+\frac{4}{5^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{3+\frac{2}{n}-\frac{1}{2^n}}{\frac{8}{n}+\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}$$

Riešené príklady – 35, 36

- $a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$
- $a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$

$$\lim_{n \rightarrow \infty} \frac{8+3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = 0$$

[35]

$$\bullet = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right] = \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8+0+0}{\infty+2} = \frac{8}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{8}{n}+\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}{3+\frac{2}{n}-\frac{1}{2^n}} = \frac{\frac{8}{\infty}+\frac{1}{\infty \cdot \infty}+\frac{4}{\infty \cdot \infty}}{3+\frac{2}{\infty}-\frac{1}{\infty}} = \frac{\frac{8}{\infty}+\frac{1}{\infty}+\frac{4}{\infty}}{3+\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0+0+0}{3+0-0} = \frac{0}{3} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{3n+2-n \cdot 2^{-n}}{8+3^{-n}+4 \cdot 5^{-n}} = \infty$$

[36]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8+\frac{1}{3^n}+\frac{4}{5^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right] = \frac{3 \cdot \infty+2-0}{8+\frac{1}{\infty}+\frac{4}{\infty}} = \frac{\infty+2}{8+0+0} = \frac{\infty}{8} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8+\frac{1}{3^n}+\frac{4}{5^n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{3+\frac{2}{n}-\frac{1}{2^n}}{\frac{8}{n}+\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}} = \frac{3+\frac{2}{\infty}-\frac{1}{\infty}}{\frac{8}{\infty}+\frac{1}{\infty \cdot \infty}+\frac{4}{\infty \cdot \infty}} = \frac{3+\frac{2}{\infty}-\frac{1}{\infty}}{\frac{8}{\infty}+\frac{1}{\infty}+\frac{4}{\infty}} = \left[n \in \mathbb{N}. \Rightarrow \frac{8}{n} + \frac{1}{n \cdot 3^n} + \frac{4}{n \cdot 5^n} > 0. \Rightarrow \frac{8}{n} + \frac{1}{n \cdot 3^n} + \frac{4}{n \cdot 5^n} \rightarrow 0^+ \text{ pre } n \rightarrow \infty \right] = \frac{3+0-0}{0+0+0} = \frac{3}{0^+} = \infty.$$

Riešené príklady – 37, 38

- $a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$
- $a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$

$$\lim_{n \rightarrow \infty} \frac{8-3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}}$$

[37]

$$\lim_{n \rightarrow \infty} \frac{3n+2-n \cdot 2^{-n}}{8-3^{-n}+4 \cdot 5^{-n}}$$

[38]

Riešené príklady – 37, 38

- $a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$
- $a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$

$$\lim_{n \rightarrow \infty} \frac{8-3^{-n}+4 \cdot 5^{-n}}{3n+2-\frac{n}{2^n}}$$

[37]

$$\bullet = \lim_{n \rightarrow \infty} \frac{8-\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{8-\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{8-\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}{3+\frac{2}{n}-\frac{1}{2^n}}$$

$$\lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8-3^{-n}+4 \cdot 5^{-n}}$$

[38]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8-\frac{1}{3^n}+\frac{4}{5^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8-\frac{1}{3^n}+\frac{4}{5^n}} \cdot \frac{1}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{3+\frac{2}{n}-\frac{1}{2^n}}{\frac{8}{n}-\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}$$

Riešené príklady – 37, 38

$$\bullet a_n = \frac{n}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{n+1}{2n} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\bullet a_n = \frac{n}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2^n} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{8-3^{-n}+4 \cdot 5^{-n}}{3n+2-n \cdot 2^{-n}} = 0$$

[37]

$$\bullet = \lim_{n \rightarrow \infty} \frac{8-\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right] = \frac{8-\frac{1}{\infty}+\frac{4}{\infty}}{3 \cdot \infty+2-0} = \frac{8-0+0}{\infty+2} = \frac{8}{\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{8-\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{\frac{8}{n}-\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}}{3+\frac{2}{n}-\frac{1}{2^n}} = \frac{\frac{8}{\infty}-\frac{1}{\infty \cdot \infty}+\frac{4}{\infty \cdot \infty}}{3+\frac{2}{\infty}-\frac{1}{\infty}} = \frac{\frac{8}{\infty}-\frac{1}{\infty}+\frac{4}{\infty}}{3+\frac{2}{\infty}-\frac{1}{\infty}} = \frac{0-0+0}{3+0-0} = \frac{0}{3} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{3n+2-n \cdot 2^{-n}}{8-3^{-n}+4 \cdot 5^{-n}} = \infty$$

[38]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8-\frac{1}{3^n}+\frac{4}{5^n}} = \left[\lim_{n \rightarrow \infty} \frac{n}{2^n} = 0 \right] = \frac{3 \cdot \infty+2-0}{8-\frac{1}{\infty}+\frac{4}{\infty}} = \frac{\infty+2}{8-0+0} = \frac{\infty}{8} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3n+2-\frac{n}{2^n}}{8-\frac{1}{3^n}+\frac{4}{5^n}} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{3+\frac{2}{n}-\frac{1}{2^n}}{\frac{8}{n}-\frac{1}{n \cdot 3^n}+\frac{4}{n \cdot 5^n}} = \frac{3+\frac{2}{\infty}-\frac{1}{\infty}}{\frac{8}{\infty}-\frac{1}{\infty \cdot \infty}+\frac{4}{\infty \cdot \infty}} = \frac{3+\frac{2}{\infty}-\frac{1}{\infty}}{\frac{8}{\infty}-\frac{1}{\infty}+\frac{4}{\infty}} = \frac{3+0-0}{0-0+0} = \frac{3}{0} = ??.$$

[Postup nemôžeme použiť.]

Riešené príklady – 39, 40

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^4} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^4} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n+1}{n}\right)^4} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}}$$

[39]

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}}$$

[40]

Riešené príklady – 39, 40

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^4} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^4} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n+1}{n}\right)^4} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}}$$

[39]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + 3\frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}}$$

[40]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2n^3}{5^n} + 3\frac{n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2n^2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2}}{\frac{2}{n^2} - \frac{1}{n} + 3 \cdot 5 \cdot \frac{5^n}{n^4}}$$

Riešené príklady – 39, 40

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^4} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^4} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim_{n \rightarrow \infty} \frac{5}{\left(\frac{n+1}{n}\right)^4} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^4} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^4} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} = \frac{2}{15}$$

[39]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{5^n} + 3 \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{\frac{2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{2}{15}.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}} = 0$$

[40]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{5^n} + 3 \frac{n^4}{5^n} - \frac{n^2}{5^n}}{\frac{2}{5^n} - \frac{n^3}{5^n} + 3 \cdot 5} = \frac{2 \cdot 0 + 3 \cdot 0 - 0}{2 \cdot 0 - 0 + 3 \cdot 5} = \frac{0}{15} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2}{2n^2 - n^3 + 3 \cdot 5^{n+1}} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n^2} + 3 - \frac{1}{n^2}}{\frac{2}{n^2} - \frac{1}{n} + 3 \cdot 5 \cdot \frac{5^n}{n^4}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty}}{\frac{2}{\infty} - \frac{1}{\infty} + 15 \cdot \infty} = \frac{0 + 3 - 0}{0 - 0 + \infty} = \frac{3}{\infty} = 0.$$

Riešené príklady – 41

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

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Riešené príklady – 41

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{n}{5})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n}{5})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n+1}{n})^k} = \lim_{n \rightarrow \infty} \frac{5}{(1 + \frac{1}{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3}$$

[41]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n^2} - \frac{1}{n} \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n}}$$

Riešené príklady – 41

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{n}{5})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n}{5})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n+1}{n})^k} = \lim_{n \rightarrow \infty} \frac{5}{(1 + \frac{1}{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} = -\infty$$

[41]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} + 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 + \infty}{0 - 1} = \frac{\infty}{-1} = -\infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 + \infty}{0 - 0} = \frac{\infty}{0} = ??.$$

[Postup nemôžeme použiť.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 (\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4})}{n^3 (\frac{2}{n} - 1)} = \lim_{n \rightarrow \infty} \frac{n (\frac{2}{n} + 3 - \frac{1}{n^2} + 2 \cdot \frac{5^n}{n^4})}{\frac{2}{n} - 1} = \frac{\infty \cdot (\frac{2}{\infty} + 3 - \frac{1}{\infty} + 2 \cdot \infty)}{\frac{1}{\infty} - 1} = \frac{\infty \cdot (0 + 3 - 0 + \infty)}{0 - 1} = \frac{\infty \cdot \infty}{-1} = -\infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n}} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0} = \frac{2}{0} = ??.$$

[Postup nemôžeme použiť.]

Riešené príklady – 42

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

[42]

Riešené príklady – 42

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{n}{5})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n}{5})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\frac{n+1}{n})^k} = \lim_{n \rightarrow \infty} \frac{5}{(1 + \frac{1}{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3}$$

[42]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} - 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n}}$$

Riešené príklady – 42

$$\bullet a_n = \frac{n^k}{5^n} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{n^k}{5^n} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \lim_{n \rightarrow \infty} \frac{1}{5} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \\ \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0.$$

$$\bullet a_n = \frac{5^n}{n^k} \geq 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{(\sqrt[n]{n})^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\bullet a_n = \frac{5^n}{n^k} > 0, k, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{5^{n+1}}{(n+1)^k}}{\frac{5^n}{n^k}} = \lim_{n \rightarrow \infty} \frac{5}{\left(1 + \frac{1}{n}\right)^k} = 5 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5^n}{n^k} = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} = \infty$$

[42]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{2 + 3n - \frac{1}{n} - 2 \cdot \frac{5^n}{n^3}}{\frac{2}{n} - 1} = \frac{2 + 3 \cdot \infty - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - 1} = \frac{2 + \infty - 0 - \infty}{0 - 1} = \frac{-\infty}{-1} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n \rightarrow \infty} \frac{\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4}}{\frac{2}{n^2} - \frac{1}{n}} = \frac{\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty}{\frac{2}{\infty} - \frac{1}{\infty}} = \frac{0 + 3 - 0 - \infty}{0 - 0} = \frac{-\infty}{0} = ??.$$

[Postup nemôžeme použiť.]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^4 \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{n^3 \left(\frac{2}{n} - 1 \right)} = \lim_{n \rightarrow \infty} \frac{n \left(\frac{2}{n} + 3 - \frac{1}{n^2} - 2 \cdot \frac{5^n}{n^4} \right)}{\frac{2}{n} - 1} = \frac{\infty \cdot \left(\frac{2}{\infty} + 3 - \frac{1}{\infty} - 2 \cdot \infty \right)}{\frac{1}{\infty} - 1} = \frac{\infty \cdot (0 + 3 - 0 - \infty)}{0 - 1} \\ = \frac{\infty \cdot (-\infty)}{-1} = \frac{-\infty}{-1} = \infty.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 - 2 \cdot 5^n}{2n^2 - n^3} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} - 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n}} = \frac{2 \cdot 0 + 3 \cdot 0 - 0 - 2}{2 \cdot 0 - 0} = \frac{-2}{0} = ??.$$

[Postup nemôžeme použiť.]

Riešené příklady – 43

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

[43]

Riešené príklady – 43

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n}$$

[43]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{7^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{7n^k} = \frac{1}{7} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{7} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{7^n} = 0. \\ a_n = \frac{n^k}{7^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{7^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{7} = \frac{1^k}{7} = \frac{1}{7} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{7^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{7^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \left(\frac{5}{7}\right)^n}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{5^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \frac{1}{5} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \\ a_n = \frac{n^k}{5^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \left(\frac{7}{5}\right)^n}$$

Riešené príklady – 43

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} = 0$$

[43]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{7^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{7n^k} = \frac{1}{7} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{7} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{7^n} = 0. \\ a_n = \frac{n^k}{7^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{7^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{7} = \frac{1^k}{7} = \frac{1}{7} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{7^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{7^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \frac{5^n}{7^n}}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{7^n} + 3 \cdot \frac{n^4}{7^n} - \frac{n^2}{7^n} + 2 \cdot \left(\frac{5}{7}\right)^n}{2 \cdot \frac{n^2}{7^n} - \frac{n^3}{7^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť } \left\{\left(\frac{5}{7}\right)^n\right\}_{n=1}^{\infty}. \\ \text{Pr. 04: } \lim_{n \rightarrow \infty} a^n = 0 \text{ pre } a \in (-1; 1). \Rightarrow \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n = 0. \end{array} \right]$$

$$= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot 0}{2 \cdot 0 - 0 + 6} = \frac{0}{6} = 0.$$

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{5^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \frac{1}{5} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \\ a_n = \frac{n^k}{5^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 7^n} \cdot \frac{1}{5^n} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{7^n}{5^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \left(\frac{7}{5}\right)^n} = \left[\begin{array}{l} \text{Geometrická postupnosť } \left\{\left(\frac{7}{5}\right)^n\right\}_{n=1}^{\infty}. \\ \text{Pr. 04: } \lim_{n \rightarrow \infty} a^n = \infty \text{ pre } a \in (1; \infty). \Rightarrow \lim_{n \rightarrow \infty} \left(\frac{7}{5}\right)^n = \infty. \end{array} \right] = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 6 \cdot \infty} = \frac{2}{\infty} = 0.$$

Riešené příklady – 44

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

[44]

Riešené príklady – 44

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n}$$

[44]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{4^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \frac{1}{4} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{4} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0. \\ a_n = \frac{n^k}{4^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \left(\frac{5}{4}\right)^n}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6}$$

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{5^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \frac{1}{5} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \\ a_n = \frac{n^k}{5^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \frac{4^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \left(\frac{4}{5}\right)^n}$$

Riešené príklady – 44

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} = \infty$$

[44]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{4^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{4n^k} = \frac{1}{4} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{4} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0. \\ a_n = \frac{n^k}{4^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{4^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{1}{\frac{1}{4^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \frac{5^n}{4^n}}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{4^n} + 3 \cdot \frac{n^4}{4^n} - \frac{n^2}{4^n} + 2 \cdot \left(\frac{5}{4}\right)^n}{2 \cdot \frac{n^2}{4^n} - \frac{n^3}{4^n} + 6} = \left[\begin{array}{l} \text{Geometrická postupnosť } \left\{\left(\frac{5}{4}\right)^n\right\}_{n=1}^{\infty}. \\ \text{Pr. 04: } \lim_{n \rightarrow \infty} a^n = \infty \text{ pre } a \in (1; \infty). \Rightarrow \lim_{n \rightarrow \infty} \left(\frac{5}{4}\right)^n = \infty. \end{array} \right]$$

$$= \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2 \cdot \infty}{2 \cdot 0 - 0 + 6} = \frac{\infty}{6} = \infty.$$

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{5^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{5n^k} = \frac{1}{5} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \\ a_n = \frac{n^k}{5^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{5^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^4 - n^2 + 2 \cdot 5^n}{2n^2 - n^3 + 6 \cdot 4^n} \cdot \frac{1}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{2 \cdot \frac{n^3}{5^n} + 3 \cdot \frac{n^4}{5^n} - \frac{n^2}{5^n} + 2}{2 \cdot \frac{n^2}{5^n} - \frac{n^3}{5^n} + 6 \cdot \left(\frac{4}{5}\right)^n}$$

$$= \left[\begin{array}{l} \text{Geometrická postupnosť } \left\{\left(\frac{4}{5}\right)^n\right\}_{n=1}^{\infty}. \\ \text{Pr. 04: } \lim_{n \rightarrow \infty} a^n = 0 \text{ pre } a \in (-1; 1). \Rightarrow \lim_{n \rightarrow \infty} \left(\frac{4}{5}\right)^n = 0. \end{array} \right] = \frac{2 \cdot 0 + 3 \cdot 0 - 0 + 2}{2 \cdot 0 - 0 + 6 \cdot 0} = \frac{2}{0} = ?? \quad \left[\text{Postup nemôžeme použiť.} \right]$$

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$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^n}$$

[45]



$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^{n+1}}$$

[46]



Riešené príklady – 45, 46

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^n}$$

[45]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} + 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} - 2}$$

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^{n+1}}$$

[46]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} + 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} - 2 \cdot 3}$$

Riešené príklady – 45, 46

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^n} = -\frac{3}{2}$$

[45]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} + 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} - 2} \\ &= \frac{3 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 + 6}{3 \cdot 0 + 4 \cdot 0 - 2} = \frac{0 + 0 + 0 + 6}{0 + 0 - 2} = \frac{6}{-2} = -\frac{3}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^{n+1}} = -1$$

[46]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{3n^2 + 4n^3 + 5n + 6 \cdot 3^n}{3n + 4n^3 - 2 \cdot 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} + 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} - 2 \cdot 3} \\ &= \frac{3 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 + 6}{3 \cdot 0 + 4 \cdot 0 - 6} = \frac{0 + 0 + 0 + 6}{0 + 0 - 6} = \frac{6}{-6} = -1. \end{aligned}$$

Riešené príklady – 47, 48

$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot 3^n}{3n + 4n^3 + 2 \cdot 3^{n+1}}$$

[47]



$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot e^n}{3n + 4n^3 + 2 \cdot e^{n+1}}$$

[48]



Riešené príklady – 47, 48

$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot 3^n}{3n + 4n^3 + 2 \cdot 3^{n+1}}$$

[47]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot 3^n}{3n + 4n^3 + 2 \cdot 3^{n+1}} \cdot \frac{1}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{-3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} - 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} + 2 \cdot 3}$$

$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot e^n}{3n + 4n^3 + 2 \cdot e^{n+1}}$$

[48]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{e^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{e^{n+1}}}{\frac{n^k}{e^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{e n^k} = \frac{1}{e} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{e} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{e^n} = 0. \\ a_n = \frac{n^k}{e^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{e^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{e} = \frac{1^k}{e} = \frac{1}{e} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{e^n} = 0. \end{array} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot e^n}{3n + 4n^3 + 2 \cdot e^{n+1}} \cdot \frac{1}{\frac{1}{e^n}} = \lim_{n \rightarrow \infty} \frac{-3 \cdot \frac{n^2}{e^n} + 4 \cdot \frac{n^3}{e^n} + 5 \cdot \frac{n}{e^n} - 6}{3 \cdot \frac{n}{e^n} + 4 \cdot \frac{n^3}{e^n} + 2 \cdot e}$$

Riešené príklady – 47, 48

$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot 3^n}{3n + 4n^3 + 2 \cdot 3^{n+1}} = -1$$

[47]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{3^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{3^{n+1}}}{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{3n^k} = \frac{1}{3} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \\ a_n = \frac{n^k}{3^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{3} = \frac{1^k}{3} = \frac{1}{3} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{3^n} = 0. \end{array} \right]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot 3^n}{3n + 4n^3 + 2 \cdot 3^{n+1}} \cdot \frac{1}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{-3 \cdot \frac{n^2}{3^n} + 4 \cdot \frac{n^3}{3^n} + 5 \cdot \frac{n}{3^n} - 6}{3 \cdot \frac{n}{3^n} + 4 \cdot \frac{n^3}{3^n} + 2 \cdot 3} \\ &= \frac{3 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 - 6}{3 \cdot 0 + 4 \cdot 0 + 6} = \frac{0 + 0 + 0 - 6}{0 + 0 + 6} = \frac{-6}{6} = -1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot e^n}{3n + 4n^3 + 2 \cdot e^{n+1}} = -\frac{3}{e}$$

[48]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^k}{e^n} > 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^k}{e^{n+1}}}{\frac{n^k}{e^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^k}{e n^k} = \frac{1}{e} \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^k = \frac{1}{e} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{e^n} = 0. \\ a_n = \frac{n^k}{e^n} \geq 0, k \in \mathbb{N}, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^k}{e^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^k}{e} = \frac{1^k}{e} = \frac{1}{e} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^k}{e^n} = 0. \end{array} \right]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{-3n^2 + 4n^3 + 5n - 6 \cdot e^n}{3n + 4n^3 + 2 \cdot e^{n+1}} \cdot \frac{1}{\frac{1}{e^n}} = \lim_{n \rightarrow \infty} \frac{-3 \cdot \frac{n^2}{e^n} + 4 \cdot \frac{n^3}{e^n} + 5 \cdot \frac{n}{e^n} - 6}{3 \cdot \frac{n}{e^n} + 4 \cdot \frac{n^3}{e^n} + 2 \cdot e} \\ &= \frac{3 \cdot 0 + 4 \cdot 0 + 5 \cdot 0 - 6}{3 \cdot 0 + 4 \cdot 0 + 2e} = \frac{0 + 0 + 0 - 6}{0 + 0 + 2e} = \frac{-6}{2e} = -\frac{3}{e}. \end{aligned}$$

Riešené príklady – 49, 50

Geometrická postupnosť $\{(\frac{2}{3})^n\}_{n=1}^{\infty}$.

• Pr. 04: $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

[49]

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

[50]

Riešené příklady – 49, 50

Geometrická postupnosť $\{(\frac{2}{3})^n\}_{n=1}^{\infty}$. • **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}}$$

[49]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{2^n}{3^n} + \frac{3^n}{3^n} \right]}{3^n \left[\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}}$$

[50]

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n}$$

$$= \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-3)^n \left[\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n} \right]}{(-3)^n \left[\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n} \right]} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3}$$

Riešené příklady – 49, 50

Geometrická postupnost $\{(\frac{2}{3})^n\}_{n=1}^{\infty}$. • **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} = \frac{1}{3}$$

[49]

$$\bullet = \lim_{n \rightarrow \infty} \frac{2^n + 3^n}{2^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{2^n}{3^n} + \frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3} = \frac{0+1}{2 \cdot 0 + 3} = \frac{1}{3}.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{2^n}{3^n} + \frac{3^n}{3^n} \right]}{3^n \left[\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{2(\frac{2}{3})^n + 3} = \frac{0+1}{2 \cdot 0 + 3} = \frac{1}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} = -\frac{1}{3}$$

[50]

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + (-3)^n}{(-2)^{n+1} + (-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} \\ = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3} = \frac{0+1}{-2 \cdot 0 - 3} = -\frac{1}{3}.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-3)^n \left[\frac{(-2)^n}{(-3)^n} + \frac{(-3)^n}{(-3)^n} \right]}{(-3)^n \left[\frac{(-2)^{n+1}}{(-3)^n} + \frac{(-3)^{n+1}}{(-3)^n} \right]} = \lim_{n \rightarrow \infty} \frac{(\frac{-2}{-3})^n + 1}{-2 \cdot (\frac{-2}{-3})^n - 3 \cdot (\frac{-3}{-3})^n} = \lim_{n \rightarrow \infty} \frac{(\frac{2}{3})^n + 1}{-2(\frac{2}{3})^n - 3} \\ = \frac{0+1}{-2 \cdot 0 - 3} = -\frac{1}{3}.$$

Riešené príklady – 51, 52

Geometrické postupnosti $\{(\pm \frac{2}{3})^n\}_{n=1}^{\infty}$.

• Pr. 04: $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n+1} - 2^{n+1}}$$

[51]

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}}$$

[52]

Riešené príklady – 51, 52

Geometrické postupnosti $\{(\pm \frac{2}{3})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n+1} - 2^{n+1}}$$

[51]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n+1} - 2^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{3^n} - \frac{2^n}{3^n}}{\frac{3^{n+1}}{3^n} - \frac{2^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{2}{3})^n}{3 - 2(\frac{2}{3})^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{3^n}{3^n} - \frac{2^n}{3^n} \right]}{3^n \left[\frac{3^{n+1}}{3^n} - \frac{2^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{2}{3})^n}{3 - 2(\frac{2}{3})^n}$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}}$$

[52]

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{3^n} + \frac{3^n}{3^n}}{\frac{(-2)^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3}$$

$$= \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{(-2)^n}{3^n} + \frac{3^n}{3^n} \right]}{3^n \left[\frac{(-2)^{n+1}}{3^n} + \frac{3^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3}$$

Riešené príklady – 51, 52

Geometrické postupnosti $\{(\pm \frac{2}{3})^n\}_{n=1}^{\infty}$.

• **Pr.04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{3})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{3})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n+1} - 2^{n+1}} = \frac{1}{3}$$

[51]

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n - 2^n}{3^{n+1} - 2^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{3^n} - \frac{2^n}{3^n}}{\frac{3^{n+1}}{3^n} - \frac{2^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{2}{3})^n}{3 - 2(\frac{2}{3})^n} = \frac{1-0}{3-2 \cdot 0} = \frac{1}{3}.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{3^n}{3^n} - \frac{2^n}{3^n} \right]}{3^n \left[\frac{3^{n+1}}{3^n} - \frac{2^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{2}{3})^n}{3 - 2(\frac{2}{3})^n} = \frac{1-0}{3-2 \cdot 0} = \frac{1}{3}.$$

$$\lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} = \frac{1}{3}$$

[52]

$$\bullet = \lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}} = \lim_{n \rightarrow \infty} \frac{\frac{(-2)^n}{3^n} + \frac{3^n}{3^n}}{\frac{(-2)^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}} = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3} \\ = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3} = \frac{0+1}{-2 \cdot 0 + 3} = \frac{1}{3}.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{3^n \left[\frac{(-2)^n}{3^n} + \frac{3^n}{3^n} \right]}{3^n \left[\frac{(-2)^{n+1}}{3^n} + \frac{3^{n+1}}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{(-\frac{2}{3})^n + 1}{-2(-\frac{2}{3})^n + 3} = \frac{0+1}{-2 \cdot 0 + 3} = \frac{1}{3}.$$

Riešené príklady – 53

Geometrické postupnosti $\{(\pm\frac{2}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\pm\frac{4}{5})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{5})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{5})^n = 0$,

$\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{4}{5})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{1 + (-2)^n + 3^{n+1} + (-4)^n + 5^{n-1}}{2^{n+1} - 3^{n-1} + 4^{n+1} - 5^{n+1}}$$

[53]

Riešené príklady – 53

Geometrické postupnosti $\{(\pm\frac{2}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\pm\frac{4}{5})^n\}_{n=1}^{\infty}$.

- **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{5})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{5})^n = 0$,
 $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{4}{5})^n = 0$.

$$\lim_{n \rightarrow \infty} \frac{1 + (-2)^n + 3^{n+1} + (-4)^n + 5^{n-1}}{2^{n+1} - 3^{n-1} + 4^{n+1} - 5^{n+1}}$$

[53]

- $$= \lim_{n \rightarrow \infty} \frac{1 + (-2)^n + 3^{n+1} + (-4)^n + 5^{n-1}}{1 + 2^{n+1} - 3^{n-1} + 4^{n+1} - 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + \frac{(-2)^n}{5^n} + \frac{3^{n+1}}{5^n} + \frac{(-4)^n}{5^n} + \frac{5^n}{5^n}}{\frac{1}{5^n} + \frac{2^{n+1}}{5^n} - \frac{3^{n-1}}{5^n} + \frac{4^{n+1}}{5^n} - \frac{5^{n+1}}{5^n}}$$
- $$= \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + (-\frac{2}{5})^n + 3(\frac{3}{5})^n + (-\frac{4}{5})^n + 1}{\frac{1}{5^n} + 2(\frac{2}{5})^n - \frac{1}{3}(\frac{3}{5})^n + 4(\frac{4}{5})^n - 5}$$
-
- $$= \lim_{n \rightarrow \infty} \frac{5^n \left[\frac{1}{5^n} + \frac{(-2)^n}{5^n} + \frac{3^{n+1}}{5^n} + \frac{(-4)^n}{5^n} + \frac{5^n}{5^n} \right]}{5^n \left[\frac{1}{5^n} + \frac{2^{n+1}}{5^n} - \frac{3^{n-1}}{5^n} + \frac{4^{n+1}}{5^n} - \frac{5^{n+1}}{5^n} \right]}$$
- $$= \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + (-\frac{2}{5})^n + 3(\frac{3}{5})^n + (-\frac{4}{5})^n + 1}{\frac{1}{5^n} + 2(\frac{2}{5})^n - \frac{1}{3}(\frac{3}{5})^n + 4(\frac{4}{5})^n - 5}$$

Riešené príklady – 53

Geometrické postupnosti $\{(\pm \frac{2}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\pm \frac{4}{5})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{2}{5})^n = 0$, $\lim_{n \rightarrow \infty} (-\frac{2}{5})^n = 0$,

$$\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0, \lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0, \lim_{n \rightarrow \infty} (-\frac{4}{5})^n = 0.$$

$$\lim_{n \rightarrow \infty} \frac{1 + (-2)^n + 3^{n+1} + (-4)^n + 5^{n-1}}{2^{n+1} - 3^{n-1} + 4^{n+1} - 5^{n+1}} = -\frac{1}{5}$$

[53]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1 + (-2)^n + 3^{n+1} + (-4)^n + 5^{n-1}}{1 + 2^{n+1} - 3^{n-1} + 4^{n+1} - 5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + \frac{(-2)^n}{5^n} + \frac{3^{n+1}}{5^n} + \frac{(-4)^n}{5^n} + \frac{5^n}{5^n}}{\frac{1}{5^n} + \frac{2^{n+1}}{5^n} - \frac{3^{n-1}}{5^n} + \frac{4^{n+1}}{5^n} - \frac{5^{n+1}}{5^n}} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + (-\frac{2}{5})^n + 3(\frac{3}{5})^n + (-\frac{4}{5})^n + 1}{\frac{1}{5^n} + 2(\frac{2}{5})^n - \frac{1}{3}(\frac{3}{5})^n + 4(\frac{4}{5})^n - 5} = \frac{\frac{1}{\infty} + 0 + 3 \cdot 0 + 0 + 1}{\frac{1}{\infty} + 2 \cdot 0 - \frac{1}{3} \cdot 0 + 4 \cdot 0 - 5} = \frac{0 + 0 + 0 + 0 + 1}{0 + 0 - 0 + 0 - 5} = -\frac{1}{5}. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{5^n \left[\frac{1}{5^n} + \frac{(-2)^n}{5^n} + \frac{3^{n+1}}{5^n} + \frac{(-4)^n}{5^n} + \frac{5^n}{5^n} \right]}{5^n \left[\frac{1}{5^n} + \frac{2^{n+1}}{5^n} - \frac{3^{n-1}}{5^n} + \frac{4^{n+1}}{5^n} - \frac{5^{n+1}}{5^n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} + (-\frac{2}{5})^n + 3(\frac{3}{5})^n + (-\frac{4}{5})^n + 1}{\frac{1}{5^n} + 2(\frac{2}{5})^n - \frac{1}{3}(\frac{3}{5})^n + 4(\frac{4}{5})^n - 5} = \frac{\frac{1}{\infty} + 0 + 3 \cdot 0 + 0 + 1}{\frac{1}{\infty} + 2 \cdot 0 - \frac{1}{3} \cdot 0 + 4 \cdot 0 - 5} = \frac{0 + 0 + 0 + 0 + 1}{0 + 0 - 0 + 0 - 5} = -\frac{1}{5}. \end{aligned}$$

Riešené príklady – 54, 55

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• Pr. 04: $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}}$$

[54]

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}}$$

[55]

Riešené príklady – 54, 55

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}}$$

[54]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\frac{4^n}{5^n} - \frac{4^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{1}{(\frac{4}{5})^n - (\frac{4}{3})^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{4^n}}{\frac{3^n}{5^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n - 1}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}}$$

[55]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\frac{4^n}{5^n}}{1 - \frac{4^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{4}{5})^n}{1 - (\frac{4}{3})^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{5^n}}{\frac{3^n}{4^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n - 1}$$

Riešené príklady – 54, 55

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} = 0$$

[54]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1}{\frac{4^n}{5^n} - \frac{4^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{1}{(\frac{4}{5})^n - (\frac{4}{3})^n} = \frac{1}{0 - \infty} = \frac{1}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{4^n}}{\frac{3^n}{5^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n - 1} = \frac{0}{0 - 1} = \frac{0}{-1} = 0.$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} = 0$$

[55]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\frac{4^n}{5^n}}{1 - \frac{4^n}{3^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{4}{5})^n}{1 - (\frac{4}{3})^n} = \frac{0}{1 - \infty} = \frac{0}{-\infty} = 0.$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n} - \frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{5^n}}{\frac{3^n}{4^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n - 1} = \frac{0}{0 - 1} = \frac{0}{-1} = 0.$$

Riešené príklady – 56, 57, 58

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• Pr. 04: $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}}$$

[56]

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{4^n}}{\frac{1}{3^n}}$$

[57]

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{3^n}}{\frac{1}{4^n}}$$

[58]

Riešené príklady – 56, 57, 58

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}}$$

[56]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\frac{4^n}{3^n}}{\frac{4^n}{5^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n - 1}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{4^n}}{\frac{1}{3^n}}$$

[57]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{4^n}}{\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \left[\frac{3^n}{5^n} - \frac{3^n}{4^n} \right] = \lim_{n \rightarrow \infty} \left[\left(\frac{3}{5}\right)^n - \left(\frac{3}{4}\right)^n \right]$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{3^n}}{\frac{1}{4^n}}$$

[58]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{3^n}}{\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \left[\frac{4^n}{5^n} - \frac{4^n}{3^n} \right] = \lim_{n \rightarrow \infty} \left[\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n \right]$$

Riešené příklady – 56, 57, 58

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}} = -\infty$$

[56]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n} - \frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{\frac{4^n}{3^n}}{\frac{4^n}{5^n} - 1} = \lim_{n \rightarrow \infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n - 1} = \frac{\infty}{0-1} = \frac{\infty}{-1} = -\infty.$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{4^n}}{\frac{1}{3^n}} = 0$$

[57]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{4^n}}{\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \left[\frac{3^n}{5^n} - \frac{3^n}{4^n} \right] = \lim_{n \rightarrow \infty} \left[\left(\frac{3}{5}\right)^n - \left(\frac{3}{4}\right)^n \right] = 0 - 0 = 0.$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{3^n}}{\frac{1}{4^n}} = -\infty$$

[58]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{5^n} - \frac{1}{3^n}}{\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \left[\frac{4^n}{5^n} - \frac{4^n}{3^n} \right] = \lim_{n \rightarrow \infty} \left[\left(\frac{4}{5}\right)^n - \left(\frac{4}{3}\right)^n \right] = 0 - \infty = -\infty.$$

Riešené príklady – 59

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}}$$

[59]

Riešené príklady – 59

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• **Pr. 04:** $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}}$$

[59]

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{3^n} [\frac{3^n}{4^n} - 1]}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{5^n}{3^n} [\frac{3^n}{4^n} - 1] = \lim_{n \rightarrow \infty} (\frac{5}{3})^n [(\frac{3}{4})^n - 1]$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot \frac{4^n}{4^n} = \lim_{n \rightarrow \infty} \frac{1 - \frac{4^n}{3^n}}{\frac{4^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{4}{3})^n}{(\frac{4}{5})^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot \frac{3^n}{3^n} = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{4^n} - 1}{\frac{3^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{4})^n - 1}{(\frac{3}{5})^n}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot \frac{5^n}{5^n} = \lim_{n \rightarrow \infty} [\frac{5^n}{4^n} - \frac{5^n}{3^n}] = \lim_{n \rightarrow \infty} [(\frac{5}{4})^n - (\frac{5}{3})^n]$$

Riešené príklady – 59

Geometrické postupnosti $\{(\frac{3}{4})^n\}_{n=1}^{\infty}$, $\{(\frac{3}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{5})^n\}_{n=1}^{\infty}$, $\{(\frac{4}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{3})^n\}_{n=1}^{\infty}$, $\{(\frac{5}{4})^n\}_{n=1}^{\infty}$.

• Pr. 04: $\lim_{n \rightarrow \infty} a^n = 0$ pre $a \in (-1; 1)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{3}{5})^n = 0$, $\lim_{n \rightarrow \infty} (\frac{4}{5})^n = 0$.

$\lim_{n \rightarrow \infty} a^n = \infty$ pre $a \in (1; \infty)$. $\Rightarrow \lim_{n \rightarrow \infty} (\frac{4}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{3})^n = \infty$, $\lim_{n \rightarrow \infty} (\frac{5}{4})^n = \infty$.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} = -\infty$$

[59]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{1}{3^n} \left[\frac{3^n}{4^n} - 1 \right]}{\frac{1}{5^n}} = \lim_{n \rightarrow \infty} \frac{5^n}{3^n} \left[\frac{3^n}{4^n} - 1 \right] = \lim_{n \rightarrow \infty} \left(\frac{5}{3} \right)^n \left[\left(\frac{3}{4} \right)^n - 1 \right] \\ &= \infty(0 - 1) = \infty \cdot (-1) = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot 4^n = \lim_{n \rightarrow \infty} \frac{1 - \frac{4^n}{3^n}}{\frac{4^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{1 - (\frac{4}{3})^n}{(\frac{4}{5})^n} = \left[n \in \mathbb{N}. \Rightarrow (\frac{4}{5})^n > 0. \Rightarrow (\frac{4}{5})^n \rightarrow 0^+ \text{ pre } n \rightarrow \infty \right] \\ &= \frac{1 - \infty}{0^+} = \frac{-\infty}{0^+} = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot 3^n = \lim_{n \rightarrow \infty} \frac{\frac{3^n}{4^n} - 1}{\frac{3^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{(\frac{3}{4})^n - 1}{(\frac{3}{5})^n} = \left[n \in \mathbb{N}. \Rightarrow (\frac{3}{5})^n > 0. \Rightarrow (\frac{3}{5})^n \rightarrow 0^+ \text{ pre } n \rightarrow \infty \right] \\ &= \frac{0 - 1}{0^+} = \frac{-1}{0^+} = -\infty. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{\frac{1}{4^n} - \frac{1}{3^n}}{\frac{1}{5^n}} \cdot 5^n = \lim_{n \rightarrow \infty} \left[\frac{5^n}{4^n} - \frac{5^n}{3^n} \right] = \lim_{n \rightarrow \infty} \left[\left(\frac{5}{4} \right)^n - \left(\frac{5}{3} \right)^n \right] = \infty - \infty = ??.$$

[Postup nemôžeme použiť.]

Riešené príklady – 60, 61

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n - 3^n}$$

[60]

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n + 3^n}$$

[61]

Riešené príklady – 60, 61

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n - 3^n}$$

[60]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{4^n \left(1 - \frac{3^n}{4^n}\right)} = \lim_{n \rightarrow \infty} 4 \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} = 4 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{3}{4}\right)^n}$$

$$\bullet = \left[\begin{aligned} &\frac{4^3}{2} = \frac{64}{2} = 32 > 27 = 3^3, n \in \mathbb{N}. \Rightarrow \frac{4^{n+3}}{2} = 4^n \cdot \frac{4^3}{2} > 3^n \cdot \frac{4^3}{2} > 3^n \cdot 3^3 = 3^{n+3}. \Rightarrow \text{Pre } n \in \mathbb{N}, n \geq 3 \text{ platí } \frac{4^n}{2} > 3^n, -\frac{4^n}{2} < -3^n. \\ &\Rightarrow 4 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{4^n}{2}} = \sqrt[n]{4^n - \frac{4^n}{2}} < \sqrt[n]{4^n - 3^n} < \sqrt[n]{4^n} = 4. \end{aligned} \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n + 3^n}$$

[61]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{4^n \left(1 + \frac{3^n}{4^n}\right)} = \lim_{n \rightarrow \infty} 4 \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} = 4 \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{3}{4}\right)^n}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow 4^n < 4^n + 3^n < 4^n + 4^n = 2 \cdot 4^n. \Rightarrow 4 = \sqrt[n]{4^n} < \sqrt[n]{4^n + 3^n} < \sqrt[n]{2 \cdot 4^n} = 4 \sqrt[n]{2}. \right]$$

Riešené príklady – 60, 61

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n - 3^n} = 4$$

[60]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{4^n \left(1 - \frac{3^n}{4^n}\right)} = \lim_{n \rightarrow \infty} 4 \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} = 4 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow \frac{3}{4} \geq \left(\frac{3}{4}\right)^n > 0. \Rightarrow \frac{1}{4} = 1 - \frac{3}{4} \leq 1 - \left(\frac{3}{4}\right)^n < 1 - 0 = 1. \Rightarrow \sqrt[n]{\frac{1}{4}} \leq \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} < \sqrt[n]{1} = 1. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{4}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} \leq \lim_{n \rightarrow \infty} 1 = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{3}{4}\right)^n} = 1. \right] = 4 \cdot 1 = 4. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{4^3}{2} = \frac{64}{2} = 32 > 27 = 3^3, n \in \mathbb{N}. \Rightarrow \frac{4^{n+3}}{2} = 4^n \cdot \frac{4^3}{2} > 3^n \cdot \frac{4^3}{2} > 3^n \cdot 3^3 = 3^{n+3}. \Rightarrow \text{Pre } n \in \mathbb{N}, n \geq 3 \text{ platí } \frac{4^n}{2} > 3^n, -\frac{4^n}{2} < -3^n. \right. \\ &\quad \left. \Rightarrow 4 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{4^n}{2}} = \sqrt[n]{4^n - \frac{4^n}{2}} < \sqrt[n]{4^n - 3^n} < \sqrt[n]{4^n} = 4. \Rightarrow 4 = 4 \cdot 1 = \lim_{n \rightarrow \infty} 4 \sqrt[n]{\frac{1}{2}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{4^n - 3^n} \leq \lim_{n \rightarrow \infty} 4 = 4. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{4^n - 3^n} = 4. \right] = 4. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{4^n + 3^n} = 4$$

[61]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{4^n \left(1 + \frac{3^n}{4^n}\right)} = \lim_{n \rightarrow \infty} 4 \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} = 4 \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow 0 < \left(\frac{3}{4}\right)^n \leq \frac{3}{4} < 1. \Rightarrow 1 = 1 + 0 < 1 + \left(\frac{3}{4}\right)^n \leq 1 + \frac{3}{4} < 1 + 1 = 2. \Rightarrow 1 = \sqrt[n]{1} \leq \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} < \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} 1 \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{2} = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{3}{4}\right)^n} = 1. \right] = 4 \cdot 1 = 4. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow 4^n < 4^n + 3^n < 4^n + 4^n = 2 \cdot 4^n. \Rightarrow 4 = \sqrt[n]{4^n} < \sqrt[n]{4^n + 3^n} < \sqrt[n]{2 \cdot 4^n} = 4 \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 4 = \lim_{n \rightarrow \infty} 4 \leq \lim_{n \rightarrow \infty} \sqrt[n]{4^n + 3^n} \leq \lim_{n \rightarrow \infty} 4 \sqrt[n]{2} = 4 \cdot 1 = 4. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{4^n + 3^n} = 4. \right] = 4. \end{aligned}$$

Riešené príklady – 62, 63

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

[62]

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

[63]

Riešené príklady – 62, 63

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n}$$

[62]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n}$$

$$\bullet = \left[\begin{array}{l} \frac{3^2}{2} = \frac{9}{2} > 4 = 2^2, n \in \mathbb{N}. \Rightarrow \frac{3^{n+2}}{2} = 3^n \cdot \frac{3^2}{2} > 2^n \cdot \frac{3^2}{2} > 2^n \cdot 2^2 = 2^{n+2}. \Rightarrow \text{Pre } n \in \mathbb{N}, n \geq 2 \text{ platí } \frac{3^n}{2} > 2^n, -\frac{3^n}{2} < -2^n. \\ \Rightarrow 3 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{3^n}{2}} = \sqrt[n]{3^n - \frac{3^n}{2}} < \sqrt[n]{3^n - 2^n} < \sqrt[n]{3^n} = 3. \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1}$$

[63]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{2^n \left(1 + \frac{1}{2^n}\right)} = \lim_{n \rightarrow \infty} 2 \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} = 2 \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{2}\right)^n}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow 2 = \sqrt[n]{2^n} < \sqrt[n]{2^n + 1} < \sqrt[n]{2 \cdot 2^n} = 2 \sqrt[n]{2}. \right]$$

Riešené příklady – 62, 63

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n} = 3$$

[62]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{2^n}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow \frac{2}{3} \geq \left(\frac{2}{3}\right)^n > 0. \Rightarrow \frac{1}{3} = 1 - \frac{2}{3} \leq 1 - \left(\frac{2}{3}\right)^n < 1 - 0 = 1. \Rightarrow \sqrt[n]{\frac{1}{3}} \leq \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} < \sqrt[n]{1} = 1. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} \leq \lim_{n \rightarrow \infty} 1 = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{2}{3}\right)^n} = 1. \right] = 3 \cdot 1 = 3. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\frac{3^2}{2} = \frac{9}{2} > 4 = 2^2, n \in \mathbb{N}. \Rightarrow \frac{3^{n+2}}{2} = 3^n \cdot \frac{3^2}{2} > 2^n \cdot \frac{3^2}{2} > 2^n \cdot 2^2 = 2^{n+2}. \Rightarrow \text{Pre } n \in \mathbb{N}, n \geq 2 \text{ platí } \frac{3^n}{2} > 2^n, -\frac{3^n}{2} < -2^n. \right. \\ &\quad \left. \Rightarrow 3 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{3^n}{2}} = \sqrt[n]{3^n - \frac{3^n}{2}} < \sqrt[n]{3^n - 2^n} < \sqrt[n]{3^n} = 3. \Rightarrow 3 = 3 \cdot 1 = \lim_{n \rightarrow \infty} 3 \sqrt[n]{\frac{1}{2}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n} \leq \lim_{n \rightarrow \infty} 3 = 3. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{3^n - 2^n} = 3. \right] = 3. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2$$

[63]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{2^n \left(1 + \frac{1}{2^n}\right)} = \lim_{n \rightarrow \infty} 2 \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} = 2 \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow 0 < \left(\frac{1}{2}\right)^n \leq \frac{1}{2} < 1. \Rightarrow 1 = 1 + 0 < 1 + \left(\frac{1}{2}\right)^n \leq 1 + \frac{1}{2} < 1 + 1 = 2. \Rightarrow \sqrt[n]{1} \leq \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} < \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[n]{1} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{2} = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{2}\right)^n} = 1. \right] = 2 \cdot 1 = 2. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow 2^n < 2^n + 1 < 2^n + 2^n = 2 \cdot 2^n. \Rightarrow 2 = \sqrt[n]{2^n} < \sqrt[n]{2^n + 1} < \sqrt[n]{2 \cdot 2^n} = 2 \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 2 = \lim_{n \rightarrow \infty} 2 \leq \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} \leq \lim_{n \rightarrow \infty} 2 \sqrt[n]{2} = 2 \cdot 1 = 2. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = 2. \right] = 2. \end{aligned}$$

Riešené príklady – 64, 65

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 1}$$

[64]

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n + 1}$$

[65]

Riešené príklady – 64, 65

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 1}$$

[64]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 - \frac{1}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - \left(\frac{1}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - \left(\frac{1}{3}\right)^n}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow \frac{3^n}{2} > 1, -\frac{3^n}{2} < -1. \Rightarrow 3 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{3^n}{2}} = \sqrt[n]{3^n - \frac{3^n}{2}} < \sqrt[n]{3^n - 1} < \sqrt[n]{3^n} = 3. \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n + 1}$$

[65]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{3^n \left(1 + \frac{1}{3^n}\right)} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 + \left(\frac{1}{3}\right)^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{1}{3}\right)^n}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow 3^n < 3^n + 1 < 3^n + 3^n = 2 \cdot 3^n. \Rightarrow 3 = \sqrt[n]{3^n} < \sqrt[n]{3^n + 1} < \sqrt[n]{2 \cdot 3^n} = 3 \sqrt[n]{2}. \right]$$

Riešené příklady – 64, 65

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n - 1} = 3$$

[64]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{3^n(1 - \frac{1}{3^n})} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 - (\frac{1}{3})^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 - (\frac{1}{3})^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow \frac{1}{3} \geq (\frac{1}{3})^n > 0. \Rightarrow \frac{2}{3} = 1 - \frac{1}{3} \leq 1 - (\frac{1}{3})^n < 1 - 0 = 1. \Rightarrow \sqrt[n]{\frac{2}{3}} \leq \sqrt[n]{1 - (\frac{1}{3})^n} < \sqrt[n]{1}. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{2}{3}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 - (\frac{1}{3})^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1} = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 - (\frac{1}{3})^n} = 1. \right] = 3 \cdot 1 = 3. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow \frac{3^n}{2} > 1, -\frac{3^n}{2} < -1. \Rightarrow 3 \sqrt[n]{\frac{1}{2}} = \sqrt[n]{\frac{3^n}{2}} = \sqrt[n]{3^n - \frac{3^n}{2}} < \sqrt[n]{3^n - 1} < \sqrt[n]{3^n} = 3. \right. \\ &\quad \left. \Rightarrow 3 = 3 \cdot 1 = \lim_{n \rightarrow \infty} 3 \sqrt[n]{\frac{1}{2}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{3^n - 1} \leq \lim_{n \rightarrow \infty} 3 = 3. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{3^n - 1} = 3. \right] = 3. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{3^n + 1} = 3$$

[65]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{3^n(1 + \frac{1}{3^n})} = \lim_{n \rightarrow \infty} 3 \sqrt[n]{1 + (\frac{1}{3})^n} = 3 \lim_{n \rightarrow \infty} \sqrt[n]{1 + (\frac{1}{3})^n} \\ &= \left[n \in \mathbb{N}. \Rightarrow 0 < (\frac{1}{3})^n \leq \frac{1}{3} < 1. \Rightarrow 1 = 1 + 0 < 1 + (\frac{1}{3})^n \leq 1 + \frac{1}{3} < 1 + 1 = 2. \Rightarrow \sqrt[n]{1} \leq \sqrt[n]{1 + (\frac{1}{3})^n} < \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[n]{1} \leq \lim_{n \rightarrow \infty} \sqrt[n]{1 + (\frac{1}{3})^n} \leq \lim_{n \rightarrow \infty} \sqrt[n]{2} = 1. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{1 + (\frac{1}{3})^n} = 1. \right] = 3. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow 3^n < 3^n + 1 < 3^n + 3^n = 2 \cdot 3^n. \Rightarrow 3 = \sqrt[n]{3^n} < \sqrt[n]{3^n + 1} < \sqrt[n]{2 \cdot 3^n} = 3 \sqrt[n]{2}. \right. \\ &\quad \left. \Rightarrow 3 = \lim_{n \rightarrow \infty} 3 \leq \lim_{n \rightarrow \infty} \sqrt[n]{3^n + 1} \leq \lim_{n \rightarrow \infty} 3 \sqrt[n]{2} = 3 \cdot 1 = 3. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{3^n + 1} = 3. \right] = 3. \end{aligned}$$

Riešené príklady – 66, 67

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}}$$

[66]

$$L = \lim_{n \rightarrow \infty} \frac{a^n}{n!} \quad \text{pre } a \in \mathbb{R}.$$

[67]

Riešené príklady – 66, 67

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}}$$

[66]

Pre $n \in \mathbb{N}$ označme $a_n = \frac{n^n}{n!}$. $\Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^n}{n!}} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}}$.

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \left[n \in \mathbb{N} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \frac{(n+1)^{n+1}}{n^n(n+1)} = \frac{(n+1)^n}{n^n} = \left(1 + \frac{1}{n}\right)^n > 1. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \right]$$

$$L = \lim_{n \rightarrow \infty} \frac{a^n}{n!} \quad \text{pre } a \in \mathbb{R}.$$

[67]

Pre $a = 0$, $n \in \mathbb{N}$ platí $\frac{a^n}{n!} = \frac{0}{n!} = 0$:

Pre $a \neq 0$, $n \in \mathbb{N}$ označme $a_n = \frac{a^n}{n!}$. $\Rightarrow |a_n| = \left| \frac{a^n}{n!} \right| = \frac{|a|^n}{n!} = \frac{|a|^n}{n!} > 0$ a $\left| \lim_{n \rightarrow \infty} \frac{a^n}{n!} \right| = \lim_{n \rightarrow \infty} \left| \frac{a^n}{n!} \right| = \lim_{n \rightarrow \infty} \frac{|a|^n}{n!}$.

Riešené príklady – 66, 67

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}} = e$$

[66]

Pre $n \in \mathbb{N}$ označme $a_n = \frac{n^n}{n!}$. $\Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^n}{n!}} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}}$.

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \left[n \in \mathbb{N} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \frac{(n+1)^{n+1}}{n^n(n+1)} = \frac{(n+1)^n}{n^n} = \left(1 + \frac{1}{n}\right)^n > 1. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \right] \\ = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = e.$$

[Pokiaľ limity $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ a $\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$ existujú. $\Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{a_n}$.]

$$L = \lim_{n \rightarrow \infty} \frac{a^n}{n!} = 0 \text{ pre } a \in \mathbb{R}.$$

[67]

Pre $a = 0$, $n \in \mathbb{N}$ platí $\frac{a^n}{n!} = \frac{0}{n!} = 0$:

$$\bullet L = \lim_{n \rightarrow \infty} \frac{0}{n!} = \lim_{n \rightarrow \infty} 0 = 0. \quad \left[\text{Postupnosť } \left\{ \frac{a^n}{n!} \right\}_{n=1}^{\infty} = \{0\}_{n=1}^{\infty} \text{ je konštantná.} \right]$$

Pre $a \neq 0$, $n \in \mathbb{N}$ označme $a_n = \frac{a^n}{n!}$. $\Rightarrow |a_n| = \left| \frac{a^n}{n!} \right| = \frac{|a|^n}{n!} = \frac{|a|^n}{n!} > 0$ a $\left| \lim_{n \rightarrow \infty} \frac{a^n}{n!} \right| = \lim_{n \rightarrow \infty} \left| \frac{a^n}{n!} \right| = \lim_{n \rightarrow \infty} \frac{|a|^n}{n!}$.

$$\bullet |L| = \left| \lim_{n \rightarrow \infty} \frac{a^n}{n!} \right| = \left[\begin{aligned} \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} &= \lim_{n \rightarrow \infty} \frac{\frac{|a|^{n+1}}{(n+1)!}}{\frac{|a|^n}{n!}} = \lim_{n \rightarrow \infty} \frac{|a|}{n+1} = \frac{|a|}{\infty} = 0 < 1. \Rightarrow \lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{|a|^n}{n!} = 0. \\ \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} &= \lim_{n \rightarrow \infty} \sqrt[n]{\frac{|a|^n}{n!}} = \lim_{n \rightarrow \infty} \frac{|a|}{\sqrt[n]{n!}} = \frac{|a|}{\infty} = 0 < 1. \Rightarrow \lim_{n \rightarrow \infty} |a_n| = \lim_{n \rightarrow \infty} \frac{|a|^n}{n!} = 0. \end{aligned} \right] = 0. \Rightarrow \bullet L = 0.$$

[Pre limity postupností $\{a_n\}_{n=1}^{\infty}$, $\{|a_n|\}_{n=1}^{\infty}$ platí: $\left| \lim_{n \rightarrow \infty} a_n \right| = \lim_{n \rightarrow \infty} |a_n| = 0. \Leftrightarrow \lim_{n \rightarrow \infty} a_n = 0$.]

Riešené príklady – 68, 69, 70

$$\lim_{n \rightarrow \infty} \frac{a^n n!}{n^n}$$

pre $a \in (0; \infty)$, $a \neq e$.

[68]



$$\lim_{n \rightarrow \infty} \frac{3^n n!}{n^n}$$

[69]



$$\lim_{n \rightarrow \infty} \frac{2^n n!}{n^n}$$

[70]



Riešené príklady – 68, 69, 70

$$\lim_{n \rightarrow \infty} \frac{a^n n!}{n^n} \quad \text{pre } a \in (0; \infty), a \neq e. \quad [68]$$

$$\bullet = \left[a_n = \frac{a^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{a^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{a^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{a^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{a^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{a}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{a}{(1+\frac{1}{n})^n} = \frac{a}{e}. \right]$$

$$\lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} \quad [69]$$

$$\bullet = \left[a_n = \frac{3^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{3}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{3}{(1+\frac{1}{n})^n} = \frac{3}{e} \approx 1,104 > 1. \right]$$

$$\lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} \quad [70]$$

$$\bullet = \left[a_n = \frac{2^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{2^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{2}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{2}{(1+\frac{1}{n})^n} = \frac{2}{e} \approx 0,736 < 1. \right]$$

Riešené príklady – 68, 69, 70

$$\lim_{n \rightarrow \infty} \frac{a^n n!}{n^n} = \begin{cases} \infty & \text{pre } a \in (e; \infty), \\ 0 & \text{pre } a \in (0; e). \end{cases} \quad [68]$$

$$\bullet = \left[a_n = \frac{a^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{a^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{a^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{a^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{a^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{a}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{a}{(1+\frac{1}{n})^n} = \frac{a}{e}. \right] = \begin{cases} \infty, & \text{pre } a > e, \\ 0, & \text{pre } a < e. \end{cases}$$

$\bullet a > e. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{a}{e} > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \infty.$
 $\bullet a < e. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{a}{e} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = 0.$

$$\lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} = \infty \quad [69]$$

$$\bullet = \left[a_n = \frac{3^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{3^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{3^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{3^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{3}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{3}{(1+\frac{1}{n})^n} = \frac{3}{e} \approx 1,104 > 1. \right] = \infty.$$

$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n n!}{n^n} = \infty.$

$$\lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} = 0 \quad [70]$$

$$\bullet = \left[a_n = \frac{2^n n!}{n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}{\frac{2^n n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{2^{n+1}(n+1)}{(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{2^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \frac{2}{(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{2}{(1+\frac{1}{n})^n} = \frac{2}{e} \approx 0,736 < 1. \right] = 0.$$

$\Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2^n n!}{n^n} = 0.$

Riešené príklady – 71, 72, 73, 74

$$\lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n}$$

[71]



$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(2n)^n}$$

[72]



$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(3n)^n}$$

[73]



$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(5n)^n}$$

[74]



Riešené příklady – 71, 72, 73, 74

$$\lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n}$$

[71]

$$\bullet = \left[a_n = \frac{8^n n!}{(3n)^n} = \frac{8^n n!}{3^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8n^n (n+1)}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \lim_{n \rightarrow \infty} \frac{8}{3\left(\frac{n+1}{n}\right)^n} = \lim_{n \rightarrow \infty} \frac{8}{3\left(1+\frac{1}{n}\right)^n} = \frac{8}{3e} \approx 0,981 < 1. \right]$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(2n)^n}$$

[72]

$$\bullet = \left[a_n = \frac{7^n n!}{(2n)^n} = \frac{7^n n!}{2^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{2^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{2^n n^n}} = \lim_{n \rightarrow \infty} \frac{7n^n (n+1)}{2(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{2(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{2\left(\frac{n+1}{n}\right)^n} = \lim_{n \rightarrow \infty} \frac{7}{2\left(1+\frac{1}{n}\right)^n} = \frac{7}{2e} \approx 1,288 > 1. \right]$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(3n)^n}$$

[73]

$$\bullet = \left[a_n = \frac{7^n n!}{(3n)^n} = \frac{7^n n!}{3^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{7n^n (n+1)}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{3(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{3\left(\frac{n+1}{n}\right)^n} = \lim_{n \rightarrow \infty} \frac{7}{3\left(1+\frac{1}{n}\right)^n} = \frac{7}{3e} \approx 0,858 < 1. \right]$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(5n)^n}$$

[74]

$$\bullet = \left[a_n = \frac{7^n n!}{(5n)^n} = \frac{7^n n!}{5^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{5^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{5^n n^n}} = \lim_{n \rightarrow \infty} \frac{7n^n (n+1)}{5(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{5(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{5\left(\frac{n+1}{n}\right)^n} = \lim_{n \rightarrow \infty} \frac{7}{5\left(1+\frac{1}{n}\right)^n} = \frac{7}{5e} \approx 0,515 < 1. \right]$$

Riešené příklady – 71, 72, 73, 74

$$\lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = 0$$

[71]

$$\bullet = \left[a_n = \frac{8^n n!}{(3n)^n} = \frac{8^n n!}{3^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{8^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}}}{\frac{8^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{8^n (n+1)}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{8n^n}{3(n+1)^n} = \lim_{n \rightarrow \infty} \frac{8}{3(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{8}{3(1+\frac{1}{n})^n} = \frac{8}{3e} \approx 0,981 < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{8^n n!}{(3n)^n} = 0. \right] = 0.$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(2n)^n} = \infty$$

[72]

$$\bullet = \left[a_n = \frac{7^n n!}{(2n)^n} = \frac{7^n n!}{2^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{2^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{2^n n^n}} = \lim_{n \rightarrow \infty} \frac{7^n (n+1)}{2(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{2(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{2(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{7}{2(1+\frac{1}{n})^n} = \frac{7}{2e} \approx 1,288 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{7^n n!}{(2n)^n} = \infty. \right] = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(3n)^n} = 0$$

[73]

$$\bullet = \left[a_n = \frac{7^n n!}{(3n)^n} = \frac{7^n n!}{3^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{3^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{3^n n^n}} = \lim_{n \rightarrow \infty} \frac{7^n (n+1)}{3(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{3(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{3(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{7}{3(1+\frac{1}{n})^n} = \frac{7}{3e} \approx 0,858 < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{7^n n!}{(3n)^n} = 0. \right] = 0.$$

$$\lim_{n \rightarrow \infty} \frac{7^n n!}{(5n)^n} = 0$$

[74]

$$\bullet = \left[a_n = \frac{7^n n!}{(5n)^n} = \frac{7^n n!}{5^n n^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{7^{n+1} (n+1)!}{5^{n+1} (n+1)^{n+1}}}{\frac{7^n n!}{5^n n^n}} = \lim_{n \rightarrow \infty} \frac{7^n (n+1)}{5(n+1)^{n+1}} = \lim_{n \rightarrow \infty} \frac{7n^n}{5(n+1)^n} = \lim_{n \rightarrow \infty} \frac{7}{5(\frac{n+1}{n})^n} = \lim_{n \rightarrow \infty} \frac{7}{5(1+\frac{1}{n})^n} = \frac{7}{5e} \approx 0,515 < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{7^n n!}{(5n)^n} = 0. \right] = 0.$$

Riešené príklady – 75, 76

$$\lim_{n \rightarrow \infty} \frac{n^q}{a^n}$$

pre $a \in (0; \infty)$, $q > 0$.

[75]



$$\lim_{n \rightarrow \infty} \frac{n^3}{2^n}$$

[76]



Riešené príklady – 75, 76

$$\lim_{n \rightarrow \infty} \frac{n^q}{a^n} \quad \text{pre } a \in (0; \infty), q > 0.$$

[75]

Pre $a > 1$, $n \in \mathbb{N}$ označme $a_n = \frac{n^q}{a^n}$.

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \left[\begin{array}{l} a_n = \frac{n^q}{a^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^q}{a^{n+1}}}{\frac{n^q}{a^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^q}{a n^q} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^q}{a} = \frac{(1+0)^q}{a} = \frac{1^q}{a} = \frac{1}{a} < 1. \\ a_n = \frac{n^q}{a^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^q}{a^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^q}{a} = \frac{1^q}{a} = \frac{1}{a} < 1. \end{array} \right]$$

Pre $a = 1$, $n \in \mathbb{N}$ platí $\frac{n^q}{a^n} = \frac{n^q}{1^n} = n^q$:

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \lim_{n \rightarrow \infty} n^q$$

Pre $a < 1$, $n \in \mathbb{N}$ platí $\frac{1}{a} > 1. \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{a^n} = \lim_{n \rightarrow \infty} (\frac{1}{a})^n = \infty$.

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \lim_{n \rightarrow \infty} \frac{1}{a^n} \cdot n^q = \lim_{n \rightarrow \infty} (\frac{1}{a})^n \cdot n^q$$

$$\lim_{n \rightarrow \infty} \frac{n^3}{2^n}$$

[76]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^3}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^3}{2^{n+1}}}{\frac{n^3}{2^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^3}{2 n^3} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^3}{2} = \frac{(1+0)^3}{2} = \frac{1^3}{2} = \frac{1}{2} < 1. \\ a_n = \frac{n^3}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^3}{2^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^3}{2} = \frac{1^3}{2} = \frac{1}{2} < 1. \end{array} \right]$$

Riešené príklady – 75, 76

$$\lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \begin{cases} 0 & \text{pre } a \in (1; \infty), q > 0. \\ \infty & \text{pre } a \in (0; 1), \end{cases}$$

[75]

Pre $a > 1$, $n \in \mathbb{N}$ označme $a_n = \frac{n^q}{a^n}$.

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \left[\begin{array}{l} a_n = \frac{n^q}{a^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^q}{a^{n+1}}}{\frac{n^q}{a^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^q}{a n^q} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^q}{a} = \frac{(1+0)^q}{a} = \frac{1^q}{a} = \frac{1}{a} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = 0. \\ a_n = \frac{n^q}{a^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^q}{a^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^q}{a} = \frac{1^q}{a} = \frac{1}{a} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = 0. \end{array} \right] = 0.$$

Pre $a = 1$, $n \in \mathbb{N}$ platí $\frac{n^q}{a^n} = \frac{n^q}{1^n} = n^q$:

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \lim_{n \rightarrow \infty} n^q = \infty. \quad \left[\text{Pr. 01: } \lim_{n \rightarrow \infty} n^q = \infty \text{ pre } q > 0. \right]$$

Pre $a < 1$, $n \in \mathbb{N}$ platí $\frac{1}{a} > 1. \Rightarrow \lim_{n \rightarrow \infty} \frac{1}{a^n} = \lim_{n \rightarrow \infty} \left(\frac{1}{a}\right)^n = \infty$.

$$\bullet \lim_{n \rightarrow \infty} \frac{n^q}{a^n} = \lim_{n \rightarrow \infty} \frac{1}{a^n} \cdot n^q = \lim_{n \rightarrow \infty} \left(\frac{1}{a}\right)^n \cdot n^q = \infty \cdot \infty = \infty.$$

$$\lim_{n \rightarrow \infty} \frac{n^3}{2^n} = 0$$

[76]

$$\bullet = \left[\begin{array}{l} a_n = \frac{n^3}{2^n} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^3}{2^{n+1}}}{\frac{n^3}{2^n}} = \lim_{n \rightarrow \infty} \frac{(n+1)^3}{2 n^3} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^3}{2} = \frac{(1+0)^3}{2} = \frac{1^3}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = 0. \\ a_n = \frac{n^3}{2^n} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^3}{2^n}} = \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^3}{2} = \frac{1^3}{2} = \frac{1}{2} < 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = 0. \end{array} \right] = 0.$$

Riešené príklady – 77, 78

$$\lim_{n \rightarrow \infty} \frac{a^n}{n^q}$$

pre $a \in (0; \infty)$, $q > 0$.

[77]



$$\lim_{n \rightarrow \infty} \frac{2^n}{n^3}$$

[78]



Riešené příklady – 77, 78

$$\lim_{n \rightarrow \infty} \frac{a^n}{n^q}$$

pre $a \in (0; \infty)$, $q > 0$.

[77]

Pre $a > 1$, $n \in \mathbb{N}$ označme $a_n = \frac{a^n}{n^q}$.

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \left[\begin{array}{l} a_n = \frac{a^n}{n^q} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{a^{n+1}}{(n+1)^q}}{\frac{a^n}{n^q}} = \lim_{n \rightarrow \infty} \frac{a^{n+1}}{\frac{(n+1)^q}{n^q}} = \lim_{n \rightarrow \infty} \frac{a}{(1+\frac{1}{n})^q} = \frac{a}{(1+0)^q} = \frac{a}{1^q} = a > 1. \\ a_n = \frac{a^n}{n^q} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[q]{a_n} = \lim_{n \rightarrow \infty} \sqrt[q]{\frac{a^n}{n^q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[q]{a^n}}{(\sqrt[q]{n})^q} = \frac{a}{1^q} = \frac{a}{1} = a > 1. \end{array} \right]$$

Pre $a = 1$, $n \in \mathbb{N}$ platí $\frac{a^n}{n^q} = \frac{1^n}{n^q} = \frac{1}{n^q}$:

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \lim_{n \rightarrow \infty} \frac{1}{n^q}$$

Pre $a < 1$, $n \in \mathbb{N}$ platí $\frac{1}{a} > 1 \Rightarrow \lim_{n \rightarrow \infty} a^n = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{a^n}} = \lim_{n \rightarrow \infty} \frac{1}{(\frac{1}{a})^n} = \frac{1}{\lim_{n \rightarrow \infty} (\frac{1}{a})^n} = \frac{1}{\infty} = 0$.

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \lim_{n \rightarrow \infty} a^n \cdot \frac{1}{n^q}$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{n^3}$$

[78]

$$\bullet = \left[\begin{array}{l} a_n = \frac{2^n}{n^3} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}}{(n+1)^3}}{\frac{2^n}{n^3}} = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{\frac{(n+1)^3}{n^3}} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^3}{2} = \frac{(1+0)^3}{2} = \frac{2}{1^3} = \frac{2}{1} = 2 > 1. \\ a_n = \frac{2^n}{n^3} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[3]{a_n} = \lim_{n \rightarrow \infty} \sqrt[3]{\frac{2^n}{n^3}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{2^n}}{(\sqrt[3]{n})^3} = \frac{2}{1^3} = \frac{2}{1} = 2 > 1. \end{array} \right]$$

Riešené príklady – 77, 78

$$\lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \begin{cases} \infty & \text{pre } a \in (1; \infty), q > 0. \\ 0 & \text{pre } a \in (0; 1), \end{cases}$$

[77]

Pre $a > 1$, $n \in \mathbb{N}$ označme $a_n = \frac{a^n}{n^q}$.

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \begin{cases} a_n = \frac{a^n}{n^q} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{a^{n+1}}{(n+1)^q}}{\frac{a^n}{n^q}} = \lim_{n \rightarrow \infty} \frac{a^{n+1} n^q}{(n+1)^q a^n} = \lim_{n \rightarrow \infty} \frac{a}{(1+\frac{1}{n})^q} = \frac{a}{(1+0)^q} = \frac{a}{1^q} = a > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \infty. \\ a_n = \frac{a^n}{n^q} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{a^n}{n^q}} = \lim_{n \rightarrow \infty} \frac{a}{(\sqrt[n]{n})^q} = \frac{a}{1^q} = \frac{a}{1} = a > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \infty. \end{cases} \quad \Rightarrow \infty.$$

Pre $a = 1$, $n \in \mathbb{N}$ platí $\frac{a^n}{n^q} = \frac{1^n}{n^q} = \frac{1}{n^q}$:

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \lim_{n \rightarrow \infty} \frac{1}{n^q} = 0. \quad \left[\text{Pr. 02: } \lim_{n \rightarrow \infty} \frac{1}{n^q} = 0 \text{ pre } q > 0. \right]$$

Pre $a < 1$, $n \in \mathbb{N}$ platí $\frac{1}{a} > 1. \Rightarrow \lim_{n \rightarrow \infty} a^n = \lim_{n \rightarrow \infty} \frac{1}{\frac{1}{a^n}} = \lim_{n \rightarrow \infty} \frac{1}{(\frac{1}{a})^n} = \frac{1}{\lim_{n \rightarrow \infty} (\frac{1}{a})^n} = \frac{1}{\infty} = 0$.

$$\bullet \lim_{n \rightarrow \infty} \frac{a^n}{n^q} = \lim_{n \rightarrow \infty} a^n \cdot \frac{1}{n^q} = 0 \cdot \frac{1}{\infty} = 0 \cdot 0 = 0.$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{n^3} = \infty$$

[78]

$$\bullet = \begin{cases} a_n = \frac{2^n}{n^3} > 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}}{(n+1)^3}}{\frac{2^n}{n^3}} = \lim_{n \rightarrow \infty} \frac{2^{n+1} n^3}{(n+1)^3 2^n} = \lim_{n \rightarrow \infty} \frac{(1+\frac{1}{n})^3}{2} = \frac{(1+0)^3}{2} = \frac{2}{2} = \frac{2}{1} = 2 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \infty. \\ a_n = \frac{2^n}{n^3} \geq 0, n \in \mathbb{N}. \Rightarrow \lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{2^n}{n^3}} = \lim_{n \rightarrow \infty} \frac{2}{(\sqrt[n]{n})^3} = \frac{2}{1^3} = \frac{2}{1} = 2 > 1. \Rightarrow \lim_{n \rightarrow \infty} a_n = \infty. \end{cases} \quad \Rightarrow \infty.$$

Riešené príklady – 79, 80, 81

$$\lim_{n \rightarrow \infty} \frac{n^3}{0,5^n}$$

[79]

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+5} \right)^4$$

[80]

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

[81]

Riešené príklady – 79, 80, 81

$$\lim_{n \rightarrow \infty} \frac{n^3}{0,5^n}$$

[79]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3}{(\frac{1}{2})^n} = \lim_{n \rightarrow \infty} \frac{n^3}{\frac{1}{2^n}} = \lim_{n \rightarrow \infty} 2^n n^3$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+5} \right)^4$$

[80]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+5-2}{n+5} \right)^4 = \lim_{n \rightarrow \infty} \left(1 - \frac{2}{n+5} \right)^4$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6$$

[81]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6$$

Riešené príklady – 79, 80, 81

$$\lim_{n \rightarrow \infty} \frac{n^3}{0,5^n} = \infty$$

[79]

$$\bullet = \lim_{n \rightarrow \infty} \frac{n^3}{(\frac{1}{2})^n} = \lim_{n \rightarrow \infty} \frac{n^3}{\frac{1}{2^n}} = \lim_{n \rightarrow \infty} 2^n n^3 = \infty \cdot \infty^3 = \infty.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+5} \right)^4 = 1$$

[80]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+5-2}{n+5} \right)^4 = \lim_{n \rightarrow \infty} \left(1 - \frac{2}{n+5} \right)^4 = \left(1 - \frac{2}{\infty+5} \right)^4 = \left(1 - \frac{2}{\infty} \right)^4 = (1-0)^4 = 1^4 = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6 = 1^6 = 1.$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^6 = 1$$

[81]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^6 = \lim_{n \rightarrow \infty} \left(1 - \frac{4}{2n+3} \right)^6 = \left(1 - \frac{4}{2 \cdot \infty + 3} \right)^6 = \left(1 - \frac{4}{\infty} \right)^6 = (1-0)^6 = 1^6 = 1.$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^6 = \lim_{n \rightarrow \infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}} \right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}} \right)^6 = \left(\frac{2-0}{2+0} \right)^6 = 1^6 = 1.$$

Riešené príklady – 82, 83

$$\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \right)^4$$

[82]

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \right)^4$$

[83]

Riešené príklady – 82, 83

$$\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \right)^4$$

[82]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+2+1}{n+1} \right)^4 = \lim_{n \rightarrow \infty} \left(2 + \frac{1}{n+1} \right)^4$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \cdot \frac{1}{1} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{2+\frac{3}{n}}{1+\frac{1}{n}} \right)^4$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \right)^4$$

[83]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n-\frac{1}{2}+3+\frac{1}{2}}{2n-1} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{1}{2} + \frac{\frac{7}{2}}{2n-1} \right)^4$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \cdot \frac{1}{1} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{1+\frac{3}{n}}{2-\frac{1}{n}} \right)^4$$

Riešené príklady – 82, 83

$$\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \right)^4 = 16$$

[82]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+2+1}{n+1} \right)^4 = \lim_{n \rightarrow \infty} \left(2 + \frac{1}{n+1} \right)^4 = \left(2 + \frac{1}{\infty+1} \right)^4 = \left(2 + \frac{1}{\infty} \right)^4 \\ &= (2 + 0)^4 = 2^4 = 16. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1} \cdot \frac{1}{n} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{2+\frac{3}{n}}{1+\frac{1}{n}} \right)^4 = \left(\frac{2+\frac{3}{\infty}}{1+\frac{1}{\infty}} \right)^4 = \left(\frac{2+0}{1+0} \right)^4 = 2^4 = 16.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \right)^4 = \frac{1}{16}$$

[83]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n-\frac{1}{2}+3+\frac{1}{2}}{2n-1} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{1}{2} + \frac{\frac{7}{2}}{2n-1} \right)^4 = \left(\frac{1}{2} + \frac{\frac{7}{2}}{2 \cdot \infty - 1} \right)^4 = \left(\frac{1}{2} + \frac{\frac{7}{2}}{\infty} \right)^4 \\ &= \left(\frac{1}{2} + 0 \right)^4 = \left(\frac{1}{2} \right)^4 = \frac{1}{16}. \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+3}{2n-1} \cdot \frac{1}{n} \right)^4 = \lim_{n \rightarrow \infty} \left(\frac{1+\frac{3}{n}}{2-\frac{1}{n}} \right)^4 = \left(\frac{1+\frac{3}{\infty}}{2-\frac{1}{\infty}} \right)^4 = \left(\frac{1+0}{2-0} \right)^4 = \left(\frac{1}{2} \right)^4 = \frac{1}{16}.$$

Riešené príklady – 84, 85, 86

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

[84]

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

[85]

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n-1}{n-5}}$$

[86]

Riešené príklady – 84, 85, 86

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}}$$

[84]

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{n(8+\frac{1}{n})}{n(4-\frac{3}{n})}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

[85]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n(5+\frac{1}{n})}{n(1+\frac{5}{n})}} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5+\frac{1}{n}}{1+\frac{5}{n}}}$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow 4 \leq 4n. \Rightarrow 1 = \frac{n+5}{n+5} = \frac{n+4+1}{n+5} = \frac{n+4n+1}{n+5} = \frac{5n+1}{n+5} < \frac{5n+25}{n+5} = \frac{5(n+5)}{n+5} = 5. \right]$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n-1}{n-5}}$$

[86]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n(5-\frac{1}{n})}{n(1-\frac{5}{n})}} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5-\frac{1}{n}}{1-\frac{5}{n}}}$$

$$\bullet = \left[n \in \mathbb{N}, n \geq 6. \quad 0 < \frac{1}{6} \leq \frac{1}{n} \leq 1. \Rightarrow 4 = 5 - 1 < 5 - \frac{1}{n} = \frac{5n-1}{n} < \frac{5n-1}{n-5} < \frac{5n-25+24}{n-5} = 5 + \frac{24}{n-5} < 5 + 24 = 29. \right]$$

Riešené príklady – 84, 85, 86

$$\lim_{n \rightarrow \infty} 4^{\frac{8n+1}{4n-3}} = 16$$

[84]

$$\bullet = 4^{\lim_{n \rightarrow \infty} \frac{8n+1}{4n-3}} = 4^{\lim_{n \rightarrow \infty} \frac{n(8+\frac{1}{n})}{n(4-\frac{3}{n})}} = 4^{\lim_{n \rightarrow \infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^{\frac{8}{4}} = 4^2 = 16.$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1$$

[85]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n(5+\frac{1}{n})}{n(1+\frac{5}{n})}} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5+\frac{1}{n}}{1+\frac{5}{n}}} = \left[n \in \mathbb{N}. \Rightarrow 0 < \frac{1}{n} \leq 1. \Rightarrow \frac{5}{6} = \frac{5+0}{1+5 \cdot 1} < \frac{5+\frac{1}{n}}{1+\frac{5}{n}} < \frac{5+1}{1+5 \cdot 0} = 6. \right. \\ \left. \Rightarrow \sqrt[6]{\frac{5}{6}} < \sqrt[n]{\frac{5+\frac{1}{n}}{1+\frac{5}{n}}} < \sqrt[6]{6}. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[6]{\frac{5}{6}} \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5+\frac{1}{n}}{1+\frac{5}{n}}} \leq \lim_{n \rightarrow \infty} \sqrt[6]{6} = 1. \right] = 1.$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow 4 \leq 4n. \Rightarrow 1 = \frac{n+5}{n+5} = \frac{n+4+1}{n+5} = \frac{n+4n+1}{n+5} = \frac{5n+1}{n+5} < \frac{5n+25}{n+5} = \frac{5(n+5)}{n+5} = 5. \right. \\ \left. \Rightarrow 1 = \sqrt[5]{1} < \sqrt[n]{\frac{5n+1}{n+5}} < \sqrt[5]{5}. \Rightarrow 1 = \lim_{n \rightarrow \infty} 1 \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim_{n \rightarrow \infty} \sqrt[5]{5} = 1. \right] = 1.$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n-1}{n-5}} = 1$$

[86]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n(5-\frac{1}{n})}{n(1-\frac{5}{n})}} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5-\frac{1}{n}}{1-\frac{5}{n}}} = \left[n \in \mathbb{N}, n \geq 6. \Rightarrow 0 < \frac{1}{n} \leq \frac{1}{6} \leq 1. \Rightarrow 4 = \frac{5-1}{1-5 \cdot 0} < \frac{5-\frac{1}{n}}{1-\frac{5}{n}} < \frac{5-0}{1-\frac{5}{6}} = \frac{5}{\frac{1}{6}} = 30. \right. \\ \left. \Rightarrow \sqrt[4]{4} < \sqrt[n]{\frac{5-\frac{1}{n}}{1-\frac{5}{n}}} < \sqrt[30]{30}. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[4]{4} \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5-\frac{1}{n}}{1-\frac{5}{n}}} \leq \lim_{n \rightarrow \infty} \sqrt[30]{30} = 1. \right] = 1.$$

$$\bullet = \left[n \in \mathbb{N}, n \geq 6. \Rightarrow 0 < \frac{1}{n} \leq \frac{1}{6} \leq 1. \Rightarrow 4 = 5-1 < 5-\frac{1}{n} = \frac{5n-1}{n-5} < \frac{5n-1}{n-5} < \frac{5n-25+24}{n-5} = 5 + \frac{24}{n-5} < 5+24 = 29. \right. \\ \left. \Rightarrow \sqrt[4]{4} < \sqrt[n]{\frac{5n-1}{n-5}} < \sqrt[29]{29}. \Rightarrow 1 = \lim_{n \rightarrow \infty} \sqrt[4]{4} \leq \lim_{n \rightarrow \infty} \sqrt[n]{\frac{5n-1}{n-5}} \leq \lim_{n \rightarrow \infty} \sqrt[29]{29} = 1. \right] = 1.$$

Riešené príklady – 87, 88

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n}$$

[87]

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+2}\right)^{n+2}$$

[88]

Riešené príklady – 87, 88

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n}$$

[87]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \cdot \left(1 + \frac{1}{n}\right)^n \right] = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right]^2$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+2}\right)^{n+2}$$

[88]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+2+1}{n+2}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2} = \left[\lim_{n+2 \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2} = e. \right]$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+2+1}{n+2}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2}$$

$$\bullet = \left[\begin{array}{l} \text{Subst. } m = n+2 \mid n \rightarrow \infty \\ n+3 = m+1 \mid m \rightarrow \infty \end{array} \right] = \lim_{m \rightarrow \infty} \left(\frac{m+1}{m}\right)^m$$

Riešené príklady – 87, 88

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n} = e^2$$

[87]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \cdot \left(1 + \frac{1}{n}\right)^n \right] = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \cdot e = e^2.$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \right]^2 = e^2.$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^2 = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right]^2 = e^2.$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+3}{n+2}\right)^{n+2} = e$$

[88]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+2+1}{n+2}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2} = \left[\lim_{n+2 \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2} = e \right] = e.$$

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+2+1}{n+2}\right)^{n+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+2}\right)^{n+2} = \left[\text{Subst. } k = n+2 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e.$$

$$\bullet = \left[\text{Subst. } m = n+2 \begin{array}{l} n \rightarrow \infty \\ n+3 = m+1 \\ m \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left(\frac{m+1}{m}\right)^m = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{m}\right)^m = e.$$

Riešené príklady – 89, 90

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$$

[89]

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

[90]

Riešené príklady – 89, 90

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$$

[89]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^n$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{n}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{n}{n+1}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6}$$

[90]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{n+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2n+3}} \end{aligned}$$

Riešené príklady – 89, 90

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n = e$$

[89]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^n = \left[\text{Subst. } k = n+1 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^{k-1} \\ &= \lim_{k \rightarrow \infty} \left[\left(1 + \frac{1}{k} \right)^k \left(1 + \frac{1}{k} \right)^{-1} \right] = e \cdot \left(1 + \frac{1}{\infty} \right)^{-1} = e \cdot (1+0)^{-1} = e. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{n}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{n}{n+1}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{n+1} = \left[\text{Subst. } k = n+1 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n}{n+1} = \lim_{n \rightarrow \infty} \frac{n}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n}} = \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1. \right] = e^1 = e. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n+6} = e^{-2}$$

[90]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{n+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2n+3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } k = 2n+3 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{6}{n})}{n(2+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{1+\frac{6}{n}}{2+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}. \right] = (e^{-4})^{\frac{1}{2}} = e^{-2}. \end{aligned}$$

Riešené príklady – 91, 92, 93, 94

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

[91]

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

[92]

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{n^2+1}$$

[93]

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{\sqrt{n}+1}$$

[94]

Riešené príklady – 91, 92, 93, 94

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1}$$

[91]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^{\frac{n^2+1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1}$$

[92]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^{\frac{\sqrt{n}+1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{n^2+1}$$

[93]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n}\right)^n \right]^{\frac{n^2+1}{n}}$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{\sqrt{n}+1}$$

[94]

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n}\right)^n \right]^{\frac{\sqrt{n}+1}{n}}$$

Riešené príklady – 91, 92, 93, 94

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2+1} = \infty \quad [91]$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^{\frac{n^2+1}{n}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \right] \left[\lim_{n \rightarrow \infty} \frac{n^2+1}{n} = \lim_{n \rightarrow \infty} \left(n + \frac{1}{n}\right) = \infty + \frac{1}{\infty} = \infty + 0 = \infty \right] = e^\infty = \infty.$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{\sqrt{n}+1} = 1 \quad [92]$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n}\right)^n \right]^{\frac{\sqrt{n}+1}{n}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \right] \left[\lim_{n \rightarrow \infty} \frac{\sqrt{n}+1}{n} = \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{n}\right) = \frac{1}{\infty} + \frac{1}{\infty} = 0 + 0 = 0 \right] = e^0 = 1.$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{n^2+1} = 0 \quad [93]$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n}\right)^n \right]^{\frac{n^2+1}{n}} = \left[\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1} \right] \left[\lim_{n \rightarrow \infty} \frac{n^2+1}{n} = \lim_{n \rightarrow \infty} \left(n + \frac{1}{n}\right) = \infty + \frac{1}{\infty} = \infty + 0 = \infty \right] = (e^{-1})^\infty = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{\sqrt{n}+1} = 1 \quad [94]$$

$$\bullet = \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n}\right)^n \right]^{\frac{\sqrt{n}+1}{n}} = \left[\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1} \right] \left[\lim_{n \rightarrow \infty} \frac{\sqrt{n}+1}{n} = \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{n}\right) = \frac{1}{\infty} + \frac{1}{\infty} = 0 + 0 = 0 \right] = (e^{-1})^0 = e^0 = 1.$$

Riešené príklady – 95, 96

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

[95]

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

[96]

Riešené príklady – 95, 96

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6}$$

[95]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{n^6+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6}$$

[96]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right]^{\frac{n^6+6}{2n-3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3}} \end{aligned}$$

Riešené příklady – 95, 96

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{n^6+6} = 0$$

[95]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{n^6+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } k = 2n+3 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4} \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^6+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{n(n^5+\frac{6}{n})}{n(2+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty \right] = (e^{-4})^\infty = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n-3} \right)^{n^6+6} = \infty$$

[96]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n-3+2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right]^{\frac{n^6+6}{2n-3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2n-3} \right)^{2n-3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{2}{2n-3} \right)^{2n-3} = \left[\text{Subst. } k = 2n-3 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{2}{k} \right)^k = e^2 \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^6+6}{2n-3} = \lim_{n \rightarrow \infty} \frac{n(n^5+\frac{6}{n})}{n(2-\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{n^5+\frac{6}{n}}{2-\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty \right] = (e^2)^\infty = e^\infty = \infty. \end{aligned}$$

Riešené príklady – 97, 98

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n+6}$$

[97]

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6 - 1}{2n^6 + 3} \right)^{n^6 + 6}$$

[98]

Riešené príklady – 97, 98

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6-1}{2n^6+3} \right)^{n+6}$$

[97]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n^6+3-4}{2n^6+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\frac{n+6}{2n^6+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2n^6+3}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6-1}{2n^6+3} \right)^{n^6+6}$$

[98]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n^6+3-4}{2n^6+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\frac{n^6+6}{2n^6+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n^6+3}} \end{aligned}$$

Riešené příklady – 97, 98

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6-1}{2n^6+3} \right)^{n+6} = 1$$

[97]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n^6+3-4}{2n^6+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\frac{n+6}{2n^6+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2n^6+3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} = \left[\text{Subst. } k = 2n^6+3 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4} \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n+6}{2n^6+3} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{6}{n})}{n(2n^5+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2\infty+\frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0 \right] = (e^{-4})^0 = e^0 = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n^6-1}{2n^6+3} \right)^{n^6+6} = e^{-2}$$

[98]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n^6+3-4}{2n^6+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{n^6+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\frac{n^6+6}{2n^6+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} \right]^{\lim_{n \rightarrow \infty} \frac{n^6+6}{2n^6+3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n^6+3} \right)^{2n^6+3} = \left[\text{Subst. } k = 2n^6+3 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4} \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^6+6}{2n^6+3} = \lim_{n \rightarrow \infty} \frac{n^6(1+\frac{6}{n^6})}{n^6(2+\frac{3}{n^6})} = \lim_{n \rightarrow \infty} \frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2} \right] = (e^{-4})^{\frac{1}{2}} = e^{-2}. \end{aligned}$$

Riešené príklady – 99, 100

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3} \right)^{\sqrt[6]{n}+6}$$

[99]

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n}+6}$$

[100]

Riešené príklady – 99, 100

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3} \right)^{\sqrt[6]{n}+6}$$

[99]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n}+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n}+3} \right)^{2\sqrt[6]{n}+3} \right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n}+3} \right)^{2\sqrt[6]{n}+3} \right] \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n}+6}$$

[100]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{\sqrt[6]{n}+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right] \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2n+3} \end{aligned}$$

Riešené príklady – 99, 100

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = e^{-2}$$

[99]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n}+3} \right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n}+3} \right] \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n}+3} = \left[\text{Subst. } k = 2\sqrt[6]{n}+3 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}(1+\frac{6}{\sqrt[6]{n}})}{\sqrt[6]{n}(2+\frac{3}{\sqrt[6]{n}})} = \lim_{n \rightarrow \infty} \frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}. \right] = (e^{-4})^{\frac{1}{2}} = e^{-2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+3} \right)^{\sqrt[6]{n}+6} = 0$$

[100]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2n+3-4}{2n+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{\sqrt[6]{n}+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right]^{\frac{\sqrt[6]{n}+6}{2n+3}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2n+3} \right)^{2n+3} \right] \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2n+3} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2n+3} \right)^{2n+3} = \left[\text{Subst. } k = 2n+3 \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}+6}{2n+3} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n}(1+\frac{6}{\sqrt[6]{n}})}{n(2+\frac{3}{n})} = \lim_{n \rightarrow \infty} \frac{1+\frac{6}{\sqrt[6]{n}}}{\sqrt[6]{n^5}(2+\frac{3}{n})} = \frac{1+\frac{6}{\infty}}{\infty(2+\frac{3}{\infty})} = \frac{1+0}{\infty(2+0)} = \frac{1}{\infty} = 0. \right] = (e^{-4})^0 = e^0 = 1. \end{aligned}$$

Riešené príklady – 101, 102

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3} \right)^{n+6}$$

[101]

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3} \right)^{n+6}$$

[102]

Riešené príklady – 101, 102

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6}$$

[101]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right]^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n-3}} \right)^{n+6}$$

[102]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n-3}+2}{2\sqrt[6]{n-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{2\sqrt[6]{n-3}} \right]^{\frac{n+6}{2\sqrt[6]{n-3}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{2\sqrt[6]{n-3}} \right]^{\lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n-3}}} \end{aligned}$$

Riešené príklady – 101, 102

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n+3}} \right)^{n+6} = 0$$

[101]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n+3}-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right]^{\frac{n+6}{2\sqrt[6]{n+3}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} \right] \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-4}{2\sqrt[6]{n+3}} \right)^{2\sqrt[6]{n+3}} = \left[\text{Subst. } k = 2\sqrt[6]{n+3} \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-4}{k} \right)^k = e^{-4}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n+3}} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{6}{n})}{\sqrt[6]{n}(2+\frac{3}{\sqrt[6]{n}})} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5}(1+\frac{6}{n})}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty(1+\frac{6}{\infty})}{2+\frac{3}{\infty}} = \frac{\infty(1+0)}{2+0} = \frac{\infty}{2} = \infty. \right] = (e^{-4})^\infty = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n-3}} \right)^{n+6} = \infty$$

[102]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{n+6} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{2\sqrt[6]{n-3}} \right]^{\frac{n+6}{2\sqrt[6]{n-3}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{2\sqrt[6]{n-3}} \right] \lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n-3}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{2}{2\sqrt[6]{n-3}} \right)^{2\sqrt[6]{n-3}} = \left[\text{Subst. } k = 2\sqrt[6]{n-3} \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{2}{k} \right)^k = e^2. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n+6}{2\sqrt[6]{n-3}} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{6}{n})}{\sqrt[6]{n}(2-\frac{3}{\sqrt[6]{n}})} = \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n^5}(1+\frac{6}{n})}{2-\frac{3}{\sqrt[6]{n}}} = \frac{\infty(1+\frac{6}{\infty})}{2-\frac{3}{\infty}} = \frac{\infty(1+0)}{2-0} = \frac{\infty}{2} = \infty. \right] = (e^2)^\infty = e^\infty = \infty. \end{aligned}$$

Riešené příklady – 103, 104

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}+1}$$

[103]

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}}$$

[104]

Riešené príklady – 103, 104

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}}$$

[103]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}}$$

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}}$$

[104]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} \right]^{\frac{\sqrt[3]{n}}{\sqrt[3]{n+1}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} \right]^{\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}}} \end{aligned}$$

Riešené príklady – 103, 104

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} = e \quad [103]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} \\ &= \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} = \left[\text{Subst. } k = \sqrt[3]{n+1} \left| \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right. \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] = e. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+2}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = e \quad [104]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \left[\text{Subst. } k = \sqrt[3]{n+1} \left| \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right. \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^{k-1} \\ &= \lim_{k \rightarrow \infty} \left[\left(1 + \frac{1}{k} \right)^k \left(1 + \frac{1}{k} \right)^{-1} \right] = e \cdot \left(1 + \frac{1}{\infty} \right)^{-1} = e \cdot (1 + 0)^{-1} = e. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n+1+1}}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n}} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} \right]^{\frac{\sqrt[3]{n}}{\sqrt[3]{n+1}}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} \right] \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{\sqrt[3]{n+1}} \right)^{\sqrt[3]{n+1}} = \left[\text{Subst. } k = \sqrt[3]{n+1} \left| \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right. \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n} \left(1 + \frac{1}{\sqrt[3]{n}} \right)} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{\sqrt[3]{n}}} = \frac{1}{1 + 0} = \frac{1}{1+0} = 1. \right] = e^1 = e. \end{aligned}$$

Riešené príklady – 105, 106

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^{\sqrt{n}}$$

[105]

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$$

[106]

Riešené príklady – 105, 106

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^{\sqrt{n}}$$

[105]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^{\sqrt{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{\sqrt{n}} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{\sqrt{n}}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+1}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n$$

[106]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^{n^2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{n^2} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{n^2}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{n^2}{n+1}} \end{aligned}$$

Riešené príklady – 105, 106

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^{\sqrt{n}} = 1$$

[105]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^{\sqrt{n}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{\sqrt{n}} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{\sqrt{n}}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+1}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{n+1} = \left[\text{Subst. } k = n+1 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+1} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}(1+\frac{1}{n})} = \frac{1}{\infty(1+\frac{1}{\infty})} = \frac{1}{\infty(1+0)} = \frac{1}{\infty} = 0. \right] = e^0 = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^n = \infty$$

[106]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+1+1}{n+1} \right)^{n^2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{n^2} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\frac{n^2}{n+1}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n+1} \right)^{n+1} \right]^{\lim_{n \rightarrow \infty} \frac{n^2}{n+1}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+1} \right)^{n+1} = \left[\text{Subst. } k = n+1 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^2}{n+1} = \lim_{n \rightarrow \infty} \frac{n^2}{n(1+\frac{1}{n})} = \lim_{n \rightarrow \infty} \frac{n}{1+\frac{1}{n}} = \frac{\infty}{1+\frac{1}{\infty}} = \frac{\infty}{1+0} = \infty. \right] = e^\infty = \infty. \end{aligned}$$

Riešené príklady – 107, 108

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+4}$$

[107]

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+6}$$

[108]

Riešené príklady – 107, 108

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+4}$$

[107]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+4}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} \right]^{\frac{n^4+4}{n^4+6}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} \right]^{\lim_{n \rightarrow \infty} \frac{n^4+4}{n^4+6}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+6}$$

[108]

$$\bullet = \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+6}$$

Riešené príklady – 107, 108

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+4} = e^{-3}$$

[107]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+4} = \left[\begin{array}{l} \text{Subst. } k = n^4+6 \mid n \rightarrow \infty \\ n^4+4 = k-2 \mid k \rightarrow \infty \end{array} \right] \\ &= \lim_{k \rightarrow \infty} \left(1 + \frac{-3}{k} \right)^{k-2} = \lim_{k \rightarrow \infty} \left[\left(1 + \frac{-3}{k} \right)^k \left(1 + \frac{-3}{k} \right)^{-2} \right] = e^{-3} \cdot \left(1 + \frac{-3}{\infty} \right)^{-2} \\ &= e^{-3} \cdot (1+0)^{-1} = e^{-3}. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+4} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} \right]^{\frac{n^4+4}{n^4+6}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} \right]^{\lim_{n \rightarrow \infty} \frac{n^4+4}{n^4+6}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} = \left[\begin{array}{l} \text{Subst. } k = n^4+6 \mid n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-3}{k} \right)^k = e^{-3} \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^4+4}{n^4+6} = \lim_{n \rightarrow \infty} \frac{n^4(1+\frac{4}{n^4})}{n^4(1+\frac{6}{n^4})} = \lim_{n \rightarrow \infty} \frac{1+\frac{4}{n^4}}{1+\frac{6}{n^4}} = \frac{1+\frac{4}{\infty}}{1+\frac{6}{\infty}} = \frac{1+0}{1+0} = 1 \right] = (e^{-3})^1 = e^{-3}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^4+3}{n^4+6} \right)^{n^4+6} = e^{-3}$$

[108]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n^4+6-3}{n^4+6} \right)^{n^4+6} = \lim_{n \rightarrow \infty} \left(1 + \frac{-3}{n^4+6} \right)^{n^4+6} = \left[\begin{array}{l} \text{Subst. } k = n^4+6 \mid n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] \\ &= \lim_{k \rightarrow \infty} \left(1 + \frac{-3}{k} \right)^k = e^{-3}. \end{aligned}$$

Riešené príklady – 109, 110

$$\lim_{n \rightarrow \infty} \left(\frac{n+5}{n+7} \right)^{n^4+1}$$

[109]

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

[110]

Riešené príklady – 109, 110

$$\lim_{n \rightarrow \infty} \left(\frac{n+5}{n+7} \right)^{n^4+1}$$

[109]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(\frac{n+7-2}{n+7} \right)^{n^4+1} = \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+7} \right)^{n^4+1} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-2}{n+7} \right)^{n+7} \right]^{\frac{n^4+1}{n+7}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-2}{n+7} \right)^{n+7} \right]^{\lim_{n \rightarrow \infty} \frac{n^4+1}{n+7}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n$$

[110]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right]^n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \end{aligned}$$

Riešené príklady – 109, 110

$$\lim_{n \rightarrow \infty} \left(\frac{n+5}{n+7} \right)^{n^4+1} = 0$$

[109]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(\frac{n+7-2}{n+7} \right)^{n^4+1} = \lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+7} \right)^{n^4+1} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-2}{n+7} \right)^{n+7} \right]^{\frac{n^4+1}{n+7}} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{-2}{n+7} \right)^{n+7} \right]^{\lim_{n \rightarrow \infty} \frac{n^4+1}{n+7}} = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+7} \right)^{n+7} = \left[\text{Subst. } k = n+7 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-2}{k} \right)^k = e^{-2}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \frac{n^4+1}{n+7} = \lim_{n \rightarrow \infty} \frac{n(n^3+\frac{1}{n})}{n(1+\frac{7}{n})} = \lim_{n \rightarrow \infty} \frac{n^3+\frac{1}{n}}{1+\frac{7}{n}} = \frac{\infty+\frac{1}{\infty}}{1+\frac{7}{\infty}} = \frac{\infty+0}{1+0} = \infty. \right] = (e^{-2})^\infty = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} \right)^n = \sqrt{6}$$

[110]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3}}{2} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^n \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} \right]^{n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2}} \\ &= \left[\lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} \right)^{\frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2}} = \left[\text{Subst. } k = \frac{2}{\sqrt[n]{2} + \sqrt[n]{3} - 2} \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \left| \begin{array}{l} \sqrt[n]{2} > 1, \sqrt[n]{2} \rightarrow 1 \\ \sqrt[n]{3} > 1, \sqrt[n]{3} \rightarrow 1 \end{array} \right| \sqrt[n]{2} + \sqrt[n]{3} - 2 > 0 \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{1}{k} \right)^k = e. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} n \frac{\sqrt[n]{2} + \sqrt[n]{3} - 2}{2} = \lim_{n \rightarrow \infty} \frac{n(\sqrt[n]{2}-1) + n(\sqrt[n]{3}-1)}{2} = \left[\text{ViL: } \lim_{\alpha \rightarrow 1} \frac{n(\sqrt[n]{\alpha}-1)}{2} = \ln \alpha \text{ pre } \alpha > 0. \right] = \frac{\ln 2 + \ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln \sqrt{6}. \right] = e^{\ln \sqrt{6}} = \sqrt{6}. \end{aligned}$$

Riešené príklady – 111, 112

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$

[111]

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

[112]

Riešené příklady – 111, 112

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right)$$

[111]

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right)$$

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot n \left(\frac{\sqrt[n]{3}}{\sqrt[n]{2}} - 1 \right) = \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) = \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right)$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

[112]

$$\bullet = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 + 1 - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} \left[n \left(\sqrt[n]{2} - 1 \right) - n \left(\sqrt[n+1]{2} - 1 \right) \right]$$

$$= \left[n \in \mathbb{N} \Rightarrow n \left(\sqrt[n+1]{2} - 1 \right) = (n+1-1) \left(\sqrt[n+1]{2} - 1 \right) = (n+1) \left(\sqrt[n+1]{2} - 1 \right) - \sqrt[n+1]{2} + 1 = 1 - \sqrt[n+1]{2} + (n+1) \left(\sqrt[n+1]{2} - 1 \right) \right]$$

$$= -1 + \lim_{n \rightarrow \infty} \sqrt[n+1]{2} - \lim_{n \rightarrow \infty} (n+1) \left(\sqrt[n+1]{2} - 1 \right) + \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right)$$

Riešené příklady – 111, 112

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - \sqrt[n]{2} \right) = \ln 3 - \ln 2 = \ln \frac{3}{2}$$

[111]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2} \right) = \lim_{n \rightarrow \infty} \left[n \left(\sqrt[n]{3} - 1 \right) - n \left(\sqrt[n]{2} - 1 \right) \right] \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\alpha} - 1 \right) = \ln \alpha \text{ pre } \alpha > 0. \right] = \ln 3 - \ln 2 = \ln \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot n \left(\frac{\sqrt[n]{3}}{\sqrt[n]{2}} - 1 \right) = \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) = \lim_{n \rightarrow \infty} \sqrt[n]{2} \cdot \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\frac{3}{2}} - 1 \right) \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} \sqrt[n]{\alpha} = 1, \lim_{n \rightarrow \infty} n \left(\sqrt[n]{\alpha} - 1 \right) = \ln \alpha \text{ pre } \alpha > 0. \right] = 1 \cdot \ln \frac{3}{2} = \ln \frac{3}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = 0$$

[112]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 + 1 - \sqrt[n+1]{2} \right) = \lim_{n \rightarrow \infty} \left[n \left(\sqrt[n]{2} - 1 \right) - n \left(\sqrt[n+1]{2} - 1 \right) \right] \\ &= \left[n \in \mathbb{N}. \Rightarrow n \left(\sqrt[n+1]{2} - 1 \right) = (n+1-1) \left(\sqrt[n+1]{2} - 1 \right) = (n+1) \left(\sqrt[n+1]{2} - 1 \right) - \sqrt[n+1]{2} + 1 = 1 - \sqrt[n+1]{2} + (n+1) \left(\sqrt[n+1]{2} - 1 \right). \right] \\ &= -1 + \lim_{n \rightarrow \infty} \sqrt[n+1]{2} - \lim_{n \rightarrow \infty} (n+1) \left(\sqrt[n+1]{2} - 1 \right) + \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) \\ &= \left[\text{Subst. } k = n+1 \left| \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right. \right] = -1 + \lim_{k \rightarrow \infty} \sqrt[k]{2} - \lim_{k \rightarrow \infty} k \left(\sqrt[k]{2} - 1 \right) + \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} \sqrt[n]{\alpha} = 1 \text{ pre } \alpha > 0. \right] \left[\lim_{k \rightarrow \infty} k \left(\sqrt[k]{2} - 1 \right) = \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right). \Rightarrow - \lim_{k \rightarrow \infty} k \left(\sqrt[k]{2} - 1 \right) + \lim_{n \rightarrow \infty} n \left(\sqrt[n]{2} - 1 \right) = 0. \right] = -1 + 1 + 0 = 0. \end{aligned}$$

Riešené příklady – 113

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

[113]

Riešené príklady – 113

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right)$$

[113]

$$\begin{aligned} \bullet &= \left[n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \right. \\ &= n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{1}{2^{\frac{1}{n(n+1)}}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n(n+1)}} - 1}{2^{\frac{1}{n(n+1)}}} \right) = n^2 \cdot \frac{2^{\frac{1}{n}}}{2^{\frac{1}{n(n+1)}}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = n^2 \cdot \frac{\sqrt[n]{2}}{n(n+1)\sqrt[n]{2}} \left(n(n+1)\sqrt[n]{2} - 1 \right) = \frac{n \cdot n(n+1)}{n+1} \cdot \frac{\sqrt[n]{2}}{n(n+1)\sqrt[n]{2}} \left(n(n+1)\sqrt[n]{2} - 1 \right). \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \left[n(n+1) \left(n(n+1)\sqrt[n]{2} - 1 \right) \right] \cdot \lim_{n \rightarrow \infty} \frac{1}{n(n+1)\sqrt[n]{2}} \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2}$$

$$\begin{aligned} \bullet &= \left[n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = n^2 \cdot 2^{\frac{1}{n+1}} \left(\frac{2^{\frac{1}{n}}}{2^{\frac{1}{n+1}}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n+1}} \left(2^{\frac{1}{n} - \frac{1}{n+1}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n+1}} \left(2^{\frac{(n+1)-n}{n(n+1)}} - 1 \right) \right. \\ &= n^2 \cdot 2^{\frac{1}{n+1}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n+1}} \left(\frac{2^{\frac{1}{n(n+1)}} - 1}{2^{\frac{1}{n(n+1)}}} \right) = \frac{n \cdot n(n+1)}{n+1} \cdot \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n(n+1)}}} \left(n(n+1)\sqrt[n]{2} - 1 \right) = \frac{n}{n+1} \cdot 2^{\frac{1}{n+1}} \cdot n(n+1) \left(n(n+1)\sqrt[n]{2} - 1 \right). \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \left[n(n+1) \left(n(n+1)\sqrt[n]{2} - 1 \right) \right] \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} 2^{\frac{1}{n+1}}$$

Riešené príklady – 113

$$\lim_{n \rightarrow \infty} n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = \ln 2$$

[113]

$$\begin{aligned} &= \left[n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{1}{n+1} - \frac{1}{n}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{n-(n+1)}{n(n+1)}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(1 - 2^{\frac{-1}{n(n+1)}} \right) \right. \\ &\quad \left. = n^2 \cdot 2^{\frac{1}{n}} \left(1 - \frac{1}{2^{\frac{n(n+1)}{1}}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{n(n+1)}{1}} - 1}{2^{\frac{n(n+1)}{1}}} \right) = n^2 \cdot \frac{2^{\frac{1}{n}}}{2^{\frac{n(n+1)}{1}}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = n^2 \cdot \frac{\sqrt[n]{2}}{2^{\frac{n(n+1)}{1}}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = \frac{n \cdot n(n+1)}{n+1} \cdot \frac{\sqrt[n]{2}}{2^{\frac{n(n+1)}{1}}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) \right] \\ &= \lim_{n \rightarrow \infty} \left[n(n+1) \left(2^{\frac{1}{n(n+1)}} - 1 \right) \right] \cdot \lim_{n \rightarrow \infty} \frac{1}{2^{\frac{n(n+1)}{1}}} \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \\ &= \left[\text{Subst. } k = n(n+1) \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] \left[n \in \mathbb{N} \Rightarrow \frac{n}{n+1} = \frac{n}{n(1+\frac{1}{n})} = \frac{1}{1+\frac{1}{n}} \right] = \left[\text{ViL: } \lim_{\alpha \rightarrow \infty} \sqrt[\alpha]{\alpha} = 1, \lim_{\alpha \rightarrow \infty} (\sqrt[\alpha]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0. \right] \\ &= \lim_{k \rightarrow \infty} k \left(\sqrt[k]{k} - 1 \right) \cdot \lim_{k \rightarrow \infty} \frac{1}{\sqrt[k]{k}} \cdot \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n}} \cdot \lim_{n \rightarrow \infty} \sqrt[n]{2} \\ &= \ln 2 \cdot \frac{1}{1} \cdot \frac{1}{1+\frac{1}{\infty}} \cdot 1 = \ln 2 \cdot 1 \cdot \frac{1}{1+0} \cdot 1 = \ln 2. \end{aligned}$$

$$\begin{aligned} &= \left[n^2 \left(\sqrt[n]{2} - \sqrt[n+1]{2} \right) = n^2 \left(2^{\frac{1}{n}} - 2^{\frac{1}{n+1}} \right) = n^2 \cdot 2^{\frac{1}{n}} \left(\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n}} \left(2^{\frac{1}{n} - \frac{1}{n+1}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n}} \left(2^{\frac{(n+1)-n}{n(n+1)}} - 1 \right) \right. \\ &\quad \left. = n^2 \cdot \sqrt[n]{2} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = n^2 \cdot 2^{\frac{1}{n}} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = \frac{n \cdot n(n+1)}{n+1} \cdot \sqrt[n+1]{2} \left(2^{\frac{1}{n(n+1)}} - 1 \right) = \frac{n}{n+1} \cdot \sqrt[n+1]{2} \cdot n(n+1) \left(2^{\frac{1}{n(n+1)}} - 1 \right) \right] \\ &= \lim_{n \rightarrow \infty} \left[n(n+1) \left(2^{\frac{1}{n(n+1)}} - 1 \right) \right] \cdot \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \lim_{n \rightarrow \infty} \sqrt[n+1]{2} = \left[\text{Subst. } k = n(n+1) \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] \\ &\quad \left[\text{Subst. } m = n+1 \mid \begin{matrix} n \rightarrow \infty \\ m \rightarrow \infty \end{matrix} \right] \left[n \in \mathbb{N} \Rightarrow \frac{n}{n+1} = \frac{n}{n(1+\frac{1}{n})} = \frac{1}{1+\frac{1}{n}} \right] = \lim_{k \rightarrow \infty} k \left(\sqrt[k]{k} - 1 \right) \cdot \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n}} \cdot \lim_{m \rightarrow \infty} \sqrt[m]{2} \\ &= \left[\text{ViL: } \lim_{\alpha \rightarrow \infty} \sqrt[\alpha]{\alpha} = 1, \lim_{\alpha \rightarrow \infty} (\sqrt[\alpha]{\alpha} - 1) = \ln \alpha \text{ pre } \alpha > 0. \right] = \ln 2 \cdot \frac{1}{1+\frac{1}{\infty}} \cdot 1 = \ln 2 \cdot \frac{1}{1+0} \cdot 1 = \ln 2. \end{aligned}$$

Riešené príklady – 114, 115, 116, 117

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} \right] \quad [114]$$

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n} \quad [115]$$

$$\lim_{n \rightarrow \infty} n \sin \frac{1}{n} \quad [116]$$

$$\lim_{n \rightarrow \infty} \sin \frac{1}{n} \quad [117]$$

Riešené príklady – 114, 115, 116, 117

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right] \quad [114]$$

Pre všetky $n \in \mathbb{N}$ platí $2^n \leq 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} = 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \leq 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n$.

$$\bullet = \left[\begin{aligned} n \in \mathbb{N}. & \Rightarrow 2^n \leq 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \leq n \cdot 2^n. \Rightarrow \ln 2^n \leq \ln [2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n}] \leq \ln (n \cdot 2^n). \\ & \Rightarrow \ln 2 = \frac{n \ln 2}{n} = \frac{\ln 2^n}{n} \leq \frac{1}{n} \ln [2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n}] \leq \frac{\ln (n \cdot 2^n)}{n} = \frac{\ln n + \ln 2^n}{n} = \frac{\ln n + n \ln 2}{n} = \frac{1}{n} \ln n + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \end{aligned} \right]$$

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n} \quad [115]$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow -1 \leq \sin n \leq 1. \Rightarrow -\frac{1}{n} \leq \frac{\sin n}{n} \leq \frac{1}{n}. \Rightarrow 0 = -\lim_{n \rightarrow \infty} \frac{1}{n} = \lim_{n \rightarrow \infty} \left(-\frac{1}{n}\right) \leq \lim_{n \rightarrow \infty} \frac{\sin n}{n} \leq \lim_{n \rightarrow \infty} \frac{1}{n} = 0. \right]$$

$$\lim_{n \rightarrow \infty} n \sin \frac{1}{n} \quad [116]$$

$$\bullet = \left[\text{Subst. } \left. \begin{aligned} k &= \frac{1}{n} \\ n &= \frac{1}{k} \end{aligned} \right| \begin{aligned} n &\rightarrow \infty \\ k &\rightarrow 0 \end{aligned} \right] = \lim_{k \rightarrow 0} \frac{1}{k} \sin k = \lim_{k \rightarrow 0} \frac{\sin k}{k}$$

$$\lim_{n \rightarrow \infty} \sin \frac{1}{n} \quad [117]$$

$$\bullet = \left[\text{Subst. } \left. \begin{aligned} k &= \frac{1}{n} \\ n &= \frac{1}{k} \end{aligned} \right| \begin{aligned} n &\rightarrow \infty \\ k &\rightarrow 0 \end{aligned} \right] = \lim_{k \rightarrow 0} \sin k$$

Riešené príklady – 114, 115, 116, 117

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \right] = \ln 2 \quad [114]$$

Pre všetky $n \in \mathbb{N}$ platí $2^n \leq 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} = 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \dots + 2^{\frac{n}{n}} \leq 2^n + 2^n + 2^n + \dots + 2^n = n \cdot 2^n$.

$$\bullet = \left[\begin{array}{l} n \in \mathbb{N}. \Rightarrow 2^n \leq 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n} \leq n \cdot 2^n. \Rightarrow \ln 2^n \leq \ln [2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n}] \leq \ln (n \cdot 2^n). \\ \Rightarrow \ln 2 = \frac{\ln 2^n}{n} = \frac{\ln 2^n}{n} \leq \frac{1}{n} \ln [2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \dots + \sqrt[n]{2^n}] \leq \frac{\ln (n \cdot 2^n)}{n} = \frac{\ln n + \ln 2^n}{n} = \frac{\ln n + n \ln 2}{n} = \frac{1}{n} \ln n + \ln 2 = \ln \sqrt[n]{n} + \ln 2. \\ \Rightarrow \ln 2 = \lim_{n \rightarrow \infty} \ln 2 \leq L \leq \lim_{n \rightarrow \infty} [\ln \sqrt[n]{n} + \ln 2] = \lim_{n \rightarrow \infty} \ln \sqrt[n]{n} + \ln 2 = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2. \Rightarrow L = \ln 2. \end{array} \right] = \ln 2.$$

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n} = 0 \quad [115]$$

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow -1 \leq \sin n \leq 1. \Rightarrow -\frac{1}{n} \leq \frac{\sin n}{n} \leq \frac{1}{n}. \Rightarrow 0 = -\lim_{n \rightarrow \infty} \frac{1}{n} = \lim_{n \rightarrow \infty} \left(-\frac{1}{n}\right) \leq \lim_{n \rightarrow \infty} \frac{\sin n}{n} \leq \lim_{n \rightarrow \infty} \frac{1}{n} = 0. \Rightarrow \lim_{n \rightarrow \infty} \frac{\sin n}{n} = 0. \right] = 0.$$

$$\lim_{n \rightarrow \infty} n \sin \frac{1}{n} = 1 \quad [116]$$

$$\bullet = \left[\text{Subst. } \left. \begin{array}{l} k = \frac{1}{n} \\ n = \frac{1}{k} \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow 0 \end{array} \right] = \lim_{k \rightarrow 0} \frac{1}{k} \sin k = \lim_{k \rightarrow 0} \frac{\sin k}{k} = \left[\text{ViL: } \lim_{\oplus \rightarrow 0} \frac{\sin \oplus}{\oplus} = 1. \right] = 1.$$

$$\lim_{n \rightarrow \infty} \sin \frac{1}{n} = 0 \quad [117]$$

$$\bullet = \left[\text{Subst. } \left. \begin{array}{l} k = \frac{1}{n} \\ n = \frac{1}{k} \end{array} \right| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow 0 \end{array} \right] = \lim_{k \rightarrow 0} \sin k = \sin 0 = 0.$$

Riešené príklady – 118, 119

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right]$$

[118]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 + 1} - n^2 \right]$$

[119]

Riešené příklady – 118, 119

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right]$$

[118]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n} \cdot \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \sqrt{n} \left[\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right] \cdot \frac{\sqrt{n+1}+n}{\sqrt{n+1}+n} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - n] \cdot [\sqrt{n+1} + n] = (n+1) - n^2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n+1-n^2}{\sqrt{n+1}+n} \\ &= \left[\begin{array}{l} n \in \mathbb{N}. \\ \Rightarrow n = \sqrt{n^2} = \sqrt{n} \cdot \sqrt{n}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n(1+\frac{1}{n}-n)}{n[\sqrt{\frac{1}{n}+\frac{1}{n^2}}+1]} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}-n}{\sqrt{\frac{1}{n}+\frac{1}{n^2}}+1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3+1} - n^2 \right]$$

[119]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3} \cdot \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \sqrt{n^3} \left[\sqrt{1 + \frac{1}{n^3}} - \sqrt{n} \right]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3+1} - n^2 \right] \cdot \frac{\sqrt{n^3+1}+n^2}{\sqrt{n^3+1}+n^2} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^3+1} - n^2] \cdot [\sqrt{n^3+1} + n^2] = (n^3+1) - n^4. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^3+1-n^4}{\sqrt{n^3+1}+n^2} \\ &= \left[\begin{array}{l} n \in \mathbb{N}. \\ \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3} \cdot \sqrt{n}. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^3(1+\frac{1}{n^3}-n)}{n^2[\sqrt{\frac{1}{n}+\frac{1}{n^4}}+1]} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{1}{n}-n)}{\sqrt{\frac{1}{n}+\frac{1}{n^4}}+1} \end{aligned}$$

Riešené příklady – 118, 119

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right] = -\infty$$

[118]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n} \cdot \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \sqrt{n} \left[\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right] = \infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \infty \right] \\ &= \infty \cdot (\sqrt{1+0} - \infty) = \infty \cdot (1 - \infty) = \infty \cdot (-\infty) = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - n \right] \cdot \frac{\sqrt{n+1}+n}{\sqrt{n+1}+n} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - n] \cdot [\sqrt{n+1} + n] = (n+1) - n^2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n+1-n^2}{\sqrt{n+1}+n} \\ &= \left[\begin{array}{l} n \in \mathbb{N}. \\ \Rightarrow n = \sqrt{n^2} = \sqrt{n} \cdot n. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n(1+\frac{1}{n}-n)}{n \left[\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{1+\frac{1}{n}-n}{\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1} = \frac{1+\frac{1}{\infty}-\infty}{\sqrt{\frac{1}{\infty} + \frac{1}{\infty}} + 1} = \frac{1+0-\infty}{\sqrt{0+0+1}} = \frac{-\infty}{1} = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3+1} - n^2 \right] = -\infty$$

[119]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3} \cdot \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \sqrt{n^3} \left[\sqrt{1 + \frac{1}{n^3}} - \sqrt{n} \right] = \infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \infty \right] \\ &= \infty \cdot (\sqrt{1+0} - \infty) = \infty \cdot (1 - \infty) = \infty \cdot (-\infty) = -\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3+1} - n^2 \right] \cdot \frac{\sqrt{n^3+1}+n^2}{\sqrt{n^3+1}+n^2} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^3+1} - n^2] \cdot [\sqrt{n^3+1} + n^2] = (n^3+1) - n^4. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^3+1-n^4}{\sqrt{n^3+1}+n^2} \\ &= \left[\begin{array}{l} n \in \mathbb{N}. \\ \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3} \cdot n. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^3(1+\frac{1}{n^3}-n)}{n^2 \left[\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{n(1+\frac{1}{n}-n)}{\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1} = \frac{\infty(1+\frac{1}{\infty}-\infty)}{\sqrt{\frac{1}{\infty} + \frac{1}{\infty}} + 1} = \frac{\infty(1+0-\infty)}{\sqrt{0+0+1}} \\ &= \frac{\infty(-\infty)}{1} = -\infty. \end{aligned}$$

Riešené příklady – 120

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n + 1 \right]$$

[120]

Riešené príklady – 120

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n + 1 \right]$$

[120]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + n + 1} + (n - 1)}{\sqrt{n^2 + n + 1} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n + 1} - (n - 1)] \cdot [\sqrt{n^2 + n + 1} + (n - 1)] = (n^2 + n + 1) - (n - 1)^2 = (n^2 + n + 1) - (n^2 - 2n + 1) = 3n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n}{\sqrt{n^2 + n + 1} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{3n}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] \cdot \frac{\sqrt{n^2 + n + 1} + n}{\sqrt{n^2 + n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n + 1} - n] \cdot [\sqrt{n^2 + n + 1} + n] = (n^2 + n + 1) - n^2 = n + 1. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt{n^2 + n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1} \end{aligned}$$

Riešené príklady – 120

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n + 1 \right] = \frac{3}{2}$$

[120]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + n + 1} + (n - 1)}{\sqrt{n^2 + n + 1} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n + 1} - (n - 1)] \cdot [\sqrt{n^2 + n + 1} + (n - 1)] = (n^2 + n + 1) - (n - 1)^2 = (n^2 + n + 1) - (n^2 - 2n + 1) = 3n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n}{\sqrt{n^2 + n + 1} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{3n}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{3}{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{3}{\sqrt{1 + 0 + 0} + 1 - 0} = \frac{3}{1 + 1} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] \cdot \frac{\sqrt{n^2 + n + 1} + n}{\sqrt{n^2 + n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n + 1} - n] \cdot [\sqrt{n^2 + n + 1} + n] = (n^2 + n + 1) - n^2 = n + 1. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt{n^2 + n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1} \\ &= 1 + \frac{1 + \frac{1}{\infty}}{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{1 + 0}{\sqrt{1 + 0 + 0} + 1} = 1 + \frac{1}{1 + 1} = 1 + \frac{1}{2} = \frac{3}{2}. \end{aligned}$$

Riešené příklady – 121

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n + 1 \right]$$

[121]

Riešené príklady – 121

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n + 1 \right]$$

[121]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + n} + (n - 1)}{\sqrt{n^2 + n} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n} - (n - 1)] \cdot [\sqrt{n^2 + n} + (n - 1)] = (n^2 + n) - (n - 1)^2 = (n^2 + n) - (n^2 - 2n + 1) = 3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n - 1}{\sqrt{n^2 + n} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(3 - \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3 - \frac{1}{n}}{\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] \cdot \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n} - n] \cdot [\sqrt{n^2 + n} + n] = (n^2 + n) - n^2 = n. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt{1 + \frac{1}{n}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} \end{aligned}$$

Riešené príklady – 121

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n + 1 \right] = \frac{3}{2}$$

[121]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + n} + (n - 1)}{\sqrt{n^2 + n} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n} - (n - 1)] \cdot [\sqrt{n^2 + n} + (n - 1)] = (n^2 + n) - (n - 1)^2 = (n^2 + n) - (n^2 - 2n + 1) = 3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n - 1}{\sqrt{n^2 + n} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(3 - \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3 - \frac{1}{n}}{\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n}} = \frac{3 - \frac{1}{\infty}}{\sqrt{1 + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{3 - 0}{\sqrt{1 + 0} + 1 - 0} = \frac{3}{1 + 1} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] \cdot \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + n} - n] \cdot [\sqrt{n^2 + n} + n] = (n^2 + n) - n^2 = n. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt{1 + \frac{1}{n}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} \\ &= 1 + \frac{1}{\sqrt{1 + \frac{1}{\infty}} + 1} = 1 + \frac{1}{\sqrt{1 + 0} + 1} = 1 + \frac{1}{1 + 1} = 1 + \frac{1}{2} = \frac{3}{2}. \end{aligned}$$

Riešené příklady – 122

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - n - 1 \right]$$

[122]

Riešené príklady – 122

$$\lim_{n \rightarrow \infty} [\sqrt{n^2 - n} - n - 1]$$

[122]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - (n + 1) \right] \cdot \frac{\sqrt{n^2 - n} + (n + 1)}{\sqrt{n^2 - n} + (n + 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - (n + 1)] \cdot [\sqrt{n^2 - n} + (n + 1)] = (n^2 - n) - (n + 1)^2 = (n^2 - n) - (n^2 + 2n + 1) = -3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-3n - 1}{\sqrt{n^2 - n} + n + 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(-3 - \frac{1}{n})}{n[\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n}]} \\ &= \lim_{n \rightarrow \infty} \frac{-3 - \frac{1}{n}}{\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= -1 + \lim_{n \rightarrow \infty} [\sqrt{n^2 - n} - n] = -1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - n \right] \cdot \frac{\sqrt{n^2 - n} + n}{\sqrt{n^2 - n} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - n] \cdot [\sqrt{n^2 - n} + n] = (n^2 - n) - n^2 = -n. \right] = -1 + \lim_{n \rightarrow \infty} \frac{-n}{\sqrt{n^2 - n} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = -1 - \lim_{n \rightarrow \infty} \frac{n}{n[\sqrt{1 - \frac{1}{n}} + 1]} = -1 - \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 - \frac{1}{n}} + 1} \end{aligned}$$

Riešené príklady – 122

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - n - 1 \right] = -\frac{3}{2}$$

[122]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - (n + 1) \right] \cdot \frac{\sqrt{n^2 - n} + (n + 1)}{\sqrt{n^2 - n} + (n + 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - (n + 1)] \cdot [\sqrt{n^2 - n} + (n + 1)] = (n^2 - n) - (n + 1)^2 = (n^2 - n) - (n^2 + 2n + 1) = -3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-3n - 1}{\sqrt{n^2 - n} + n + 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(-3 - \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{-3 - \frac{1}{n}}{\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n}} = \frac{-3 - \frac{1}{\infty}}{\sqrt{1 + \frac{1}{\infty}} + 1 + \frac{1}{\infty}} = \frac{-3 - 0}{\sqrt{1 + 0} + 1 - 0} = \frac{-3}{1 + 1} = -\frac{3}{2}. \end{aligned}$$

$$\begin{aligned} \bullet &= -1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - n \right] = -1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n} - n \right] \cdot \frac{\sqrt{n^2 - n} + n}{\sqrt{n^2 - n} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - n] \cdot [\sqrt{n^2 - n} + n] = (n^2 - n) - n^2 = -n. \right] = -1 + \lim_{n \rightarrow \infty} \frac{-n}{\sqrt{n^2 - n} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = -1 - \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt{1 - \frac{1}{n}} + 1 \right]} = -1 - \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 - \frac{1}{n}} + 1} \\ &= -1 - \frac{1}{\sqrt{1 - \frac{1}{\infty}} + 1} = -1 - \frac{1}{\sqrt{1 - 0} + 1} = -1 - \frac{1}{1 + 1} = -1 - \frac{1}{2} = -\frac{3}{2}. \end{aligned}$$

Riešené příklady – 123

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n - 1} - n + 2 \right]$$

[123]

Riešené príklady – 123

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n - 1} - n + 2 \right]$$

[123]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n - 1} - (n - 2) \right] \cdot \frac{\sqrt{n^2 + 2n - 1} + (n - 2)}{\sqrt{n^2 + 2n - 1} + (n - 2)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - (n - 2)] \cdot [\sqrt{n^2 + 2n - 1} + (n - 2)] = (n^2 + 2n - 1) - (n - 2)^2 = (n^2 + 2n - 1) - (n^2 - 4n + 4) = 6n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{6n - 5}{\sqrt{n^2 + 2n - 1} + n - 2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(6 - \frac{5}{n})}{n \left[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{6 - \frac{5}{n}}{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n}} \end{aligned}$$

$$\begin{aligned} &= 2 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n - 1} - n \right] = 2 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n - 1} - n \right] \cdot \frac{\sqrt{n^2 + 2n - 1} + n}{\sqrt{n^2 + 2n - 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - n] \cdot [\sqrt{n^2 + 2n - 1} + n] = (n^2 + 2n - 1) - n^2 = 2n - 1. \right] = 2 + \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 + 2n - 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 2 + \lim_{n \rightarrow \infty} \frac{2n - 1}{n \left[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 \right]} = 2 + \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1} \end{aligned}$$

Riešené příklady – 123

$$\lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n - 1} - n + 2] = 3$$

[123]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n - 1} - (n - 2)] \cdot \frac{\sqrt{n^2 + 2n - 1} + (n - 2)}{\sqrt{n^2 + 2n - 1} + (n - 2)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - (n - 2)] \cdot [\sqrt{n^2 + 2n - 1} + (n - 2)] = (n^2 + 2n - 1) - (n - 2)^2 = (n^2 + 2n - 1) - (n^2 - 4n + 4) = 6n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{6n - 5}{\sqrt{n^2 + 2n - 1} + n - 2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(6 - \frac{5}{n})}{n[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n}]} \\ &= \lim_{n \rightarrow \infty} \frac{6 - \frac{5}{n}}{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n}} = \frac{6 - \frac{5}{\infty}}{\sqrt{1 + \frac{2}{\infty} - \frac{1}{\infty}} + 1 - \frac{2}{\infty}} = \frac{6 - 0}{\sqrt{1 + 0 - 0} + 1 - 0} = \frac{6}{1 + 1} = 3. \end{aligned}$$

$$\begin{aligned} &= 2 + \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n - 1} - n] = 2 + \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n - 1} - n] \cdot \frac{\sqrt{n^2 + 2n - 1} + n}{\sqrt{n^2 + 2n - 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - n] \cdot [\sqrt{n^2 + 2n - 1} + n] = (n^2 + 2n - 1) - n^2 = 2n - 1. \right] = 2 + \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 + 2n - 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 2 + \lim_{n \rightarrow \infty} \frac{2n - 1}{n[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1]} = 2 + \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1} \\ &= 2 + \frac{2 - \frac{1}{\infty}}{\sqrt{1 + \frac{2}{\infty} - \frac{1}{\infty}} + 1} = 2 + \frac{2 - 0}{\sqrt{1 + 0 - 0} + 1} = 2 + \frac{2}{1 + 1} = 2 + 1 = 3. \end{aligned}$$

Riešené příklady – 124

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n + 4} - n - 3 \right]$$

[124]

Riešené príklady – 124

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n + 4} - n - 3 \right]$$

[124]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n + 4} - (n + 3) \right] \cdot \frac{\sqrt{n^2 + 2n + 4} + (n + 3)}{\sqrt{n^2 + 2n + 4} + (n + 3)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n + 4} - (n + 3)] \cdot [\sqrt{n^2 + 2n + 4} + (n + 3)] = (n^2 + 2n + 4) - (n + 3)^2 = (n^2 + 2n + 4) - (n^2 + 6n + 9) = -4n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-4n - 5}{\sqrt{n^2 + 2n + 4} + n + 3} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(-4 - \frac{5}{n})}{n \left[\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1 + \frac{3}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{-4 - \frac{5}{n}}{\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1 + \frac{3}{n}} \end{aligned}$$

$$\begin{aligned} &= -3 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n + 4} - n \right] = -3 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2n + 4} - n \right] \cdot \frac{\sqrt{n^2 + 2n + 4} + n}{\sqrt{n^2 + 2n + 4} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n + 4} - n] \cdot [\sqrt{n^2 + 2n + 4} + n] = (n^2 + 2n + 4) - n^2 = 2n + 4. \right] = -3 + \lim_{n \rightarrow \infty} \frac{2n + 4}{\sqrt{n^2 + 2n + 4} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = -3 + \lim_{n \rightarrow \infty} \frac{2n + 4}{n \left[\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1 \right]} = -3 + \lim_{n \rightarrow \infty} \frac{2 + \frac{4}{n}}{\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1} \end{aligned}$$

Riešené příklady – 124

$$\lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n + 4} - n - 3] = -2$$

[124]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n + 4} - (n + 3)] \cdot \frac{\sqrt{n^2 + 2n + 4} + (n + 3)}{\sqrt{n^2 + 2n + 4} + (n + 3)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n + 4} - (n + 3)] \cdot [\sqrt{n^2 + 2n + 4} + (n + 3)] = (n^2 + 2n + 4) - (n + 3)^2 = (n^2 + 2n + 4) - (n^2 + 6n + 9) = -4n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-4n - 5}{\sqrt{n^2 + 2n + 4} + n + 3} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(-4 - \frac{5}{n})}{n[\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1 + \frac{3}{n}]} \\ &= \lim_{n \rightarrow \infty} \frac{-4 - \frac{5}{n}}{\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1 + \frac{3}{n}} = \frac{-4 - \frac{5}{\infty}}{\sqrt{1 + \frac{2}{\infty} + \frac{4}{\infty}} + 1 + \frac{3}{\infty}} = \frac{-4 - 0}{\sqrt{1 + 0 + 0} + 1 + 0} = \frac{-4}{1 + 1} = -2. \end{aligned}$$

$$\begin{aligned} &= -3 + \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n + 4} - n] = -3 + \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2n + 4} - n] \cdot \frac{\sqrt{n^2 + 2n + 4} + n}{\sqrt{n^2 + 2n + 4} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n + 4} - n] \cdot [\sqrt{n^2 + 2n + 4} + n] = (n^2 + 2n + 4) - n^2 = 2n + 4. \right] = -3 + \lim_{n \rightarrow \infty} \frac{2n + 4}{\sqrt{n^2 + 2n + 4} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = -3 + \lim_{n \rightarrow \infty} \frac{2n + 4}{n[\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1]} = -3 + \lim_{n \rightarrow \infty} \frac{2 + \frac{4}{n}}{\sqrt{1 + \frac{2}{n} + \frac{4}{n^2}} + 1} \\ &= -3 + \frac{2 + \frac{4}{\infty}}{\sqrt{1 + \frac{2}{\infty} + \frac{4}{\infty}} + 1} = -3 + \frac{2 + 0}{\sqrt{1 + 0 + 0} + 1} = -3 + \frac{2}{1 + 1} = -3 + 1 = -2. \end{aligned}$$

Riešené příklady – 125

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n + 1 \right]$$

[125]

Riešené príklady – 125

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n + 1 \right]$$

[125]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 4n + 1} - (n - 1)] \cdot [\sqrt{n^2 + 4n + 1} + (n - 1)] = (n^2 + 4n + 1) - (n - 1)^2 = (n^2 + 4n + 1) - (n^2 - 2n + 1) = 6n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{6n}{n \left[\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n \right] \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 4n + 1} - n] \cdot [\sqrt{n^2 + 4n + 1} + n] = (n^2 + 4n + 1) - n^2 = 4n + 1. \right] = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{n \left[\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} \end{aligned}$$

Riešené príklady – 125

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n + 1 \right] = 3$$

[125]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - (n - 1) \right] \cdot \frac{\sqrt{n^2 + 4n + 1} + (n - 1)}{\sqrt{n^2 + 4n + 1} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 4n + 1} - (n - 1)] \cdot [\sqrt{n^2 + 4n + 1} + (n - 1)] = (n^2 + 4n + 1) - (n - 1)^2 = (n^2 + 4n + 1) - (n^2 - 2n + 1) = 6n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{6n}{\sqrt{n^2 + 4n + 1} + n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{6n}{n \left[\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{6}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n}} = \frac{6}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty}} = \frac{6}{\sqrt{1 + 0 + 0} + 1 - 0} = \frac{6}{1 + 1} = 3. \end{aligned}$$

$$\begin{aligned} \bullet &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 4n + 1} - n \right] \cdot \frac{\sqrt{n^2 + 4n + 1} + n}{\sqrt{n^2 + 4n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 4n + 1} - n] \cdot [\sqrt{n^2 + 4n + 1} + n] = (n^2 + 4n + 1) - n^2 = 4n + 1. \right] = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 1 + \lim_{n \rightarrow \infty} \frac{4n + 1}{n \left[\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{\sqrt{1 + \frac{4}{n} + \frac{1}{n^2}} + 1} \\ &= 1 + \frac{4 + \frac{1}{\infty}}{\sqrt{1 + \frac{4}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} = 1 + \frac{4}{1 + 1} = 1 + 2 = 3. \end{aligned}$$

Riešené příklady – 126

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 3n + 1} - n + 3 \right]$$

[126]

Riešené príklady – 126

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 3n + 1} - n + 3 \right]$$

[126]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 3n + 1} - (n - 3) \right] \cdot \frac{\sqrt{n^2 - 3n + 1} + (n - 3)}{\sqrt{n^2 - 3n + 1} + (n - 3)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - 3n + 1} - (n - 3)] \cdot [\sqrt{n^2 - 3n + 1} + (n - 3)] = (n^2 - 3n + 1) - (n - 3)^2 = (n^2 - 3n + 1) - (n^2 - 6n + 9) = 3n - 8. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n - 8}{\sqrt{n^2 - 3n + 1} + n - 3} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(3 - \frac{8}{n})}{n \left[\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3 - \frac{8}{n}}{\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n}} \end{aligned}$$

$$\begin{aligned} &= 3 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 3n + 1} - n \right] = 3 + \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - 3n + 1} - n \right] \cdot \frac{\sqrt{n^2 - 3n + 1} + n}{\sqrt{n^2 - 3n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - 3n + 1} - n] \cdot [\sqrt{n^2 - 3n + 1} + n] = (n^2 - 3n + 1) - n^2 = -3n + 1. \right] = 3 + \lim_{n \rightarrow \infty} \frac{-3n + 1}{\sqrt{n^2 - 3n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 3 + \lim_{n \rightarrow \infty} \frac{-3n + 1}{n \left[\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 \right]} = 3 + \lim_{n \rightarrow \infty} \frac{-3 + \frac{1}{n}}{\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1} \end{aligned}$$

Riešené příklady – 126

$$\lim_{n \rightarrow \infty} [\sqrt{n^2 - 3n + 1} - n + 3] = \frac{3}{2}$$

[126]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} [\sqrt{n^2 - 3n + 1} - (n - 3)] \cdot \frac{\sqrt{n^2 - 3n + 1} + (n - 3)}{\sqrt{n^2 - 3n + 1} + (n - 3)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - 3n + 1} - (n - 3)] \cdot [\sqrt{n^2 - 3n + 1} + (n - 3)] = (n^2 - 3n + 1) - (n - 3)^2 = (n^2 - 3n + 1) - (n^2 - 6n + 9) = 3n - 8. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n - 8}{\sqrt{n^2 - 3n + 1} + n - 3} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(3 - \frac{8}{n})}{n[\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n}]} \\ &= \lim_{n \rightarrow \infty} \frac{3 - \frac{8}{n}}{\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n}} = \frac{3 - \frac{5}{\infty}}{\sqrt{1 - \frac{3}{\infty} + \frac{1}{\infty}} + 1 - \frac{3}{\infty}} = \frac{3 - 0}{\sqrt{1 - 0 + 0} + 1 - 0} = \frac{3}{1 + 1} = \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} &= 3 + \lim_{n \rightarrow \infty} [\sqrt{n^2 - 3n + 1} - n] = 3 + \lim_{n \rightarrow \infty} [\sqrt{n^2 - 3n + 1} - n] \cdot \frac{\sqrt{n^2 - 3n + 1} + n}{\sqrt{n^2 - 3n + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - 3n + 1} - n] \cdot [\sqrt{n^2 - 3n + 1} + n] = (n^2 - 3n + 1) - n^2 = -3n + 1. \right] = 3 + \lim_{n \rightarrow \infty} \frac{-3n + 1}{\sqrt{n^2 - 3n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = 3 + \lim_{n \rightarrow \infty} \frac{-3n + 1}{n[\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1]} = 3 + \lim_{n \rightarrow \infty} \frac{-3 + \frac{1}{n}}{\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1} \\ &= 3 + \frac{-3 + \frac{1}{\infty}}{\sqrt{1 - \frac{3}{\infty} + \frac{1}{\infty}} + 1} = 3 + \frac{-3 + 0}{\sqrt{1 - 0 + 0} + 1} = 3 + \frac{-3}{1 + 1} = 3 - \frac{3}{2} = \frac{3}{2}. \end{aligned}$$

Riešené příklady – 127, 128

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right]$$

[127]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right]$$

[128]

Riešené príklady – 127, 128

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right]$$

[127]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] \cdot \frac{\sqrt{n^2 + n + 1} + n}{\sqrt{n^2 + n + 1} + n} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + n + 1} - n] \cdot [\sqrt{n^2 + n + 1} + n] = (n^2 + n + 1) - n^2 = n + 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt{n^2 + n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right]$$

[128]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] \cdot \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + n} - n] \cdot [\sqrt{n^2 + n} + n] = (n^2 + n) - n^2 = n. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n} + n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt{1 + \frac{1}{n}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} \end{aligned}$$

Riešené príklady – 127, 128

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] = \frac{1}{2}$$

[127]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - n \right] \cdot \frac{\sqrt{n^2 + n + 1} + n}{\sqrt{n^2 + n + 1} + n} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + n + 1} - n] \cdot [\sqrt{n^2 + n + 1} + n] = (n^2 + n + 1) - n^2 = n + 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt{n^2 + n + 1} + n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1} \\ &= \frac{1 + \frac{1}{\infty}}{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = \frac{1 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{1}{1 + 1} = \frac{1}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] = \frac{1}{2}$$

[128]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n} - n \right] \cdot \frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + n} - n] \cdot [\sqrt{n^2 + n} + n] = (n^2 + n) - n^2 = n. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n} + n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt{1 + \frac{1}{n}} + 1 \right]} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} \\ &= \frac{1}{\sqrt{1 + \frac{1}{\infty}} + 1} = \frac{1}{\sqrt{1 + 0} + 1} = \frac{1}{1 + 1} = \frac{1}{2}. \end{aligned}$$

Riešené příklady – 129, 130, 131

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right] \quad \text{pre } p, q \in \mathbb{R}, p \neq q.$$

[129]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right]$$

[130]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right]$$

[131]

Riešené príklady – 129, 130, 131

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right] \quad \text{pre } p, q \in \mathbb{R}, p \neq q.$$

[129]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right] \cdot \frac{\sqrt{n+p} + \sqrt{n+q}}{\sqrt{n+p} + \sqrt{n+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+p} - \sqrt{n+q}] \cdot [\sqrt{n+p} + \sqrt{n+q}] = (n+p) - (n+q) = p - q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt{n+p} + \sqrt{n+q}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right]$$

[130]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right] \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n}] \cdot [\sqrt{n+1} + \sqrt{n}] = (n+1) - n = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right]$$

[131]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right] \cdot \frac{\sqrt{n+1} + \sqrt{n-1}}{\sqrt{n+1} + \sqrt{n-1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n-1}] \cdot [\sqrt{n+1} + \sqrt{n-1}] = (n+1) - (n-1) = 2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n+1} + \sqrt{n-1}} \end{aligned}$$

Riešené príklady – 129, 130, 131

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right] = 0 \text{ pre } p, q \in \mathbb{R}, p \neq q. \quad [129]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+p} - \sqrt{n+q} \right] \cdot \frac{\sqrt{n+p} + \sqrt{n+q}}{\sqrt{n+p} + \sqrt{n+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+p} - \sqrt{n+q}] \cdot [\sqrt{n+p} + \sqrt{n+q}] = (n+p) - (n+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p-q}{\sqrt{n+p} + \sqrt{n+q}} = \frac{p-q}{\infty + \infty} = \frac{p-q}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right] = 0 \quad [130]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n} \right] \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n}] \cdot [\sqrt{n+1} + \sqrt{n}] = (n+1) - n = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right] = 0 \quad [131]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n+1} - \sqrt{n-1} \right] \cdot \frac{\sqrt{n+1} + \sqrt{n-1}}{\sqrt{n+1} + \sqrt{n-1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n-1}] \cdot [\sqrt{n+1} + \sqrt{n-1}] = (n+1) - (n-1) = 2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n+1} + \sqrt{n-1}} = \frac{2}{\infty + \infty} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené príklady – 132

$$\lim_{n \rightarrow \infty} \left[\sqrt{an + p} - \sqrt{an + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[132]

Riešené príklady – 132

$$\lim_{n \rightarrow \infty} \left[\sqrt{an+p} - \sqrt{an+q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[132]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{an+p} - \sqrt{an+q} \right] \cdot \frac{\sqrt{an+p} + \sqrt{an+q}}{\sqrt{an+p} + \sqrt{an+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{an+p} - \sqrt{an+q}] \cdot [\sqrt{an+p} + \sqrt{an+q}] = (an+p) - (an+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p-q}{\sqrt{an+p} + \sqrt{an+q}} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = an \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+p} - \sqrt{k+q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k+p} - \sqrt{k+q} \right] \cdot \frac{\sqrt{k+p} + \sqrt{k+q}}{\sqrt{k+p} + \sqrt{k+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p-q}{\sqrt{k+p} + \sqrt{k+q}} \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{\alpha + \frac{p}{n}} - \sqrt{\alpha + \frac{q}{n}} \right]$$

Riešené príklady – 132

$$\lim_{n \rightarrow \infty} \left[\sqrt{an+p} - \sqrt{an+q} \right] = 0 \text{ pre } p, q \in R, p \neq q, a > 0.$$

[132]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{an+p} - \sqrt{an+q} \right] \cdot \frac{\sqrt{an+p} + \sqrt{an+q}}{\sqrt{an+p} + \sqrt{an+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt{an+p} - \sqrt{an+q}] \cdot [\sqrt{an+p} + \sqrt{an+q}] = (an+p) - (an+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p-q}{\sqrt{an+p} + \sqrt{an+q}} \\ &= \frac{p-q}{\infty + \infty} = \frac{p-q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = an \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+p} - \sqrt{k+q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k+p} - \sqrt{k+q} \right] \cdot \frac{\sqrt{k+p} + \sqrt{k+q}}{\sqrt{k+p} + \sqrt{k+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{p-q}{\sqrt{k+p} + \sqrt{k+q}} \\ &= \frac{p-q}{\infty + \infty} = \frac{p-q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{\alpha + \frac{p}{n}} - \sqrt{\alpha + \frac{q}{n}} \right] = \sqrt{\infty} \cdot \left[\sqrt{\alpha + \frac{p}{\infty}} - \sqrt{\alpha + \frac{q}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{\alpha + 0} - \sqrt{\alpha + 0} \right] = \infty \cdot 0 = ?? \text{ [Postup nemôžeme použiť.]} \end{aligned}$$

Riešené příklady – 133, 134

$$\lim_{n \rightarrow \infty} \left[\sqrt{2n+24} - \sqrt{2n+33} \right]$$

[133]

$$\lim_{n \rightarrow \infty} \left[\sqrt{3n} - \sqrt{2n} \right]$$

[134]

Riešené príklady – 133, 134

$$\lim_{n \rightarrow \infty} [\sqrt{2n+24} - \sqrt{2n+33}]$$

[133]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} [\sqrt{2n+24} - \sqrt{2n+33}] \cdot \frac{\sqrt{2n+24} + \sqrt{2n+33}}{\sqrt{2n+24} + \sqrt{2n+33}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{2n+24} - \sqrt{2n+33}] \cdot [\sqrt{2n+24} + \sqrt{2n+33}] = (2n+24) - (2n+33) = 24 - 33 = -9. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{-9}{\sqrt{2n+24} + \sqrt{2n+33}} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } k = 2n \mid \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \end{array} \right] = \lim_{n \rightarrow \infty} [\sqrt{k+24} - \sqrt{k+33}] \\ &= \lim_{n \rightarrow \infty} [\sqrt{k+24} - \sqrt{k+33}] \cdot \frac{\sqrt{k+24} + \sqrt{k+33}}{\sqrt{k+24} + \sqrt{k+33}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+24} - \sqrt{k+33}] \cdot [\sqrt{k+24} + \sqrt{k+33}] = (k+24) - (k+33) = 24 - 33 = -9. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{-9}{\sqrt{k+24} + \sqrt{k+33}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} [\sqrt{3n} - \sqrt{2n}]$$

[134]

$$= \lim_{n \rightarrow \infty} \sqrt{n} \cdot [\sqrt{3} - \sqrt{2}]$$

Riešené príklady – 133, 134

$$\lim_{n \rightarrow \infty} [\sqrt{2n+24} - \sqrt{2n+33}] = 0$$

[133]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} [\sqrt{2n+24} - \sqrt{2n+33}] \cdot \frac{\sqrt{2n+24} + \sqrt{2n+33}}{\sqrt{2n+24} + \sqrt{2n+33}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{2n+24} - \sqrt{2n+33}] \cdot [\sqrt{2n+24} + \sqrt{2n+33}] = (2n+24) - (2n+33) = 24-33 = -9. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{-9}{\sqrt{2n+24} + \sqrt{2n+33}} = \frac{-9}{\infty + \infty} = -\frac{9}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = 2n \mid \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} [\sqrt{k+24} - \sqrt{k+33}] \\ &= \lim_{n \rightarrow \infty} [\sqrt{k+24} - \sqrt{k+33}] \cdot \frac{\sqrt{k+24} + \sqrt{k+33}}{\sqrt{k+24} + \sqrt{k+33}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+24} - \sqrt{k+33}] \cdot [\sqrt{k+24} + \sqrt{k+33}] = (k+24) - (k+33) = 24-33 = -9. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{-9}{\sqrt{k+24} + \sqrt{k+33}} = \frac{-9}{\infty + \infty} = -\frac{9}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} [\sqrt{3n} - \sqrt{2n}] = \infty$$

[134]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot [\sqrt{3} - \sqrt{2}] = \infty \cdot [\sqrt{3} - \sqrt{2}] = \infty.$$

Riešené příklady – 135

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^a + p} - \sqrt{n^a + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[135]

Riešené príklady – 135

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^a + p} - \sqrt{n^a + q} \right]$$

pre $p, q \in R, p \neq q, a > 0$.

[135]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^a + p} - \sqrt{n^a + q} \right] \cdot \frac{\sqrt{n^a + p} + \sqrt{n^a + q}}{\sqrt{n^a + p} + \sqrt{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^a + p} - \sqrt{n^a + q}] \cdot [\sqrt{n^a + p} + \sqrt{n^a + q}] = (n^a + p) - (n^a + q) = p - q. \right] = \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt{n^a + p} + \sqrt{n^a + q}} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k + p} - \sqrt{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k + p} - \sqrt{k + q} \right] \cdot \frac{\sqrt{k + p} + \sqrt{k + q}}{\sqrt{k + p} + \sqrt{k + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{k + p} - \sqrt{k + q}] \cdot [\sqrt{k + p} + \sqrt{k + q}] = (k + p) - (k + q) = p - q. \right] = \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt{k + p} + \sqrt{k + q}} \end{aligned}$$

$$= \lim_{n \rightarrow \infty} \sqrt{n^a} \cdot \left[\sqrt{1 + \frac{p}{n^a}} - \sqrt{1 + \frac{q}{n^a}} \right]$$

Riešené príklady – 135

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^a + p} - \sqrt{n^a + q} \right] = 0 \text{ pre } p, q \in R, p \neq q, a > 0.$$

[135]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^a + p} - \sqrt{n^a + q} \right] \cdot \frac{\sqrt{n^a + p} + \sqrt{n^a + q}}{\sqrt{n^a + p} + \sqrt{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^a + p} - \sqrt{n^a + q}] \cdot [\sqrt{n^a + p} + \sqrt{n^a + q}] = (n^a + p) - (n^a + q) = p - q. \right] = \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt{n^a + p} + \sqrt{n^a + q}} \\ &= \frac{p - q}{\infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k + p} - \sqrt{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k + p} - \sqrt{k + q} \right] \cdot \frac{\sqrt{k + p} + \sqrt{k + q}}{\sqrt{k + p} + \sqrt{k + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{k + p} - \sqrt{k + q}] \cdot [\sqrt{k + p} + \sqrt{k + q}] = (k + p) - (k + q) = p - q. \right] = \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt{k + p} + \sqrt{k + q}} \\ &= \frac{p - q}{\infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \sqrt{n^a} \cdot \left[\sqrt{1 + \frac{p}{n^a}} - \sqrt{1 + \frac{q}{n^a}} \right] = \sqrt{\infty} \cdot \left[\sqrt{1 + \frac{p}{\infty}} - \sqrt{1 + \frac{q}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{1 + 0} - \sqrt{1 + 0} \right] = \infty \cdot 0 = ?? \text{ [Postup nemôžeme použiť.]} \end{aligned}$$

Riešené príklady – 136, 137

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2} - \sqrt{n^2 + 1} \right]$$

[136]

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n} + 2} - \sqrt{\sqrt{n} + 1} \right]$$

[137]

Riešené príklady – 136, 137

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2} - \sqrt{n^2 + 1} \right]$$

[136]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 2} - \sqrt{n^2 + 1} \right] \cdot \frac{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}}{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + 2} - \sqrt{n^2 + 1}] \cdot [\sqrt{n^2 + 2} + \sqrt{n^2 + 1}] = (n^2 + 2) - (n^2 + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^2 \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}} \right]$$

[137]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}} \right] \cdot \frac{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}}{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] \cdot [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] = (\sqrt{n+2}) - (\sqrt{n+1}) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} \end{aligned}$$

Riešené príklady – 136, 137

$$\lim_{n \rightarrow \infty} [\sqrt{n^2 + 2} - \sqrt{n^2 + 1}] = 0$$

[136]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} [\sqrt{n^2 + 2} - \sqrt{n^2 + 1}] \cdot \frac{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}}{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2 + 2} - \sqrt{n^2 + 1}] \cdot [\sqrt{n^2 + 2} + \sqrt{n^2 + 1}] = (n^2 + 2) - (n^2 + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2} + \sqrt{n^2 + 1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^2 \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} [\sqrt{k+2} - \sqrt{k+1}] = \lim_{n \rightarrow \infty} [\sqrt{k+2} - \sqrt{k+1}] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] = 0$$

[137]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] \cdot \frac{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}}{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] \cdot [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] = (\sqrt{n+2}) - (\sqrt{n+1}) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} [\sqrt{k+2} - \sqrt{k+1}] = \lim_{n \rightarrow \infty} [\sqrt{k+2} - \sqrt{k+1}] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 138, 139

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right]$$

[138]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right]$$

[139]

Riešené příklady – 138, 139

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right]$$

[138]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}}{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1} \right] = (n^4 + n^3 - n - 2) - (n^4 + n^3 - n^2 - 1) = n^2 - n - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^2 - n - 1}{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} \right] = \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n} - \frac{1}{n^2})}{n^2 \left[\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}} \right]} = \lim_{n \rightarrow \infty} \frac{1 - \frac{1}{n} - \frac{1}{n^2}}{\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right]$$

[139]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}}{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1} \right] = (n^3 - n - 2) - (n^3 - n^2 - 1) = n^2 - n - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^2 - n - 1}{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n} - \frac{1}{n^2})}{\sqrt{n^3} \left[\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^4}}{\sqrt{n^3}} = \sqrt{n} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n}(1 - \frac{1}{n} - \frac{1}{n^2})}{\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}}} \end{aligned}$$

Riešené príklady – 138, 139

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right] = \frac{1}{2} \quad [138]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}}{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1} \right] = (n^4 + n^3 - n - 2) - (n^4 + n^3 - n^2 - 1) = n^2 - n - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^2 - n - 1}{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}} \\ &= \left[n \in \mathbb{N} \Rightarrow n^2 = \sqrt{n^4} \right] = \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n} - \frac{1}{n^2})}{n^2 \left[\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}} \right]} = \lim_{n \rightarrow \infty} \frac{1 - \frac{1}{n} - \frac{1}{n^2}}{\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}}} \\ &= \frac{1 - \frac{1}{\infty} - \frac{1}{\infty}}{\sqrt{1 + \frac{1}{\infty} - \frac{1}{\infty} - \frac{2}{\infty}} + \sqrt{1 + \frac{1}{\infty} - \frac{1}{\infty} - \frac{1}{\infty}}} = \frac{1 + 0 - 0}{\sqrt{1 + 0 - 0 - 0} + \sqrt{1 + 0 - 0 - 0}} = \frac{1}{1 + 1} = \frac{1}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right] = \infty \quad [139]$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}}{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1} \right] = (n^3 - n - 2) - (n^3 - n^2 - 1) = n^2 - n - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^2 - n - 1}{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n} - \frac{1}{n^2})}{\sqrt{n^3} \left[\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^4}}{\sqrt{n^3}} = \sqrt{n} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n}(1 - \frac{1}{n} - \frac{1}{n^2})}{\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}}} \\ &= \frac{\infty(1 - \frac{1}{\infty} - \frac{1}{\infty})}{\sqrt{1 - \frac{1}{\infty} - \frac{2}{\infty}} + \sqrt{1 - \frac{1}{\infty} - \frac{1}{\infty}}} = \frac{\infty(1 + 0 - 0)}{\sqrt{1 - 0 - 0} + \sqrt{1 - 0 - 0}} = \frac{\infty}{1 + 1} = \infty. \end{aligned}$$

Riešené príklady – 140, 141

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right]$$

[140]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right]$$

[141]

Riešené príklady – 140, 141

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right]$$

[140]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}}{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1} \right] = (n^3 - 2) - (n^3 - n^2 - 1) = n^2 - 1. \right] = \lim_{n \rightarrow \infty} \frac{n^2 - 1}{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n^2})}{\sqrt{n^3} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^4}}{\sqrt{n^3}} = \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n}(1 - \frac{1}{n^2})}{\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right]$$

[141]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right] \cdot \frac{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}}{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right] \cdot \left[\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1} \right] = (n^3 - 2) - (n^3 - n - 1) = n - 1. \right] = \lim_{n \rightarrow \infty} \frac{n - 1}{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n(1 - \frac{1}{n})}{\sqrt{n^3} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n^2} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}. \right] = \lim_{n \rightarrow \infty} \frac{1 - \frac{1}{n}}{\sqrt{n} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n^2} - \frac{1}{n^3}} \right]} \end{aligned}$$

Riešené príklady – 140, 141

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right] = \infty$$

[140]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \frac{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}}{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n^2 - 1} \right] \cdot \left[\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1} \right] = (n^3 - 2) - (n^3 - n^2 - 1) = n^2 - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n^2 - 1}{\sqrt{n^3 - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n^2(1 - \frac{1}{n^2})}{\sqrt{n^3} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^4}}{\sqrt{n^3}} = \sqrt{n} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n}(1 - \frac{1}{n^2})}{\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}}} \\ &= \frac{\infty(1 - \frac{1}{\infty})}{\sqrt{1 - \frac{2}{\infty}} + \sqrt{1 - \frac{1}{\infty} - \frac{1}{\infty}}} = \frac{\infty(1 - 0)}{\sqrt{1 - 0} + \sqrt{1 - 0 - 0}} = \frac{\infty}{1 + 1} = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right] = 0$$

[141]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right] \cdot \frac{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}}{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^3 - 2} - \sqrt{n^3 - n - 1} \right] \cdot \left[\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1} \right] = (n^3 - 2) - (n^3 - n - 1) = n - 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{n - 1}{\sqrt{n^3 - 2} + \sqrt{n^3 - n - 1}} \\ &= \lim_{n \rightarrow \infty} \frac{n(1 - \frac{1}{n})}{\sqrt{n^3} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n^2} - \frac{1}{n^3}} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \frac{1}{\sqrt{n}} \right] = \lim_{n \rightarrow \infty} \frac{1 - \frac{1}{n}}{\sqrt{n} \left[\sqrt{1 - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n^2} - \frac{1}{n^3}} \right]} \\ &= \frac{1 - \frac{1}{\infty}}{\infty \left[\sqrt{1 - \frac{2}{\infty}} + \sqrt{1 - \frac{1}{\infty} - \frac{1}{\infty}} \right]} = \frac{1 - 0}{\infty \left[\sqrt{1 - 0} + \sqrt{1 - 0 - 0} \right]} = \frac{1}{\infty(1 + 1)} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 142, 143

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right]$$

[142]

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt[5]{n} + 2} - \sqrt{\sqrt[5]{n} + 1} \right]$$

[143]

Riešené príklady – 142, 143

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right]$$

[142]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right] \cdot \frac{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^5 + 2} - \sqrt{n^5 + 1}] \cdot [\sqrt{n^5 + 2} + \sqrt{n^5 + 1}] = (n^5 + 2) - (n^5 + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[5]{\sqrt[5]{n} + 2} - \sqrt[5]{\sqrt[5]{n} + 1} \right]$$

[143]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[5]{\sqrt[5]{n} + 2} - \sqrt[5]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[5]{\sqrt[5]{n} + 2} + \sqrt[5]{\sqrt[5]{n} + 1}}{\sqrt[5]{\sqrt[5]{n} + 2} + \sqrt[5]{\sqrt[5]{n} + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt[5]{\sqrt[5]{n} + 2} - \sqrt[5]{\sqrt[5]{n} + 1}] \cdot [\sqrt[5]{\sqrt[5]{n} + 2} + \sqrt[5]{\sqrt[5]{n} + 1}] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{\sqrt[5]{n} + 2} + \sqrt[5]{\sqrt[5]{n} + 1}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} \end{aligned}$$

Riešené príklady – 142, 143

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right] = 0$$

[142]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right] \cdot \frac{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^5 + 2} - \sqrt{n^5 + 1}] \cdot [\sqrt{n^5 + 2} + \sqrt{n^5 + 1}] = (n^5 + 2) - (n^5 + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k+2} - \sqrt{k+1} \right] \cdot \frac{\sqrt{k+2} + \sqrt{k+1}}{\sqrt{k+2} + \sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2} + \sqrt{k+1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[5]{n^5 + 2} - \sqrt[5]{n^5 + 1} \right] = 0$$

[143]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[5]{n^5 + 2} - \sqrt[5]{n^5 + 1} \right] \cdot \frac{\sqrt[5]{n^5 + 2} + \sqrt[5]{n^5 + 1}}{\sqrt[5]{n^5 + 2} + \sqrt[5]{n^5 + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt[5]{n^5 + 2} - \sqrt[5]{n^5 + 1}] \cdot [\sqrt[5]{n^5 + 2} + \sqrt[5]{n^5 + 1}] = (n^5 + 2) - (n^5 + 1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5 + 2} + \sqrt[5]{n^5 + 1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[5]{k+2} - \sqrt[5]{k+1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[5]{k+2} - \sqrt[5]{k+1} \right] \cdot \frac{\sqrt[5]{k+2} + \sqrt[5]{k+1}}{\sqrt[5]{k+2} + \sqrt[5]{k+1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt[5]{k+2} - \sqrt[5]{k+1}] \cdot [\sqrt[5]{k+2} + \sqrt[5]{k+1}] = (k+2) - (k+1) = 2 - 1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{k+2} + \sqrt[5]{k+1}} = \frac{1}{\infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené příklady – 144, 145

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right]$$

[144]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right]$$

[145]

Riešené príklady – 144, 145

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right]$$

[144]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right] \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2}] \cdot [\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}] = (n^2 + n + 1) - (n^2 + n + 2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\begin{array}{l} \text{Subst. } k = n^2 + n + 1 \mid n \rightarrow \infty \\ n^2 + n + 2 = k + 1 \mid k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k + 1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k + 1} \right] \cdot \frac{\sqrt{k} + \sqrt{k + 1}}{\sqrt{k} + \sqrt{k + 1}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k} - \sqrt{k + 1}] \cdot [\sqrt{k} + \sqrt{k + 1}] = k - (k + 1) = -1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{k} + \sqrt{k + 1}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right]$$

[145]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right]$$

Riešené příklady – 144, 145

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right] = 0$$

[144]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2} \right] \cdot \frac{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2 + n + 1} - \sqrt{n^2 + n + 2}] \cdot [\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}] = (n^2 + n + 1) - (n^2 + n + 2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n^2 + n + 1} + \sqrt{n^2 + n + 2}} = \frac{-1}{\sqrt{\infty + \infty + 1} + \sqrt{\infty + \infty + 2}} = \frac{-1}{\infty + \infty} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^2 + n + 1 \mid n \rightarrow \infty \right. \\ &\quad \left. n^2 + n + 2 = k + 1 \mid k \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k + 1} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k + 1} \right] \cdot \frac{\sqrt{k} + \sqrt{k + 1}}{\sqrt{k} + \sqrt{k + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k} - \sqrt{k + 1}] \cdot [\sqrt{k} + \sqrt{k + 1}] = k - (k + 1) = -1. \right] = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{k} + \sqrt{k + 1}} = \frac{-1}{\infty + \infty} = \frac{-1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n - 1} \right] = \infty$$

[145]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right] = \infty \cdot \left[\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{\infty + 1 + 0} - \sqrt{1 - 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

Riešené príklady – 146

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1} \right]$$

[146]

Riešené príklady – 146

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1} \right]$$

[146]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1} \right] \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}] = (n^2 - n + 1) - (n^2 - n - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n(n-1)+1} + \sqrt{n(n-1)-1}} \end{aligned}$$

$$\begin{aligned} &= \left[\begin{array}{l} \text{Subst. } k = n^2 - n + 1 = n(n-1) + 1 \mid n \rightarrow \infty \\ n^2 - n - 1 = n(n-1) - 1 = k - 2 \mid k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k-2} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k-2} \right] \cdot \frac{\sqrt{k} + \sqrt{k-2}}{\sqrt{k} + \sqrt{k-2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{k} - \sqrt{k-2}] \cdot [\sqrt{k} + \sqrt{k-2}] = k - (k-2) = 2. \right] = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{k} + \sqrt{k-2}} \end{aligned}$$

Riešené príklady – 146

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1} \right] = 0$$

[146]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1} \right] \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}] = (n^2 - n + 1) - (n^2 - n - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n(n-1)+1} + \sqrt{n(n-1)-1}} \\ &= \frac{2}{\sqrt{\infty(\infty-1)+1} + \sqrt{\infty(\infty-1)-1}} = \frac{2}{\sqrt{\infty+1} + \sqrt{\infty-1}} = \frac{2}{\infty+\infty} = \frac{2}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^2 - n + 1 = n(n-1) + 1 \mid n \rightarrow \infty \right. \\ &\quad \left. n^2 - n - 1 = n(n-1) - 1 = k - 2 \mid k \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k-2} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt{k} - \sqrt{k-2} \right] \cdot \frac{\sqrt{k} + \sqrt{k-2}}{\sqrt{k} + \sqrt{k-2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{k} - \sqrt{k-2}] \cdot [\sqrt{k} + \sqrt{k-2}] = k - (k-2) = 2. \right] = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{k} + \sqrt{k-2}} \\ &= \frac{2}{\sqrt{\infty} + \sqrt{\infty-1}} = \frac{2}{\infty+\infty} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené příklady – 147, 148

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right]$$

[147]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + 1} - \sqrt{n + 1} \right]$$

[148]

Riešené príklady – 147, 148

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right]$$

[147]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right] \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}] = (n^2 - n + 1) - (n^2 - 3n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[n \in \mathbb{N} \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{n \left[\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + 1} - \sqrt{n + 1} \right]$$

[148]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n^3 + \frac{1}{n}} - \sqrt{1 + \frac{1}{n}} \right]$$

Riešené príklady – 147, 148

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right] = 1$$

[147]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2} \right] \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}] = (n^2 - n + 1) - (n^2 - 3n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{n \left[\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}} = \frac{2 - 0}{\sqrt{1 - 0 + 0} + \sqrt{1 - 0 + 0}} = \frac{2}{1 + 1} = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^4 + 1} - \sqrt{n + 1} \right] = \infty$$

[148]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n^3 + \frac{1}{n}} - \sqrt{1 + \frac{1}{n}} \right] = \infty \cdot \left[\sqrt{\infty + \frac{1}{\infty}} - \sqrt{1 + \frac{1}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{\infty + 0} - \sqrt{1 + 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

Riešené příklady – 149, 150, 151

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n - 3} \right]$$

[149]

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n} + 1} - \sqrt{n - 3} \right]$$

[150]

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n^3 - 3} \right]$$

[151]

Riešené príklady – 149, 150, 151

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n - 3} \right]$$

[149]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n + \frac{1}{n}} - \sqrt{1 - \frac{3}{n}} \right]$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n} + 1} - \sqrt{n - 3} \right]$$

[150]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{\sqrt{n}} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right] = \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right]$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n^3 - 3} \right]$$

[151]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right] = \lim_{n \rightarrow \infty} n \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right]$$

Riešené príklady – 149, 150, 151

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n - 3} \right] = \infty$$

[149]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n + \frac{1}{n}} - \sqrt{1 - \frac{3}{n}} \right] = \infty \cdot \left[\sqrt{\infty + \frac{1}{\infty}} - \sqrt{1 - \frac{3}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{\infty + 0} - \sqrt{1 - 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{\sqrt{n} + 1} - \sqrt{n - 3} \right] = -\infty$$

[150]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{\sqrt{n}} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right] = \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right] \\ &= \infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \sqrt{\infty - \frac{3}{\infty}} \right] = \infty \cdot \left[\sqrt{1 + 0} - \sqrt{\infty - 0} \right] = \infty \cdot (1 - \infty) = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt{n^2 + 1} - \sqrt{n^3 - 3} \right] = -\infty$$

[151]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n^2} \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right] = \lim_{n \rightarrow \infty} n \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right] \\ &= \infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \sqrt{\infty - \frac{3}{\infty}} \right] = \infty \cdot \left[\sqrt{1 + 0} - \sqrt{\infty - 0} \right] = \infty \cdot (1 - \infty) = -\infty. \end{aligned}$$

Riešené příklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right]$$

[152]

Riešené príklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right]$$

[152]

$$\begin{aligned} & \bullet = \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3 + 1)^2} + \sqrt[3]{n^3 + 1} \cdot (n - 1) + (n - 1)^2}{\sqrt[3]{(n^3 + 1)^2} + \sqrt[3]{n^3 + 1} \cdot (n - 1) + (n - 1)^2} \\ & = \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + 1} - (n - 1) \right] \cdot \left[\sqrt[3]{(n^3 + 1)^2} + \sqrt[3]{n^3 + 1} \cdot (n - 1) + (n - 1)^2 \right] \right. \\ & \quad \left. = (n^3 + 1) - (n - 1)^3 = (n^3 + 1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 3n + 2. \right] \\ & = \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3 + 1)^2} + \sqrt[3]{n^3 + 1} \cdot (n - 1) + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ & = \lim_{n \rightarrow \infty} \frac{n^2 \left(3 - \frac{3}{n} + \frac{2}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2 \right]} \\ & = \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2} \end{aligned}$$

Riešené príklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right] = 1$$

[152]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2 \right] \right. \\
 &\quad \left. = (n^3+1) - (n-1)^3 = (n^3+1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 3n + 2. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{3n^2 - 3n + 2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^2 \left(3 - \frac{3}{n} + \frac{2}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{3 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2} \\
 &= \frac{3 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty} \right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty} \right) + \left(1 - \frac{1}{\infty} \right)^2} \\
 &= \frac{3 - 0 + 0}{\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} \cdot (1-0) + (1-0)^2} = \frac{3}{1+1 \cdot 1+1} = \frac{3}{1+1+1} = \frac{3}{3} = 1.
 \end{aligned}$$

Riešené príklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right]$$

[\[Iné riešenie 152\]](#)

Riešené príklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right]$$

[Iné riešenie 152]

$$\begin{aligned}
 & \bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\
 & = \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+1} - n \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = (n^3+1) - n^3 = 1. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right]}
 \end{aligned}$$

Riešené príklady – 152

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n + 1 \right] = 1$$

[Iné riešenie 152]

$$\begin{aligned}
 & \bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\
 & = \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{n^3+1} - n \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = (n^3+1) - n^3 = 1. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} = \left[n \in N. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{1}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right]} \\
 & = 1 + \frac{1}{\infty \left[\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right]} = 1 + \frac{1}{\infty \left[\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right]} \\
 & = 1 + \frac{1}{\infty(1+1+1)} = 1 + \frac{1}{\infty} = 1 + 0 = 1.
 \end{aligned}$$

Riešené príklady – 153, 154

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right]$$

[153]

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^4 + 1} - n \right]$$

[154]

Riešené príklady – 153, 154

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right]$$

[153]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^3+1} - n \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = (n^3+1) - n^3 = 1. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] = \lim_{n \rightarrow \infty} \frac{1}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right]} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^4 + 1} - n \right]$$

[154]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] = \lim_{n \rightarrow \infty} n \cdot \left[\sqrt{n + \frac{1}{n^3}} - 1 \right]$$

Riešené príklady – 153, 154

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] = 0$$

[153]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^3+1} - n \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = (n^3+1) - n^3 = 1. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] = \lim_{n \rightarrow \infty} \frac{1}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right]} \\ &= \frac{1}{\infty \left[\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right]} = \frac{1}{\infty \left[\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right]} = \frac{1}{\infty(1+1+1)} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^4 + 1} - n \right] = \infty$$

[154]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] = \lim_{n \rightarrow \infty} n \cdot \left[\sqrt{n + \frac{1}{n^3}} - 1 \right] = \infty \cdot \left[\sqrt{\infty + \frac{1}{\infty}} - 1 \right] \\ &= \infty \cdot \left[\sqrt{\infty + 0} - 1 \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

Riešené příklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right]$$

[155]

Riešené príklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right]$$

[155]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2 \right] \right. \\ &\quad \left. = (n^3 + n + 1) - (n - 1)^3 = (n^3 + n + 1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 2n + 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{3n^2 - 2n + 2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left(3 - \frac{2}{n} + \frac{2}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{3 - \frac{2}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2} \end{aligned}$$

Riešené príklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right] = 1$$

[155]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2 \right] \right. \\
 &\quad \left. = (n^3 + n + 1) - (n - 1)^3 = (n^3 + n + 1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 2n + 2. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{3n^2 - 2n + 2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot (n - 1) + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^2 \left(3 - \frac{2}{n} + \frac{2}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{3 - \frac{2}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2} \\
 &= \frac{3 - \frac{2}{\infty} + \frac{2}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty} \right)^2} + \sqrt[3]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty} \right) + \left(1 - \frac{1}{\infty} \right)^2} \\
 &= \frac{3 - 0 + 0}{\sqrt[3]{(1 + 0 + 0)^2} + \sqrt[3]{1 + 0 + 0} \cdot (1 - 0) + (1 - 0)^2} = \frac{3}{1 + 1 \cdot 1 + 1} = \frac{3}{3} = 1.
 \end{aligned}$$

Riešené príklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right]$$

[Iné riešenie 155]

Riešené príklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right]$$

[Iné riešenie 155]

$$\begin{aligned}
 & \bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2} \\
 & = \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n + 1} - n \right] \cdot \left[\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2 \right] = (n^3 + n + 1) - n^3 = n + 1. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{n+1}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{n^2 \left(\frac{1}{n} + \frac{1}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} + 1 \right]} \\
 & = 1 + \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} + 1}
 \end{aligned}$$

Riešené príklady – 155

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n + 1 \right] = 1$$

[Iné riešenie 155]

$$\begin{aligned}
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n + 1} - n \right] \cdot \left[\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2 \right] = (n^3 + n + 1) - n^3 = n + 1. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt[3]{(n^3 + n + 1)^2} + \sqrt[3]{n^3 + n + 1} \cdot n + n^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n^2 \left(\frac{1}{n} + \frac{1}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} + 1 \right]} \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} + 1} \\
 &= 1 + \frac{\frac{1}{\infty} + \frac{1}{\infty}}{\sqrt[3]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{0 + 0}{\sqrt[3]{(1 + 0 + 0)^2} + \sqrt[3]{1 + 0 + 0} + 1} = 1 + \frac{0}{1 + 1 + 1} = 1.
 \end{aligned}$$

Riešené příklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right]$$

[156]

Riešené príklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right]$$

[156]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2 \right] \right. \\ &\quad \left. = (n^3 + n^2 + 1) - (n - 1)^3 = (n^3 + n^2 + 1) - (n^3 - 3n^2 + 3n - 1) = 4n^2 - 3n + 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^2 - 3n + 2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left(4 - \frac{3}{n} + \frac{2}{n^2} \right)}{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3} \right)^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n} \right) + \left(1 - \frac{1}{n} \right)^2} \end{aligned}$$

Riešené príklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right] = \frac{4}{3}$$

[156]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2 \right] \right. \\ &\quad \left. = (n^3 + n^2 + 1) - (n - 1)^3 = (n^3 + n^2 + 1) - (n^3 - 3n^2 + 3n - 1) = 4n^2 - 3n + 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^2 - 3n + 2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n - 1) + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2(4 - \frac{3}{n} + \frac{2}{n^2})}{n^2 \left[\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot (1 - \frac{1}{n}) + (1 - \frac{1}{n})^2 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{3}{n} + \frac{2}{n^2}}{\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot (1 - \frac{1}{n}) + (1 - \frac{1}{n})^2} \\ &= \frac{4 - \frac{3}{\infty} + \frac{2}{\infty}}{\sqrt[3]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + \sqrt[3]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot (1 - \frac{1}{\infty}) + (1 - \frac{1}{\infty})^2} \\ &= \frac{4 - 0 + 0}{\sqrt[3]{(1 + 0 + 0)^2} + \sqrt[3]{1 + 0 + 0} \cdot (1 - 0) + (1 - 0)^2} = \frac{4}{1 + 1 \cdot 1 + 1} = \frac{4}{3}. \end{aligned}$$

Riešené príklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 156]

Riešené príklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 156]

$$\begin{aligned}
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\
 &\quad \left. \Rightarrow \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \cdot \left[\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2 \right] = (n^3 + n^2 + 1) - n^3 = n^2 + 1. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^2})}{n^2 \left[\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} + 1 \right]} \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^2}}{\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} + 1}
 \end{aligned}$$

Riešené príklady – 156

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n + 1 \right] = \frac{4}{3}$$

[Iné riešenie 156]

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \\ &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \cdot \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^3 + n^2 + 1} - n \right] \cdot \left[\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2 \right] = (n^3 + n^2 + 1) - n^3 = n^2 + 1. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot n + n^2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^2})}{n^2 \left[\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} + 1 \right]} \\ &= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^2}}{\sqrt[3]{(1 + \frac{1}{n} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} + 1} \\ &= 1 + \frac{1 + \frac{1}{\infty}}{\sqrt[3]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + \sqrt[3]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{1 + 0}{\sqrt[3]{(1 + 0 + 0)^2} + \sqrt[3]{1 + 0 + 0} + 1} \\ &= 1 + \frac{1}{1 + 1 + 1} = 1 + \frac{1}{3} = \frac{4}{3}. \end{aligned}$$

Riešené príklady – 157, 158

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] \quad \text{pre } p, q \in R, p \neq q.$$

[157]

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n+2} \right]$$

[158]

Riešené príklady – 157, 158

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] \quad \text{pre } p, q \in R, p \neq q.$$

[157]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] \cdot \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n+p} - \sqrt[3]{n+q}] \cdot [\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}] = (n+p) - (n+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n+2} \right]$$

[158]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n+2} \right] \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{n+1} - \sqrt[3]{n+2}] \cdot [\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}] = (n+1) - (n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \end{aligned}$$

Riešené príklady – 157, 158

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] = 0 \text{ pre } p, q \in R, p \neq q. \quad [157]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] \cdot \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n+p} - \sqrt[3]{n+q}] \cdot [\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}] = (n+p) - (n+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n+2} \right] = 0 \quad [158]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n+1} - \sqrt[3]{n+2} \right] \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{n+1} - \sqrt[3]{n+2}] \cdot [\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}] = (n+1) - (n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-1}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} = \frac{-1}{\infty + \infty \cdot \infty + \infty} = -\frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 159, 160

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right]$$

[159]

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right]$$

[160]

Riešené príklady – 159, 160

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right]$$

[159]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right] \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right] \cdot \left[\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2} \right] = (n-1) - (n-2) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right]$$

[160]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right] \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right] \cdot \left[\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2} \right] = (n-1) - (n+3) = -4. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-4}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} \end{aligned}$$

Riešené príklady – 159, 160

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right] = 0$$

[159]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right] \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n-1} - \sqrt[3]{n-2} \right] \cdot \left[\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2} \right] = (n-1) - (n-2) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} = \frac{1}{\infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right] = 0$$

[160]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right] \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n-1} - \sqrt[3]{n+3} \right] \cdot \left[\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2} \right] = (n-1) - (n+3) = -4. \right] \\ &= \lim_{n \rightarrow \infty} \frac{-4}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} = \frac{-4}{\infty + \infty \cdot \infty + \infty} = -\frac{4}{\infty} = 0. \end{aligned}$$

Riešené příklady – 161

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[161]

Riešené príklady – 161

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q} \right]$$

pre $p, q \in R, p \neq q, a > 0$.

[161]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q} \right] \cdot \frac{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}}{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q}] \cdot [\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{k + p} - \sqrt[3]{k + q}] \cdot [\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \end{aligned}$$

$$\bullet = \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right]$$

Riešené príklady – 161

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q} \right] = 0 \text{ pre } p, q \in R, p \neq q, a > 0.$$

[161]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q} \right] \cdot \frac{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}}{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q}] \cdot [\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n^a + p)^2} + \sqrt[3]{n^a + p} \cdot \sqrt[3]{n^a + q} + \sqrt[3]{(n^a + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{k + p} - \sqrt[3]{k + q}] \cdot [\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^a \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \left[\text{Pr. 157: } \lim_{n \rightarrow \infty} [\sqrt[3]{n + p} - \sqrt[3]{n + q}] = 0 \text{ pre } p, q \in R, p \neq q. \right] = 0. \end{aligned}$$

Riešené příklady – 162

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] \quad \text{pre } p, q \in \mathbb{R}, p \neq q.$$

[162]

Riešené príklady – 162

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right]$$

pre $p, q \in R, p \neq q.$

[162]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] \cdot \frac{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}}{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] \cdot \left[\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2} \right] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \left[\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2} \right] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \end{aligned}$$

$$\bullet = \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right]$$

Riešené príklady – 162

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] = 0 \text{ pre } p, q \in R, p \neq q. \quad [162]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] \cdot \frac{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}}{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^5 + p} - \sqrt[3]{n^5 + q} \right] \cdot \left[\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2} \right] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(n^5 + p)^2} + \sqrt[3]{n^5 + p} \cdot \sqrt[3]{n^5 + q} + \sqrt[3]{(n^5 + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \left[\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2} \right] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \left[\text{Pr. 157: } \lim_{n \rightarrow \infty} \left[\sqrt[3]{n + p} - \sqrt[3]{n + q} \right] = 0 \text{ pre } p, q \in R, p \neq q. \right] = 0. \end{aligned}$$

Riešené příklady – 163

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1} \right]$$

[163]

Riešené príklady – 163

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1} \right]$$

[163]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1} \right] \cdot \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1} \right] \cdot \left[\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2} \right] = (n^5+2) - (n^5+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \left[\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2} \right] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \end{aligned}$$

$$\bullet = \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right]$$

Riešené príklady – 163

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1} \right] = 0$$

[163]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^5 + 2} - \sqrt[3]{n^5 + 1} \right] \cdot \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1} \right] \cdot \left[\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2} \right] = (n^5+2) - (n^5+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} = \frac{1}{\infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \left[\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2} \right] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} = \frac{1}{\infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \\ &= \left[\text{Pr. 157: } \lim_{n \rightarrow \infty} \left[\sqrt[3]{n+p} - \sqrt[3]{n+q} \right] = 0 \text{ pre } p, q \in \mathbb{R}, p \neq q. \right] = 0. \end{aligned}$$

Riešené príklady – 164

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q$.

[164]

Riešené príklady – 164

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q$.

[164]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right] \cdot \frac{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}}{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2} \right] = (\sqrt[5]{n} + p) - (\sqrt[5]{n} + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \left[\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2} \right] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \end{aligned}$$

$$= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right]$$

Riešené príklady – 164

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right] = 0 \text{ pre } p, q \in R, p \neq q.$$

[164]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right] \cdot \frac{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}}{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n} + p} - \sqrt[3]{\sqrt[5]{n} + q} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2} \right] = (\sqrt[5]{n} + p) - (\sqrt[5]{n} + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(\sqrt[5]{n} + p)^2} + \sqrt[3]{\sqrt[5]{n} + p} \cdot \sqrt[3]{\sqrt[5]{n} + q} + \sqrt[3]{(\sqrt[5]{n} + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \frac{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \cdot \left[\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2} \right] = (k + p) - (k + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[3]{(k + p)^2} + \sqrt[3]{k + p} \cdot \sqrt[3]{k + q} + \sqrt[3]{(k + q)^2}} = \frac{p - q}{\infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + p} - \sqrt[3]{k + q} \right] \\ &= \left[\text{Pr. 157: } \lim_{n \rightarrow \infty} \left[\sqrt[3]{n + p} - \sqrt[3]{n + q} \right] = 0 \text{ pre } p, q \in R, p \neq q. \right] = 0. \end{aligned}$$

Riešené příklady – 165

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + 2} - \sqrt[3]{\sqrt[5]{n} + 1} \right]$$

[165]

Riešené príklady – 165

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + 2} - \sqrt[3]{\sqrt[5]{n} + 1} \right]$$

[165]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + 2} - \sqrt[3]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[3]{(\sqrt[5]{n}+2)^2} + \sqrt[3]{\sqrt[5]{n}+2} \cdot \sqrt[3]{\sqrt[5]{n}+1} + \sqrt[3]{(\sqrt[5]{n}+1)^2}}{\sqrt[3]{(\sqrt[5]{n}+2)^2} + \sqrt[3]{\sqrt[5]{n}+2} \cdot \sqrt[3]{\sqrt[5]{n}+1} + \sqrt[3]{(\sqrt[5]{n}+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n}+2} - \sqrt[3]{\sqrt[5]{n}+1} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n}+2)^2} + \sqrt[3]{\sqrt[5]{n}+2} \cdot \sqrt[3]{\sqrt[5]{n}+1} + \sqrt[3]{(\sqrt[5]{n}+1)^2} \right] = (\sqrt[5]{n}+2) - (\sqrt[5]{n}+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(\sqrt[5]{n}+2)^2} + \sqrt[3]{\sqrt[5]{n}+2} \cdot \sqrt[3]{\sqrt[5]{n}+1} + \sqrt[3]{(\sqrt[5]{n}+1)^2}} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \left[\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2} \right] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \end{aligned}$$

$$= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right]$$

Riešené príklady – 165

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + 2} - \sqrt[3]{\sqrt[5]{n} + 1} \right] = 0$$

[165]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{\sqrt[5]{n} + 2} - \sqrt[3]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[3]{(\sqrt[5]{n} + 2)^2} + \sqrt[3]{\sqrt[5]{n} + 2} \cdot \sqrt[3]{\sqrt[5]{n} + 1} + \sqrt[3]{(\sqrt[5]{n} + 1)^2}}{\sqrt[3]{(\sqrt[5]{n} + 2)^2} + \sqrt[3]{\sqrt[5]{n} + 2} \cdot \sqrt[3]{\sqrt[5]{n} + 1} + \sqrt[3]{(\sqrt[5]{n} + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[5]{\sqrt[5]{n} + 2} - \sqrt[5]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n} + 2)^2} + \sqrt[3]{\sqrt[5]{n} + 2} \cdot \sqrt[3]{\sqrt[5]{n} + 1} + \sqrt[3]{(\sqrt[5]{n} + 1)^2} \right] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(\sqrt[5]{n} + 2)^2} + \sqrt[3]{\sqrt[5]{n} + 2} \cdot \sqrt[3]{\sqrt[5]{n} + 1} + \sqrt[3]{(\sqrt[5]{n} + 1)^2}} = \frac{1}{\infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + 2} - \sqrt[3]{k + 1} \right] \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + 2} - \sqrt[3]{k + 1} \right] \cdot \frac{\sqrt[3]{(k + 2)^2} + \sqrt[3]{k + 2} \cdot \sqrt[3]{k + 1} + \sqrt[3]{(k + 1)^2}}{\sqrt[3]{(k + 2)^2} + \sqrt[3]{k + 2} \cdot \sqrt[3]{k + 1} + \sqrt[3]{(k + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k + 2} - \sqrt[3]{k + 1} \right] \cdot \left[\sqrt[3]{(k + 2)^2} + \sqrt[3]{k + 2} \cdot \sqrt[3]{k + 1} + \sqrt[3]{(k + 1)^2} \right] = (k + 2) - (k + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{(k + 2)^2} + \sqrt[3]{k + 2} \cdot \sqrt[3]{k + 1} + \sqrt[3]{(k + 1)^2}} = \frac{1}{\infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \left[\sqrt[3]{k + 2} - \sqrt[3]{k + 1} \right] \\ &= \left[\text{Pr. 157: } \lim_{n \rightarrow \infty} \left[\sqrt[3]{n + p} - \sqrt[3]{n + q} \right] = 0 \text{ pre } p, q \in R, p \neq q. \right] = 0. \end{aligned}$$

Riešené příklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right]$$

[166]

Riešené príklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right]$$

[166]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2} \right] = (n^2 + 3n + 1) - (n^2 + n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n}{\sqrt[3]{n^2 \cdot n^2}} = \frac{\sqrt[3]{n^3}}{\sqrt[3]{n^4}} = \frac{1}{\sqrt[3]{n}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]} \end{aligned}$$

Riešené príklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] = 0$$

[166]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2} \right] = (n^2 + 3n + 1) - (n^2 + n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]} = \left[n \in \mathbb{N} \Rightarrow \frac{n}{\sqrt[3]{n^2 \cdot n^2}} = \frac{\sqrt[3]{n^3}}{\sqrt[3]{n^4}} = \frac{1}{\sqrt[3]{n}} \right] \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{\infty} \left[\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty})^2} \right]} \\ &= \frac{2 - 0}{\infty \left[\sqrt[3]{(1 + 0 + 0)^2} + \sqrt[3]{1 + 0 + 0} \sqrt[3]{1 + 0 + 0} + \sqrt[3]{(1 + 0 + 0)^2} \right]} = \frac{2}{\infty \cdot (1 + 1 \cdot 1 + 1)} = \frac{2}{\infty \cdot 3} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené príklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right]$$

[Iné riešenie 166]

Riešené príklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right]$$

[Iné riešenie 166]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2} \right] = (n^2 + 3n + 1) - (n^2 + n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n + 3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n + 1 + \frac{2}{n})} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n + 3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n + 1 + \frac{2}{n})}} \end{aligned}$$

Riešené príklady – 166

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] = 0$$

[Iné riešenie 166]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + n + 2} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2} \right] = (n^2 + 3n + 1) - (n^2 + n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n - 1}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + n + 2} + \sqrt[3]{(n^2 + n + 2)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n(2 - \frac{1}{n})}{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n + 3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n + 1 + \frac{2}{n})} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{2 - \frac{1}{n}}{\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n + 3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n + 1 + \frac{2}{n})}} \\ &= \frac{2 - \frac{1}{\infty}}{\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty}) \cdot (\infty + 3 + \frac{1}{\infty})} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty + 1 + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty}) \cdot (\infty + 1 + \frac{2}{\infty})}} \\ &= \frac{2 - 0}{\sqrt[3]{(1 + 0 + 0) \cdot (\infty + 3 + 0)} + \sqrt[3]{1 + 0 + 0} \sqrt[3]{\infty + 1 + 0} + \sqrt[3]{(1 + 0 + 0) \cdot (\infty + 1 + 0)}} \\ &= \frac{2}{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}} = \frac{2}{\infty + \infty + \infty} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené příklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right]$$

[167]

Riešené príklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right]$$

[167]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2} \right] = (n^2 + 3n + 1) - (n^2 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]} = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{\sqrt[3]{n^2 \cdot n^2}} = \frac{\sqrt[3]{n^3}}{\sqrt[3]{n^4}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]} \end{aligned}$$

Riešené príklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] = 0$$

[167]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2} \right] = (n^2 + 3n + 1) - (n^2 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]} = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{\sqrt[3]{n^2 \cdot n^2}} = \frac{\sqrt[3]{n^3}}{\sqrt[3]{n^4}} = \frac{1}{\sqrt[3]{n}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]} \\ &= \frac{1}{\sqrt[3]{\infty} \left[\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{2}{\infty} + \frac{1}{\infty}} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right)^2} \right]} \\ &= \frac{1}{\infty \left[\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \sqrt[3]{1+0+0} + \sqrt[3]{(1+0+0)^2} \right]} = \frac{1}{\infty \cdot (1+1 \cdot 1+1)} = \frac{1}{\infty \cdot 3} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right]$$

[Iné riešenie 167]

Riešené príklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right]$$

[Iné riešenie 167]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2} \right] = (n^2 + 3n + 1) - (n^2 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)}} \end{aligned}$$

Riešené príklady – 167

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] = 0$$

[Iné riešenie 167]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2 + 3n + 1} - \sqrt[3]{n^2 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2} \right] = (n^2 + 3n + 1) - (n^2 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2 + 3n + 1)^2} + \sqrt[3]{n^2 + 3n + 1} \cdot \sqrt[3]{n^2 + 2n + 1} + \sqrt[3]{(n^2 + 2n + 1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)}} \\ &= \frac{1}{\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right) \cdot \left(\infty + 3 + \frac{1}{\infty}\right)} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty + 2 + \frac{1}{\infty}} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right) \cdot \left(\infty + 2 + \frac{1}{\infty}\right)}} \\ &= \frac{1}{\sqrt[3]{(1+0+0) \cdot (\infty+3+0)} + \sqrt[3]{1+0+0} \sqrt[3]{\infty+2+0} + \sqrt[3]{(1+0+0) \cdot (\infty+2+0)}} \\ &= \frac{1}{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}} = \frac{1}{\infty + \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené příklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right]$$

[168]

Riešené príklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right]$$

[168]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2} \right] = (n^3 + 3n + 1) - (n^3 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n}{n^2 \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]} \end{aligned}$$

Riešené príklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] = 0$$

[168]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2} \right] = (n^3 + 3n + 1) - (n^3 + 2n + 1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\ &= \lim_{n \rightarrow \infty} \frac{n}{n^2 \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]} \\ &= \frac{1}{\infty \left[\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{2}{\infty} + \frac{1}{\infty}} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right)^2} \right]} \\ &= \frac{1}{\infty \left[\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \sqrt[3]{1+0+0} + \sqrt[3]{(1+0+0)^2} \right]} = \frac{1}{\infty \cdot (1+1 \cdot 1+1)} = \frac{1}{\infty \cdot 3} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right]$$

[Iné riešenie 168]

Riešené príklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right]$$

[Iné riešenie 168]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2} \right] = (n^3 + 3n + 1) - (n^3 + 2n + 1) = n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 3n + 1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 2n + 1)} \right]} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 3n + 1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 2n + 1)}} \end{aligned}$$

Riešené príklady – 168

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] = 0$$

[Iné riešenie 168]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \frac{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\
 &\quad \left. \Rightarrow \left[\sqrt[3]{n^3 + 3n + 1} - \sqrt[3]{n^3 + 2n + 1} \right] \cdot \left[\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2} \right] = (n^3 + 3n + 1) - (n^3 + 2n + 1) = n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^3 + 3n + 1)^2} + \sqrt[3]{n^3 + 3n + 1} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{(n^3 + 2n + 1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n}{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 3n + 1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 2n + 1)} \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 3n + 1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3 + 2n + 1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3 + 2n + 1)}} \\
 &= \frac{1}{\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right) \cdot (\infty + 3\infty + 1)} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty + 2\infty + 1} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right) \cdot (\infty + 2\infty + 1)}} \\
 &= \frac{1}{\sqrt[3]{(1+0+0) \cdot (\infty + \infty + 1)} + \sqrt[3]{1+0+0} \sqrt[3]{\infty + \infty + 1} + \sqrt[3]{(1+0+0) \cdot (\infty + \infty + 1)}} \\
 &= \frac{1}{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}} = \frac{1}{\infty + \infty + \infty} = \frac{1}{\infty} = 0.
 \end{aligned}$$

Riešené příklady – 169, 170, 171

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1} \right]$$

[169]

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right]$$

[170]

$$\lim_{n \rightarrow \infty} \left[\sqrt[5]{n^5 + 1} - \sqrt[5]{n^4 + 1} \right]$$

[171]

Riešené príklady – 169, 170, 171

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1} \right]$$

[169]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n^2 + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right]$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right]$$

[170]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left[\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right]$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[5]{n^5 + 1} - \sqrt[5]{n^4 + 1} \right]$$

[171]

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[5]{n^4} \cdot \left[\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right]$$

Riešené príklady – 169, 170, 171

$$\lim_{n \rightarrow \infty} \left[\sqrt[3]{n^3 + n + 1} - \sqrt[3]{n - 1} \right] = \infty$$

[169]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt{n} \cdot \left[\sqrt{n^2 + 1 + \frac{1}{n}} - \sqrt{1 - \frac{1}{n}} \right] = \infty \cdot \left[\sqrt{\infty + 1 + \frac{1}{\infty}} - \sqrt{1 - \frac{1}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt{\infty + 1 + 0} - \sqrt{1 - 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n + 1} \right] = \infty$$

[170]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[4]{n} \cdot \left[\sqrt[4]{n^3 + \frac{1}{n}} - \sqrt[4]{1 + \frac{1}{n}} \right] = \infty \cdot \left[\sqrt[4]{\infty + \frac{1}{\infty}} - \sqrt[4]{1 + \frac{1}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt[4]{\infty + 0} - \sqrt[4]{1 + 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \left[\sqrt[5]{n^5 + 1} - \sqrt[5]{n^4 + 1} \right] = \infty$$

[171]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \sqrt[5]{n^4} \cdot \left[\sqrt[5]{n + \frac{1}{n^4}} - \sqrt[5]{1 + \frac{1}{n^4}} \right] = \infty \cdot \left[\sqrt[5]{\infty + \frac{1}{\infty}} - \sqrt[5]{1 + \frac{1}{\infty}} \right] \\ &= \infty \cdot \left[\sqrt[5]{\infty + 0} - \sqrt[5]{1 + 0} \right] = \infty \cdot (\infty - 1) = \infty. \end{aligned}$$

Riešené příklady – 172

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right]$$

[172]

Riešené príklady – 172

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right]$$

[172]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \frac{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}}{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \left[\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3} \right] = (n^4 + 1) - (n^4 - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \frac{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \left[\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \right] = \sqrt[4]{(n^4 + 1)^2} - \sqrt[4]{(n^4 - 1)^2} = \sqrt{n^4 + 1} - \sqrt{n^4 - 1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + 1} - \sqrt{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + 1} - \sqrt{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \cdot \frac{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}}{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + 1} - \sqrt{n^4 - 1} \right] \cdot \left[\sqrt{n^4 + 1} + \sqrt{n^4 - 1} \right] = (n^4 + 1) - (n^4 - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \cdot \frac{1}{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}} \end{aligned}$$

Riešené príklady – 172

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] = 0$$

[172]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \frac{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}}{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \left[\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3} \right] = (n^4 + 1) - (n^4 - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt[4]{(n^4 + 1)^3} + \sqrt[4]{(n^4 + 1)^2} \sqrt[4]{n^4 - 1} + \sqrt[4]{n^4 + 1} \sqrt[4]{(n^4 - 1)^2} + \sqrt[4]{(n^4 - 1)^3}} \\ &= \frac{2}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{2}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \frac{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + 1} - \sqrt[4]{n^4 - 1} \right] \cdot \left[\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1} \right] = \sqrt[4]{(n^4 + 1)^2} - \sqrt[4]{(n^4 - 1)^2} = \sqrt{n^4 + 1} - \sqrt{n^4 - 1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + 1} - \sqrt{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + 1} - \sqrt{n^4 - 1}}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \cdot \frac{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}}{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + 1} - \sqrt{n^4 - 1} \right] \cdot \left[\sqrt{n^4 + 1} + \sqrt{n^4 - 1} \right] = (n^4 + 1) - (n^4 - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2}{\sqrt[4]{n^4 + 1} + \sqrt[4]{n^4 - 1}} \cdot \frac{1}{\sqrt{n^4 + 1} + \sqrt{n^4 - 1}} = \frac{2}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{2}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené příklady – 173

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q} \right]$$

pre $p, q \in R, p \neq q, a > 0.$

[173]

Riešené příklady – 173

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q} \right]$$

pre $p, q \in R, p \neq q, a > 0$.

[173]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q} \right] \cdot \frac{\sqrt[4]{(n^a + p)^3} + \sqrt[4]{(n^a + p)^2} \sqrt[4]{n^a + q} + \sqrt[4]{n^a + p} \sqrt[4]{(n^a + q)^2} + \sqrt[4]{(n^a + q)^3}}{\sqrt[4]{(n^a + p)^3} + \sqrt[4]{(n^a + p)^2} \sqrt[4]{n^a + q} + \sqrt[4]{n^a + p} \sqrt[4]{(n^a + q)^2} + \sqrt[4]{(n^a + q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q}] \cdot [\sqrt[4]{(n^a + p)^3} + \sqrt[4]{(n^a + p)^2} \sqrt[4]{n^a + q} + \sqrt[4]{n^a + p} \sqrt[4]{(n^a + q)^2} + \sqrt[4]{(n^a + q)^3}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{(n^a + p)^3} + \sqrt[4]{(n^a + p)^2} \sqrt[4]{n^a + q} + \sqrt[4]{n^a + p} \sqrt[4]{(n^a + q)^2} + \sqrt[4]{(n^a + q)^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q} \right] \cdot \frac{\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}}{\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[4]{n^a + p} - \sqrt[4]{n^a + q}] \cdot [\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}] = \sqrt[4]{(n^a + p)^2} - \sqrt[4]{(n^a + q)^2} = \sqrt{n^a + p} - \sqrt{n^a + q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^a + p} - \sqrt{n^a + q}}{\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^a + p} - \sqrt{n^a + q}}{\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}} \cdot \frac{\sqrt{n^a + p} + \sqrt{n^a + q}}{\sqrt{n^a + p} + \sqrt{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^a + p} - \sqrt{n^a + q}] \cdot [\sqrt{n^a + p} + \sqrt{n^a + q}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{n^a + p} + \sqrt[4]{n^a + q}} \cdot \frac{1}{\sqrt{n^a + p} + \sqrt{n^a + q}} \end{aligned}$$

Riešené příklady – 173

$$\lim_{n \rightarrow \infty} \left[\sqrt[n]{n^a + p} - \sqrt[n]{n^a + q} \right] = 0 \text{ pre } p, q \in R, p \neq q, a > 0.$$

[173]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[n]{n^a + p} - \sqrt[n]{n^a + q} \right] \cdot \frac{\sqrt[n]{(n^a + p)^3} + \sqrt[n]{(n^a + p)^2} \sqrt[n]{n^a + q} + \sqrt[n]{n^a + p} \sqrt[n]{(n^a + q)^2} + \sqrt[n]{(n^a + q)^3}}{\sqrt[n]{(n^a + p)^3} + \sqrt[n]{(n^a + p)^2} \sqrt[n]{n^a + q} + \sqrt[n]{n^a + p} \sqrt[n]{(n^a + q)^2} + \sqrt[n]{(n^a + q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[n]{n^a + p} - \sqrt[n]{n^a + q}] \cdot [\sqrt[n]{(n^a + p)^3} + \sqrt[n]{(n^a + p)^2} \sqrt[n]{n^a + q} + \sqrt[n]{n^a + p} \sqrt[n]{(n^a + q)^2} + \sqrt[n]{(n^a + q)^3}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[n]{(n^a + p)^3} + \sqrt[n]{(n^a + p)^2} \sqrt[n]{n^a + q} + \sqrt[n]{n^a + p} \sqrt[n]{(n^a + q)^2} + \sqrt[n]{(n^a + q)^3}} \\ &= \frac{p - q}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[n]{n^a + p} - \sqrt[n]{n^a + q} \right] \cdot \frac{\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}}{\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[n]{n^a + p} - \sqrt[n]{n^a + q}] \cdot [\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}] = \sqrt[n]{(n^a + p)^2} - \sqrt[n]{(n^a + q)^2} = \sqrt{n^a + p} - \sqrt{n^a + q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^a + p} - \sqrt{n^a + q}}{\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^a + p} - \sqrt{n^a + q}}{\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}} \cdot \frac{\sqrt{n^a + p} + \sqrt{n^a + q}}{\sqrt{n^a + p} + \sqrt{n^a + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^a + p} - \sqrt{n^a + q}] \cdot [\sqrt{n^a + p} + \sqrt{n^a + q}] = (n^a + p) - (n^a + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[n]{n^a + p} + \sqrt[n]{n^a + q}} \cdot \frac{1}{\sqrt{n^a + p} + \sqrt{n^a + q}} = \frac{p - q}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{p - q}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené příklady – 174

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \quad \text{pre } p, q \in \mathbb{R}, p \neq q.$$

[174]

Riešené príklady – 174

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right]$$

pre $p, q \in R, p \neq q.$

[174]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \frac{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}}{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \left[\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3} \right] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \frac{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \left[\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q} \right] = \sqrt[4]{(n^5 + p)^2} - \sqrt[4]{(n^5 + q)^2} = \sqrt{n^5 + p} - \sqrt{n^5 + q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + p} - \sqrt{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + p} - \sqrt{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \cdot \frac{\sqrt{n^5 + p} + \sqrt{n^5 + q}}{\sqrt{n^5 + p} + \sqrt{n^5 + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt{n^5 + p} - \sqrt{n^5 + q} \right] \cdot \left[\sqrt{n^5 + p} + \sqrt{n^5 + q} \right] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \cdot \frac{1}{\sqrt{n^5 + p} + \sqrt{n^5 + q}} \end{aligned}$$

Riešené príklady – 174

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] = 0 \text{ pre } p, q \in R, p \neq q.$$

[174]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \frac{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}}{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q}] \cdot [\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{(n^5 + p)^3} + \sqrt[4]{(n^5 + p)^2} \sqrt[4]{n^5 + q} + \sqrt[4]{n^5 + p} \sqrt[4]{(n^5 + q)^2} + \sqrt[4]{(n^5 + q)^3}} \\ &= \frac{p - q}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q} \right] \cdot \frac{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[4]{n^5 + p} - \sqrt[4]{n^5 + q}] \cdot [\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}] = \sqrt[4]{(n^5 + p)^2} - \sqrt[4]{(n^5 + q)^2} = \sqrt{n^5 + p} - \sqrt{n^5 + q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + p} - \sqrt{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + p} - \sqrt{n^5 + q}}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \cdot \frac{\sqrt{n^5 + p} + \sqrt{n^5 + q}}{\sqrt{n^5 + p} + \sqrt{n^5 + q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^5 + p} - \sqrt{n^5 + q}] \cdot [\sqrt{n^5 + p} + \sqrt{n^5 + q}] = (n^5 + p) - (n^5 + q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{n^5 + p} + \sqrt[4]{n^5 + q}} \cdot \frac{1}{\sqrt{n^5 + p} + \sqrt{n^5 + q}} = \frac{p - q}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{p - q}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené príklady – 175

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right]$$

[175]

Riešené príklady – 175

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right]$$

[175]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \frac{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}}{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \left[\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3} \right] = (n^5 + 2) - (n^5 + 1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}} \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \frac{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \left[\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1} \right] = \sqrt[4]{(n^5 + 2)^2} - \sqrt[4]{(n^5 + 1)^2} = \sqrt{n^5 + 2} - \sqrt{n^5 + 1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + 2} - \sqrt{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + 2} - \sqrt{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \cdot \frac{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right] \cdot \left[\sqrt{n^5 + 2} + \sqrt{n^5 + 1} \right] = (n^5 + 2) - (n^5 + 1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \cdot \frac{1}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \end{aligned}$$

Riešené príklady – 175

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] = 0$$

[175]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \frac{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}}{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \left[\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3} \right] = (n^5 + 2) - (n^5 + 1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(n^5 + 2)^3} + \sqrt[4]{(n^5 + 2)^2} \sqrt[4]{n^5 + 1} + \sqrt[4]{n^5 + 2} \sqrt[4]{(n^5 + 1)^2} + \sqrt[4]{(n^5 + 1)^3}} \\ &= \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \frac{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^5 + 2} - \sqrt[4]{n^5 + 1} \right] \cdot \left[\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1} \right] = \sqrt[4]{(n^5 + 2)^2} - \sqrt[4]{(n^5 + 1)^2} = \sqrt{n^5 + 2} - \sqrt{n^5 + 1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + 2} - \sqrt{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^5 + 2} - \sqrt{n^5 + 1}}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \cdot \frac{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^5 + 2} - \sqrt{n^5 + 1} \right] \cdot \left[\sqrt{n^5 + 2} + \sqrt{n^5 + 1} \right] = (n^5 + 2) - (n^5 + 1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5 + 2} + \sqrt[4]{n^5 + 1}} \cdot \frac{1}{\sqrt{n^5 + 2} + \sqrt{n^5 + 1}} = \frac{1}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{1}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené príklady – 176

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + p} - \sqrt[4]{\sqrt[5]{n} + q} \right]$$

pre $p, q \in \mathbb{R}, p \neq q$.

[176]

Riešené príklady – 176

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right]$$

pre $p, q \in R, p \neq q$.

[176]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \frac{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}}{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \left[\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3} \right] = (\sqrt[5]{n+p}) - (\sqrt[5]{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \frac{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \left[\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}} \right] = \sqrt[4]{(\sqrt[5]{n+p})^2} - \sqrt[4]{(\sqrt[5]{n+q})^2} = \sqrt[5]{n+p} - \sqrt[5]{n+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n+p} - \sqrt[5]{n+q}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n+p} - \sqrt[5]{n+q}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \cdot \frac{\sqrt[5]{n+p} + \sqrt[5]{n+q}}{\sqrt[5]{n+p} + \sqrt[5]{n+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[5]{n+p} - \sqrt[5]{n+q} \right] \cdot \left[\sqrt[5]{n+p} + \sqrt[5]{n+q} \right] = (\sqrt[5]{n+p})^2 - (\sqrt[5]{n+q})^2 = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \cdot \frac{1}{\sqrt[5]{n+p} + \sqrt[5]{n+q}} \end{aligned}$$

Riešené príklady – 176

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] = 0 \text{ pre } p, q \in R, p \neq q.$$

[176]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \frac{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}}{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \left[\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3} \right] = (\sqrt[5]{n+p}) - (\sqrt[5]{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{(\sqrt[5]{n+p})^3} + \sqrt[4]{(\sqrt[5]{n+p})^2} \sqrt[4]{\sqrt[5]{n+q}} + \sqrt[4]{\sqrt[5]{n+p}} \sqrt[4]{(\sqrt[5]{n+q})^2} + \sqrt[4]{(\sqrt[5]{n+q})^3}} \\ &= \frac{p - q}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{p - q}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \frac{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n+p}} - \sqrt[4]{\sqrt[5]{n+q}} \right] \cdot \left[\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}} \right] = \sqrt[4]{(\sqrt[5]{n+p})^2} - \sqrt[4]{(\sqrt[5]{n+q})^2} = \sqrt[5]{n+p} - \sqrt[5]{n+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n+p} - \sqrt[5]{n+q}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n+p} - \sqrt[5]{n+q}}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \cdot \frac{\sqrt[5]{n+p} + \sqrt[5]{n+q}}{\sqrt[5]{n+p} + \sqrt[5]{n+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[5]{n+p} - \sqrt[5]{n+q} \right] \cdot \left[\sqrt[5]{n+p} + \sqrt[5]{n+q} \right] = (\sqrt[5]{n+p})^2 - (\sqrt[5]{n+q})^2 = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{p - q}{\sqrt[4]{\sqrt[5]{n+p}} + \sqrt[4]{\sqrt[5]{n+q}}} \cdot \frac{1}{\sqrt[5]{n+p} + \sqrt[5]{n+q}} = \frac{p - q}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{p - q}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené příklady – 177

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right]$$

[177]

Riešené príklady – 177

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right]$$

[177]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}}{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3} \right] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1} \right] = \sqrt[4]{(\sqrt[5]{n} + 2)^2} - \sqrt[4]{(\sqrt[5]{n} + 1)^2} = \sqrt[5]{n} + 2 - \sqrt[5]{n} + 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1}}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1}}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \cdot \frac{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1} \right] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \cdot \frac{1}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \end{aligned}$$

Riešené príklady – 177

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] = 0$$

[177]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}}{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3} \right] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{(\sqrt[5]{n} + 2)^3} + \sqrt[4]{(\sqrt[5]{n} + 2)^2} \sqrt[4]{\sqrt[5]{n} + 1} + \sqrt[4]{\sqrt[5]{n} + 2} \sqrt[4]{(\sqrt[5]{n} + 1)^2} + \sqrt[4]{(\sqrt[5]{n} + 1)^3}} \\ &= \frac{1}{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \frac{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{\sqrt[5]{n} + 2} - \sqrt[4]{\sqrt[5]{n} + 1} \right] \cdot \left[\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1} \right] = \sqrt[4]{(\sqrt[5]{n} + 2)^2} - \sqrt[4]{(\sqrt[5]{n} + 1)^2} = \sqrt[5]{n} + 2 - \sqrt[5]{n} + 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n} + 2 - \sqrt[5]{n} + 1}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[5]{n} + 2 - \sqrt[5]{n} + 1}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \cdot \frac{\sqrt[5]{n} + 2 + \sqrt[5]{n} + 1}{\sqrt[5]{n} + 2 + \sqrt[5]{n} + 1} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[5]{n} + 2 - \sqrt[5]{n} + 1 \right] \cdot \left[\sqrt[5]{n} + 2 + \sqrt[5]{n} + 1 \right] = (\sqrt[5]{n} + 2) - (\sqrt[5]{n} + 1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n} + 2} + \sqrt[4]{\sqrt[5]{n} + 1}} \cdot \frac{1}{\sqrt[5]{n} + 2 + \sqrt[5]{n} + 1} = \frac{1}{\infty + \infty} \cdot \frac{1}{\infty + \infty} = \frac{1}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0. \end{aligned}$$

Riešené příklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[178]

Riešené příklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[178]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &\quad \quad \quad = (n^4 + n + 1) - (n - 1)^4 = (n^4 + n + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \left. \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^3 - 6n^2 + 5n}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^4 \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left(4 - \frac{6}{n} + \frac{5}{n^2} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{6}{n} + \frac{5}{n^2}}{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3} \end{aligned}$$

Riešené příklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right] = 1$$

[178]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3} \\
 &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &= (n^4 + n + 1) - (n - 1)^4 = (n^4 + n + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \end{aligned} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{4n^3 - 6n^2 + 5n}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n + 1} \cdot (n - 1)^2 + (n - 1)^3} \\
 &= \left[n \in \mathbb{N} \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^3 \left(4 - \frac{6}{n} + \frac{5}{n^2} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{4 - \frac{6}{n} + \frac{5}{n^2}}{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3} \\
 &= \frac{4 - \frac{6}{\infty} + \frac{5}{\infty}}{\sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty} \right)^2} \cdot \left(1 - \frac{1}{\infty} \right) + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty} \right)^2 + \left(1 - \frac{1}{\infty} \right)^3} \\
 &= \frac{4 - 0 + 0}{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} \cdot (1-0) + \sqrt[4]{1+0+0} \cdot (1-0)^2 + (1-0)^3} = \frac{4}{1+1 \cdot 1+1 \cdot 1+1} = \frac{4}{4} = 1.
 \end{aligned}$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3}$$

$$= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ \left. \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3 \right] = (n^4 + n + 1) - n^4 = n + 1. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3} = \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3 \left(\frac{1}{n^2} + \frac{1}{n^3} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^2} + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^2} + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1}$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right] = 1$$

[Iné riešenie 178]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3}$$

$$= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ \left. \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3 \right] = (n^4 + n + 1) - n^4 = n + 1. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n+1}{\sqrt[4]{(n^4 + n + 1)^3} + \sqrt[4]{(n^4 + n + 1)^2} \cdot n + \sqrt[4]{n^4 + n + 1} \cdot n^2 + n^3} = \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3 \left(\frac{1}{n^2} + \frac{1}{n^3} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^2} + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{\frac{1}{n^2} + \frac{1}{n^3}}{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^2} + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1}$$

$$= 1 + \frac{\frac{1}{\infty} + \frac{1}{\infty}}{\sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1}$$

$$= 1 + \frac{0+0}{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} + \sqrt[4]{1+0+0+1}} = 1 + \frac{0}{1+1+1+1} = 1.$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n + 1} + (n - 1)}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \\
 &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n + 1} - (n - 1)^2. \end{array} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n + 1} + (n - 1)^2}{\sqrt{n^4 + n + 1} + (n - 1)^2} \\
 &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^4 + n + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n + 1} + (n - 1)^2 \right] = (n^4 + n + 1) - (n - 1)^4 = (n^4 + n + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \end{array} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{4n^3 - 6n^2 + 5n}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4} \cdot \sqrt[4]{n^4} = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^3 \left(4 - \frac{6}{n} + \frac{5}{n^2} \right)}{n \left[\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + \left(1 - \frac{1}{n} \right) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} + \left(1 - \frac{1}{n} \right)^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{4 - \frac{6}{n} + \frac{5}{n^2}}{\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + \left(1 - \frac{1}{n} \right)} \cdot \frac{1}{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4} \right)^2} + \left(1 - \frac{1}{n} \right)^2}
 \end{aligned}$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right] = 1$$

[Iné riešenie 178]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n + 1} + (n - 1)}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n + 1} - (n - 1)^2. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n + 1} + (n - 1)^2}{\sqrt{n^4 + n + 1} + (n - 1)^2} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt{n^4 + n + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n + 1} + (n - 1)^2 \right] = (n^4 + n + 1) - (n - 1)^4 = (n^4 + n + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^3 - 6n^2 + 5n}{\sqrt[4]{n^4 + n + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3(4 - \frac{6}{n} + \frac{5}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + (1 - \frac{1}{n}) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{(1 + \frac{1}{n^3} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{6}{n} + \frac{5}{n^2}}{\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + (1 - \frac{1}{n})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{n^3} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2} = \frac{4 - \frac{6}{\infty} + \frac{5}{\infty}}{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + (1 - \frac{1}{\infty})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + (1 - \frac{1}{\infty})^2} \\ &= \frac{4 - 0 + 0}{\sqrt[4]{1 + 0 + 0} + (1 - 0)} \cdot \frac{1}{\sqrt[4]{(1 + 0 + 0)^2} + (1 - 0)^2} = \frac{4}{1 + 1} \cdot \frac{1}{1 + 1} = \frac{4}{2 \cdot 2} = 1. \end{aligned}$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right]$$

[Iné riešenie 178]

$$\begin{aligned}
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n + 1} + n}{\sqrt[4]{n^4 + n + 1} + n} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n + 1} + n \right] = \sqrt[4]{(n^4 + n + 1)^2} - n^2 = \sqrt{n^4 + n + 1} - n^2. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - n^2}{\sqrt[4]{n^4 + n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - n^2}{\sqrt[4]{n^4 + n + 1} + n} \cdot \frac{\sqrt{n^4 + n + 1} + n^2}{\sqrt{n^4 + n + 1} + n^2} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n + 1} + n^2 \right] = (n^4 + n + 1) - n^4 = (n^4 + n + 1) - n^4 = n + 1. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt[4]{n^4 + n + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2} \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2}
 \end{aligned}$$

Riešené príklady – 178

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n + 1 \right] = 1$$

[Iné riešenie 178]

$$\begin{aligned}
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n + 1} + n}{\sqrt[4]{n^4 + n + 1} + n} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n + 1} + n \right] = \sqrt[4]{(n^4 + n + 1)^2} - n^2 = \sqrt{n^4 + n + 1} - n^2. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - n^2}{\sqrt[4]{n^4 + n + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n + 1} - n^2}{\sqrt[4]{n^4 + n + 1} + n} \cdot \frac{\sqrt{n^4 + n + 1} + n^2}{\sqrt{n^4 + n + 1} + n^2} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n + 1} + n^2 \right] = (n^4 + n + 1) - n^4 = (n^4 + n + 1) - n^4 = n + 1. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n + 1}{\sqrt[4]{n^4 + n + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n(1 + \frac{1}{n})}{n \left[\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2} \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{\sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} + 1} \cdot \frac{1}{\sqrt{n^4 + n + 1} + n^2} \\
 &= 1 + \frac{1 + \frac{1}{\infty}}{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} \cdot \frac{1}{\sqrt{\infty + \infty + 1} + \infty} = 1 + \frac{1 + 0}{\sqrt[4]{1 + 0 + 0 + 1}} \cdot \frac{1}{\infty + \infty} = 1 + \frac{1}{1 + 1} \cdot \frac{1}{\infty} = 1 + \frac{1}{2} \cdot 0 = 1.
 \end{aligned}$$

Riešené příklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[179]

Riešené příklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[179]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &= (n^4 + n^2 + 1) - (n - 1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^3 - 5n^2 + 4n}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[n \in \mathbb{N} \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left(4 - \frac{5}{n} + \frac{4}{n^2} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{5}{n} + \frac{4}{n^2}}{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3} \end{aligned}$$

Riešené příklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right] = 1$$

[179]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &\quad \left. = (n^4 + n^2 + 1) - (n - 1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^3 - 5n^2 + 4n}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[n \in \mathbb{N} \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^4 \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3(4 - \frac{5}{n} + \frac{4}{n^2})}{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} \cdot (1 - \frac{1}{n}) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot (1 - \frac{1}{n})^2 + (1 - \frac{1}{n})^3 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{5}{n} + \frac{4}{n^2}}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} \cdot (1 - \frac{1}{n}) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot (1 - \frac{1}{n})^2 + (1 - \frac{1}{n})^3} \\ &= \frac{4 - \frac{5}{\infty} + \frac{4}{\infty}}{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^3} + \sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} \cdot (1 - \frac{1}{\infty}) + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot (1 - \frac{1}{\infty})^2 + (1 - \frac{1}{\infty})^3} \\ &= \frac{4 - 0 + 0}{\sqrt[4]{(1 + 0 + 0)^3} + \sqrt[4]{(1 + 0 + 0)^2} \cdot (1 - 0) + \sqrt[4]{1 + 0 + 0} \cdot (1 - 0)^2 + (1 - 0)^3} = \frac{4}{1 + 1 \cdot 1 + 1 \cdot 1 + 1} = \frac{4}{4} = 1. \end{aligned}$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[\[Iné riešenie 179\]](#)

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 179]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^3}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^3}$$

$$= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ \left. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^3 \right] = (n^4 + n^2 + 1) - n^4 = n^2 + 1. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^3} = \left[n \in \mathbb{N}. \right. \\ \left. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^3})}{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{n \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right] = 1$$

[Iné riešenie 179]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^2}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^2}$$

$$= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ \left. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^2 \right] = (n^4 + n^2 + 1) - n^4 = n^2 + 1. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^2 + 1} \cdot n^2 + n^2} = \left[n \in \mathbb{N}. \right. \\ \left. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^3})}{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{n \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \frac{1 + \frac{1}{\infty}}{\infty \left[\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^3} + \sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1 \right]}$$

$$= 1 + \frac{1 + 0}{\infty \left[\sqrt[4]{(1 + 0 + 0)^3} + \sqrt[4]{(1 + 0 + 0)^2} + \sqrt[4]{1 + 0 + 0} + 1 \right]} = 1 + \frac{1}{\infty(1 + 1 + 1 + 1)} = 1 + \frac{1}{\infty} = 1 + 0 = 1.$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 179]

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 179]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n^2 + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n^2 + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n^2 + 1} - (n - 1)^2 \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n - 1)^2}{\sqrt{n^4 + n^2 + 1} + (n - 1)^2} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt{n^4 + n^2 + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n^2 + 1} + (n - 1)^2 \right] = (n^4 + n^2 + 1) - (n - 1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{4n^3 - 5n^2 + 4n}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n^2 + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4} \cdot n^4 = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3(4 - \frac{5}{n} + \frac{4}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n}) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{4 - \frac{5}{n} + \frac{4}{n^2}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2} \end{aligned}$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right] = 1$$

[Iné riešenie 179]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n^2 + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n^2 + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n^2 + 1} - (n - 1)^2 \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n - 1)^2}{\sqrt{n^4 + n^2 + 1} + (n - 1)^2} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\
 &\quad \left. \Rightarrow \left[\sqrt{n^4 + n^2 + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n^2 + 1} + (n - 1)^2 \right] = (n^4 + n^2 + 1) - (n - 1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n \right] \\
 &= \lim_{n \rightarrow \infty} \frac{4n^3 - 5n^2 + 4n}{\sqrt[4]{n^4 + n^2 + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n^2 + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^3(4 - \frac{5}{n} + \frac{4}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n}) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{4 - \frac{5}{n} + \frac{4}{n^2}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2} = \frac{4 - \frac{5}{\infty} + \frac{4}{\infty}}{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + (1 - \frac{1}{\infty})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + (1 - \frac{1}{\infty})^2} \\
 &= \frac{4 - 0 + 0}{\sqrt[4]{1 + 0 + 0} + (1 - 0)} \cdot \frac{1}{\sqrt[4]{(1 + 0 + 0)^2} + (1 - 0)^2} = \frac{4}{1 + 1} \cdot \frac{1}{1 + 1} = \frac{4}{2 \cdot 2} = \frac{4}{4} = 1.
 \end{aligned}$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 179]

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right]$$

[Iné riešenie 179]

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + n}{\sqrt[4]{n^4 + n^2 + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n^2 + 1} + n \right] = \sqrt[4]{(n^4 + n^2 + 1)^2} - n^2 = \sqrt{n^4 + n^2 + 1} - n^2. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - n^2}{\sqrt[4]{n^4 + n^2 + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - n^2}{\sqrt[4]{n^4 + n^2 + 1} + n} \cdot \frac{\sqrt{n^4 + n^2 + 1} + n^2}{\sqrt{n^4 + n^2 + 1} + n^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n^2 + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n^2 + 1} + n^2 \right] = (n^4 + n^2 + 1) - n^4 = (n^4 + n^2 + 1) - n^4 = n^2 + 1. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[4]{n^4 + n^2 + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n^2 + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt{n^4}. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{n^2 \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \end{aligned}$$

Riešené príklady – 179

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n + 1 \right] = 1$$

[Iné riešenie 179]

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + n}{\sqrt[4]{n^4 + n^2 + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n^2 + 1} + n \right] = \sqrt[4]{(n^4 + n^2 + 1)^2} - n^2 = \sqrt{n^4 + n^2 + 1} - n^2. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - n^2}{\sqrt[4]{n^4 + n^2 + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^2 + 1} - n^2}{\sqrt[4]{n^4 + n^2 + 1} + n} \cdot \frac{\sqrt{n^4 + n^2 + 1} + n^2}{\sqrt{n^4 + n^2 + 1} + n^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n^2 + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n^2 + 1} + n^2 \right] = (n^4 + n^2 + 1) - n^4 = (n^4 + n^2 + 1) - n^4 = n^2 + 1. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2 + 1}{\sqrt[4]{n^4 + n^2 + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n^2 + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt{n^4}. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^2(1 + \frac{1}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{n^2 \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \\ &= 1 + \frac{1 + \frac{1}{\infty}}{\infty \left[\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1 \right]} \cdot \frac{1}{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{1 + 0}{\infty \left[\sqrt[4]{1 + 0 + 0} + 1 \right]} \cdot \frac{1}{\sqrt{1 + 0 + 0} + 1} \\ &= 1 + \frac{1}{\infty(1+1)} \cdot \frac{1}{1+1} = 1 + \frac{1}{\infty} \cdot \frac{1}{2} = 1 + 0 \cdot \frac{1}{2} = 1 + 0 = 1. \end{aligned}$$

Riešené příklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[180]

Riešené příklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[180]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &= (n^4 + n^3 + 1) - (n - 1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{5n^3 - 6n^2 + 4n}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left(5 - \frac{6}{n} + \frac{4}{n^2} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{5 - \frac{6}{n} + \frac{4}{n^2}}{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3} \end{aligned}$$

Riešené příklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right] = \frac{5}{4}$$

[180]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3 \right] \\ &= (n^4 + n^3 + 1) - (n - 1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{5n^3 - 6n^2 + 4n}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n - 1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n - 1)^2 + (n - 1)^3} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^4 \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left(5 - \frac{6}{n} + \frac{4}{n^2} \right)}{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3 \right]} \\ &= \lim_{n \rightarrow \infty} \frac{5 - \frac{6}{n} + \frac{4}{n^2}}{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4} \right)^2} \cdot \left(1 - \frac{1}{n} \right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n} \right)^2 + \left(1 - \frac{1}{n} \right)^3} \\ &= \frac{5 - \frac{6}{\infty} + \frac{4}{\infty}}{\sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty} \right)^3} + \sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty} \right)^2} \cdot \left(1 - \frac{1}{\infty} \right) + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty} \right)^2 + \left(1 - \frac{1}{\infty} \right)^3} \\ &= \frac{5 - 0 + 0}{\sqrt[4]{(1 + 0 + 0)^3} + \sqrt[4]{(1 + 0 + 0)^2} \cdot (1 - 0) + \sqrt[4]{1 + 0 + 0} \cdot (1 - 0)^2 + (1 - 0)^3} = \frac{5}{1 + 1 \cdot 1 + 1 \cdot 1 + 1} = \frac{5}{4}. \end{aligned}$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[\[Iné riešenie 180\]](#)

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[Iné riešenie 180]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3}$$

$$= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3 \right] = (n^4 + n^3 + 1) - n^4 = n^3 + 1. \end{aligned} \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3 + 1}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3} = \left[\begin{aligned} &n \in \mathbb{N}. \\ &\Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \end{aligned} \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3(1 + \frac{1}{n^3})}{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1}$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right] = \frac{5}{4}$$

[Iné riešenie 180]

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right]$$

$$\bullet = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3}$$

$$= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \left[\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3 \right] = (n^4 + n^3 + 1) - n^4 = n^3 + 1. \end{aligned} \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3 + 1}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot n + \sqrt[4]{n^4 + n^3 + 1} \cdot n^2 + n^3} = \left[\begin{aligned} &n \in \mathbb{N}. \\ &\Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \end{aligned} \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{n^3(1 + \frac{1}{n^3})}{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]}$$

$$= 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1}$$

$$= 1 + \frac{1 + \frac{1}{\infty}}{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^3} + \sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1}$$

$$= 1 + \frac{1+0}{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} + \sqrt[4]{1+0+0} + 1} = 1 + \frac{1}{1+1+1+1} = 1 + \frac{1}{4} = \frac{5}{4}.$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[Iné riešenie 180]

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[Iné riešenie 180]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n^3 + 1} - (n - 1)^2 \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n - 1)^2}{\sqrt{n^4 + n^3 + 1} + (n - 1)^2} \\
 &= \left[\begin{aligned} &(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow \left[\sqrt{n^4 + n^3 + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + (n - 1)^2 \right] = (n^4 + n^3 + 1) - (n - 1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \end{aligned} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{5n^3 - 6n^2 + 4n}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n^3 + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^3(5 - \frac{6}{n} + \frac{4}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n}) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{5 - \frac{6}{n} + \frac{4}{n^2}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2}
 \end{aligned}$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right] = \frac{5}{4}$$

[Iné riešenie 180]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n - 1) \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + (n - 1) \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - (n - 1)^2 = \sqrt{n^4 + n^3 + 1} - (n - 1)^2 \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - (n - 1)^2}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n - 1)^2}{\sqrt{n^4 + n^3 + 1} + (n - 1)^2} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\
 &\quad \left. \Rightarrow \left[\sqrt{n^4 + n^3 + 1} - (n - 1)^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + (n - 1)^2 \right] = (n^4 + n^3 + 1) - (n - 1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n \right] \\
 &= \lim_{n \rightarrow \infty} \frac{5n^3 - 6n^2 + 4n}{\sqrt[4]{n^4 + n^3 + 1} + (n - 1)} \cdot \frac{1}{\sqrt{n^4 + n^3 + 1} + (n - 1)^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^3(5 - \frac{6}{n} + \frac{4}{n^2})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n}) \right]} \cdot \frac{1}{n^2 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2 \right]} \\
 &= \lim_{n \rightarrow \infty} \frac{5 - \frac{6}{n} + \frac{4}{n^2}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + (1 - \frac{1}{n})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} + (1 - \frac{1}{n})^2} = \frac{5 - \frac{6}{\infty} + \frac{4}{\infty}}{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + (1 - \frac{1}{\infty})} \cdot \frac{1}{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} + (1 - \frac{1}{\infty})^2} \\
 &= \frac{5 - 0 + 0}{\sqrt[4]{1 + 0 + 0} + (1 - 0)} \cdot \frac{1}{\sqrt[4]{(1 + 0 + 0)^2} + (1 - 0)^2} = \frac{5}{1 + 1} \cdot \frac{1}{1 + 1} = \frac{5}{2 \cdot 2} = \frac{5}{4}.
 \end{aligned}$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[\[Iné riešenie 180\]](#)

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right]$$

[Iné riešenie 180]

$$\begin{aligned} &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + n}{\sqrt[4]{n^4 + n^3 + 1} + n} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + n \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - n^2 = \sqrt{n^4 + n^3 + 1} - n^2. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - n^2}{\sqrt[4]{n^4 + n^3 + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - n^2}{\sqrt[4]{n^4 + n^3 + 1} + n} \cdot \frac{\sqrt{n^4 + n^3 + 1} + n^2}{\sqrt{n^4 + n^3 + 1} + n^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n^3 + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + n^2 \right] = (n^4 + n^3 + 1) - n^4 = (n^4 + n^3 + 1) - n^4 = n^3 + 1. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^3 + 1}{\sqrt[4]{n^4 + n^3 + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n^3 + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt{n^4}. \right] \\ &= 1 + \lim_{n \rightarrow \infty} \frac{n^3(1 + \frac{1}{n^3})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{n^2 \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \cdot \frac{1}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \end{aligned}$$

Riešené príklady – 180

$$\lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n + 1 \right] = \frac{5}{4}$$

[Iné riešenie 180]

$$\begin{aligned}
 &= 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] = 1 + \lim_{n \rightarrow \infty} \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + n}{\sqrt[4]{n^4 + n^3 + 1} + n} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - n \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + n \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - n^2 = \sqrt{n^4 + n^3 + 1} - n^2. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - n^2}{\sqrt[4]{n^4 + n^3 + 1} + n} = 1 + \lim_{n \rightarrow \infty} \frac{\sqrt{n^4 + n^3 + 1} - n^2}{\sqrt[4]{n^4 + n^3 + 1} + n} \cdot \frac{\sqrt{n^4 + n^3 + 1} + n^2}{\sqrt{n^4 + n^3 + 1} + n^2} \\
 &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt{n^4 + n^3 + 1} - n^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + n^2 \right] = (n^4 + n^3 + 1) - n^4 = (n^4 + n^3 + 1) - n^4 = n^3 + 1. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n^3 + 1}{\sqrt[4]{n^4 + n^3 + 1} + n} \cdot \frac{1}{\sqrt{n^4 + n^3 + 1} + n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt{n^4}. \right] \\
 &= 1 + \lim_{n \rightarrow \infty} \frac{n^3(1 + \frac{1}{n^3})}{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} \cdot \frac{1}{n^2 \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1 \right]} = 1 + \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n^3}}{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \cdot \frac{1}{\sqrt{1 + \frac{1}{n} + \frac{1}{n^4}} + 1} \\
 &= 1 + \frac{1 + \frac{1}{\infty}}{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} \cdot \frac{1}{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1} = 1 + \frac{1 + 0}{\sqrt[4]{1 + 0 + 0 + 1}} \cdot \frac{1}{\sqrt{1 + 0 + 0 + 1}} \\
 &= 1 + \frac{1}{1 + 1} \cdot \frac{1}{1 + 1} = 1 + \frac{1}{2} \cdot \frac{1}{2} = 1 + \frac{1}{4} = \frac{5}{4}.
 \end{aligned}$$

Riešené príklady – 181, 182

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n}$$

[181]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+1}-n^2}$$

[182]

Riešené príklady – 181, 182

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n}$$

[181]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \left[\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right]}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n} \cdot \frac{\sqrt{n+1}+n}{\sqrt{n+1}+n} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \frac{1}{\sqrt{n+1}-n} \cdot [\sqrt{n+1}+n] = (n+1) - n^2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}+n}{n+1-n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1 \right]}{n \left(1 + \frac{1}{n} - n \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1}{1 + \frac{1}{n} - n} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+1}-n^2}$$

[182]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3 \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3} \left[\sqrt{1 + \frac{1}{n^3}} - \sqrt{n} \right]}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+1}-n^2} \cdot \frac{\sqrt{n^3+1}+n^2}{\sqrt{n^3+1}+n^2} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^3+1}-n] \cdot [\sqrt{n^3+1}+n] = (n^3+1) - n^4. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}+n^2}{n^3+1-n^4} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3 \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1 \right]}{n^3 \left(1 + \frac{1}{n^3} - n \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1}{n \left(1 + \frac{1}{n} - n \right)} \end{aligned}$$

Riešené príklady – 181, 182

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n} = 0$$

[181]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \left[\sqrt{1 + \frac{1}{n}} - \sqrt{n} \right]} = \frac{1}{\infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \infty \right]} \\ &= \frac{1}{\infty \cdot (\sqrt{1+0} - \infty)} = \frac{1}{\infty \cdot (1 - \infty)} = \frac{1}{\infty \cdot (-\infty)} = \frac{1}{-\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-n} \cdot \frac{\sqrt{n+1}+n}{\sqrt{n+1}+n} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \frac{1}{\sqrt{n+1}-n} \cdot [\sqrt{n+1}+n] = (n+1) - n^2. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}+n}{n+1-n^2} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n} \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1 \right]}{n \left(1 + \frac{1}{n} - n \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^2}} + 1}{1 + \frac{1}{n} - n} = \frac{\sqrt{\frac{1}{\infty} + \frac{1}{\infty}} + 1}{1 + \frac{1}{\infty} - \infty} = \frac{\sqrt{0+0}+1}{1+0-\infty} = \frac{1}{-\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+1}-n^2} = 0$$

[182]

$$\begin{aligned} \bullet &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3 \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3} \left[\sqrt{1 + \frac{1}{n^3}} - \sqrt{n} \right]} = \frac{1}{\infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \infty \right]} \\ &= \frac{1}{\infty \cdot (\sqrt{1+0} - \infty)} = \frac{1}{\infty \cdot (1 - \infty)} = \frac{1}{\infty \cdot (-\infty)} = \frac{1}{-\infty} = 0. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3+1}-n^2} \cdot \frac{\sqrt{n^3+1}+n^2}{\sqrt{n^3+1}+n^2} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^3+1}-n^2] \cdot [\sqrt{n^3+1}+n^2] = (n^3+1) - n^4. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3+1}+n^2}{n^3+1-n^4} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4} = \sqrt{n^3 \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1 \right]}{n^3 \left(1 + \frac{1}{n^3} - n \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{1}{n} + \frac{1}{n^4}} + 1}{n \left(1 + \frac{1}{n^3} - n \right)} = \frac{\sqrt{\frac{1}{\infty} + \frac{1}{\infty}} + 1}{\infty \left(1 + \frac{1}{\infty} - \infty \right)} = \frac{\sqrt{0+0}+1}{\infty(1+0-\infty)} \\ &= \frac{1}{\infty(-\infty)} = \frac{1}{-\infty} = 0. \end{aligned}$$

Riešené príklady – 183, 184

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + n + 1} - n + 1}$$

[183]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + n} - n + 1}$$

[184]

Riešené príklady – 183, 184

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n+1}$$

[183]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-(n-1)} \cdot \frac{\sqrt{n^2+n+1}+(n-1)}{\sqrt{n^2+n+1}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n+1}-(n-1)] \cdot [\sqrt{n^2+n+1}+(n-1)] = (n^2+n+1) - (n-1)^2 = (n^2+n+1) - (n^2-2n+1) = 3n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+n-1}{3n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \frac{1}{3} \lim_{n \rightarrow \infty} \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right] \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n+1}$$

[184]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-(n-1)} \cdot \frac{\sqrt{n^2+n}+(n-1)}{\sqrt{n^2+n}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n}-(n-1)] \cdot [\sqrt{n^2+n}+(n-1)] = (n^2+n) - (n-1)^2 = (n^2+n) - (n^2-2n+1) = 3n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n}+n-1}{3n-1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n} \right]}{n \left(3 - \frac{1}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n}}{3 - \frac{1}{n}} \end{aligned}$$

Riešené príklady – 183, 184

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n+1} = \frac{2}{3}$$

[183]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-(n-1)} \cdot \frac{\sqrt{n^2+n+1}+(n-1)}{\sqrt{n^2+n+1}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n+1}-(n-1)] \cdot [\sqrt{n^2+n+1}+(n-1)] = (n^2+n+1) - (n-1)^2 = (n^2+n+1) - (n^2-2n+1) = 3n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+n-1}{3n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \frac{1}{3} \lim_{n \rightarrow \infty} \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 - \frac{1}{n} \right] \\ &= \frac{1}{3} \left[\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1 - \frac{1}{\infty} \right] = \frac{1}{3} \cdot [\sqrt{1+0+0} + 1 - 0] = \frac{1}{3} \cdot 2 = \frac{2}{3}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n+1} = \frac{2}{3}$$

[184]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-(n-1)} \cdot \frac{\sqrt{n^2+n}+(n-1)}{\sqrt{n^2+n}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n}-(n-1)] \cdot [\sqrt{n^2+n}+(n-1)] = (n^2+n) - (n-1)^2 = (n^2+n) - (n^2-2n+1) = 3n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n}+n-1}{3n-1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n} \right]}{n \left(3 - \frac{1}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n}} + 1 - \frac{1}{n}}{3 - \frac{1}{n}} = \frac{\sqrt{1 + \frac{1}{\infty}} + 1 - \frac{1}{\infty}}{3 - \frac{1}{\infty}} = \frac{\sqrt{1+0} + 1 - 0}{3-0} = \frac{1+1}{3} = \frac{2}{3}. \end{aligned}$$

Riešené príklady – 185, 186

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n} - n - 1}$$

[185]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n - 1} - n + 2}$$

[186]

Riešené príklady – 185, 186

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n} - n - 1}$$

[185]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n} - (n+1)} \cdot \frac{\sqrt{n^2 - n} + (n+1)}{\sqrt{n^2 - n} + (n+1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - (n+1)] \cdot [\sqrt{n^2 - n} + (n+1)] = (n^2 - n) - (n+1)^2 = (n^2 - n) - (n^2 + 2n + 1) = -3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n} + n + 1}{-3n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n} \right]}{n \left(-3 - \frac{1}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n}}{-3 - \frac{1}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n - 1} - n + 2}$$

[186]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n - 1} - (n-2)} \cdot \frac{\sqrt{n^2 + 2n - 1} + (n-2)}{\sqrt{n^2 + 2n - 1} + (n-2)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - (n-2)] \cdot [\sqrt{n^2 + 2n - 1} + (n-2)] = (n^2 + 2n - 1) - (n-2)^2 = (n^2 + 2n - 1) - (n^2 - 4n + 4) = 6n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 2n - 1} + n - 2}{6n - 5} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n} \right]}{n \left(6 - \frac{5}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n}}{6 - \frac{5}{n}} \end{aligned}$$

Riešené príklady – 185, 186

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n} - n - 1} = -\frac{2}{3}$$

[185]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n} - (n+1)} \cdot \frac{\sqrt{n^2 - n} + (n+1)}{\sqrt{n^2 - n} + (n+1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n} - (n+1)] \cdot [\sqrt{n^2 - n} + (n+1)] = (n^2 - n) - (n+1)^2 = (n^2 - n) - (n^2 + 2n + 1) = -3n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n} + n + 1}{-3n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n} \right]}{n \left(-3 - \frac{1}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n}} + 1 + \frac{1}{n}}{-3 - \frac{1}{n}} = \frac{\sqrt{1 + \frac{1}{\infty}} + 1 + \frac{1}{\infty}}{-3 - \frac{1}{\infty}} = \frac{\sqrt{1+0} + 1 - 0}{-3 - 0} = \frac{1+1}{-3} = -\frac{2}{3}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n - 1} - n + 2} = \frac{1}{3}$$

[186]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n - 1} - (n-2)} \cdot \frac{\sqrt{n^2 + 2n - 1} + (n-2)}{\sqrt{n^2 + 2n - 1} + (n-2)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 + 2n - 1} - (n-2)] \cdot [\sqrt{n^2 + 2n - 1} + (n-2)] = (n^2 + 2n - 1) - (n-2)^2 = (n^2 + 2n - 1) - (n^2 - 4n + 4) = 6n - 5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 2n - 1} + n - 2}{6n - 5} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n} \right]}{n \left(6 - \frac{5}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{n} - \frac{1}{n^2}} + 1 - \frac{2}{n}}{6 - \frac{5}{n}} = \frac{\sqrt{1 + \frac{2}{\infty} - \frac{1}{\infty^2}} + 1 - \frac{2}{\infty}}{6 - \frac{5}{\infty}} = \frac{\sqrt{1+0-0} + 1 - 0}{6 - 0} = \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3}. \end{aligned}$$

Riešené příklady – 187, 188

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n + 4} - n - 3}$$

[187]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 4n + 1} - n + 1}$$

[188]

Riešené príklady – 187, 188

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n+4}-n-3}$$

[187]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n+4}-(n+3)} \cdot \frac{\sqrt{n^2+2n+4}+(n+3)}{\sqrt{n^2+2n+4}+(n+3)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+2n+4}-(n+3)] \cdot [\sqrt{n^2+2n+4}+(n+3)] = (n^2+2n+4) - (n+3)^2 = (n^2+2n+4) - (n^2+6n+9) = -4n-5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+2n+4}+n+3}{-4n-5} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1+\frac{2}{n}+\frac{4}{n^2}} + 1 + \frac{3}{n} \right]}{n \left(-4 - \frac{5}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{2}{n}+\frac{4}{n^2}} + 1 + \frac{3}{n}}{-4 - \frac{5}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n+1}-n+1}$$

[188]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n+1}-(n-1)} \cdot \frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+4n+1}-(n-1)] \cdot [\sqrt{n^2+4n+1}+(n-1)] = (n^2+4n+1) - (n-1)^2 = (n^2+4n+1) - (n^2-2n+1) = 6n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+4n+1}+n-1}{6n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \frac{1}{6} \lim_{n \rightarrow \infty} \left[\sqrt{1+\frac{4}{n}+\frac{1}{n^2}} + 1 - \frac{1}{n} \right] \\ &= \frac{1}{6} \left[\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}} + 1 - \frac{1}{\infty} \right] \end{aligned}$$

Riešené příklady – 187, 188

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n+4}-n-3} = -\frac{1}{2}$$

[187]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2n+4}-(n+3)} \cdot \frac{\sqrt{n^2+2n+4}+(n+3)}{\sqrt{n^2+2n+4}+(n+3)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+2n+4}-(n+3)] \cdot [\sqrt{n^2+2n+4}+(n+3)] = (n^2+2n+4) - (n+3)^2 = (n^2+2n+4) - (n^2+6n+9) = -4n-5. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+2n+4}+n+3}{-4n-5} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}+1+\frac{3}{n} \right]}{n \left(-4-\frac{5}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{2}{n}+\frac{4}{n^2}}+1+\frac{3}{n}}{-4-\frac{5}{n}} = \frac{\sqrt{1+\frac{2}{\infty}+\frac{4}{\infty}}+1+\frac{3}{\infty}}{-4-\frac{5}{\infty}} = \frac{\sqrt{1+0+0}+1+0}{-4-0} = \frac{1+1}{-4} = \frac{2}{-4} = -\frac{1}{2}. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n+1}-n+1} = \frac{1}{3}$$

[188]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n+1}-(n-1)} \cdot \frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+4n+1}-(n-1)] \cdot [\sqrt{n^2+4n+1}+(n-1)] = (n^2+4n+1) - (n-1)^2 = (n^2+4n+1) - (n^2-2n+1) = 6n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+4n+1}+n-1}{6n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \frac{1}{6} \lim_{n \rightarrow \infty} \left[\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n} \right] \\ &= \frac{1}{6} \left[\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty} \right] = \frac{1}{6} [\sqrt{1+0+0}+1-0] = \frac{1}{6} \cdot 2 = \frac{1}{3}. \end{aligned}$$

Riešené príklady – 189

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - 3n + 1} - n + 3}$$

[189]

Riešené príklady – 189

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - 3n + 1} - n + 3}$$

[189]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - 3n + 1} - (n - 3)} \cdot \frac{\sqrt{n^2 - 3n + 1} + (n - 3)}{\sqrt{n^2 - 3n + 1} + (n - 3)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - 3n + 1} - (n - 3)] \cdot [\sqrt{n^2 - 3n + 1} + (n - 3)] = (n^2 - 3n + 1) - (n - 3)^2 = (n^2 - 3n + 1) - (n^2 - 6n + 9) = 3n - 8. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - 3n + 1} + n - 3}{3n - 8} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n} \right]}{n(3 - \frac{8}{n})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 - \frac{3}{n} + \frac{1}{n^2}} + 1 - \frac{3}{n}}{6 - \frac{8}{n}} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n - 3 \mid n \rightarrow \infty \mid \begin{array}{l} n^2 - 3n + 1 = (k + 3)^2 - 3(k + 3) + 1 \\ n = k + 3 \mid k \rightarrow \infty \end{array} \right. \\ &\quad \left. = k^2 + 6k + 9 - 3k - 9 + 1 = k^2 + 3k + 1 \right] = \lim_{k \rightarrow \infty} \frac{1}{\sqrt{k^2 + 3k + 1} - k} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k^2 + 3k + 1} - k} \cdot \frac{\sqrt{k^2 + 3k + 1} + k}{\sqrt{k^2 + 3k + 1} + k} = \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k^2 + 3k + 1} - k] \cdot [\sqrt{k^2 + 3k + 1} + k] = (k^2 + 3k + 1) - k^2 = 3k + 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{k^2 + 3k + 1} + k}{3k + 1} = \left[k \in \mathbb{N}. \Rightarrow k = \sqrt{k^2} = \sqrt{k \cdot k}. \right] = \lim_{n \rightarrow \infty} \frac{k \left[\sqrt{1 + \frac{3}{k} + \frac{1}{k^2}} + 1 \right]}{k(3 + \frac{1}{k})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{3}{k} + \frac{1}{k^2}} + 1}{3 + \frac{1}{k}} \end{aligned}$$

Riešené příklady – 189

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-3n+1}-n+3} = \frac{2}{3}$$

[189]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-3n+1}-(n-3)} \cdot \frac{\sqrt{n^2-3n+1}+(n-3)}{\sqrt{n^2-3n+1}+(n-3)} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2-3n+1} - (n-3)] \cdot [\sqrt{n^2-3n+1} + (n-3)] = (n^2-3n+1) - (n-3)^2 = (n^2-3n+1) - (n^2-6n+9) = 3n-8. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2-3n+1}+n-3}{3n-8} = \left[n \in \mathbb{N} \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n} \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1-\frac{3}{n}+\frac{1}{n^2}}+1-\frac{3}{n} \right]}{n(3-\frac{8}{n})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1-\frac{3}{n}+\frac{1}{n^2}}+1-\frac{3}{n}}{6-\frac{8}{n}} = \frac{\sqrt{1-\frac{3}{\infty}+\frac{1}{\infty}}+1-\frac{3}{\infty}}{3-\frac{8}{\infty}} = \frac{\sqrt{1-0+0}+1-0}{3-0} = \frac{1+1}{3} = \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n-3 \mid n \rightarrow \infty \right. \\ &\quad \left. n = k+3 \mid k \rightarrow \infty \right] \frac{n^2-3n+1 = (k+3)^2-3(k+3)+1}{= k^2+6k+9-3k-9+1 = k^2+3k+1} = \lim_{k \rightarrow \infty} \frac{1}{\sqrt{k^2+3k+1}-k} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k^2+3k+1}-k} \cdot \frac{\sqrt{k^2+3k+1}+k}{\sqrt{k^2+3k+1}+k} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k^2+3k+1} - k] \cdot [\sqrt{k^2+3k+1} + k] = (k^2+3k+1) - k^2 = 3k+1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{k^2+3k+1}+k}{3k+1} = \left[k \in \mathbb{N} \Rightarrow k = \sqrt{k^2} = \sqrt{k \cdot k} \right] = \lim_{n \rightarrow \infty} \frac{k \left[\sqrt{1+\frac{3}{k}+\frac{1}{k^2}}+1 \right]}{k(3+\frac{1}{k})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{3}{k}+\frac{1}{k^2}}+1}{3+\frac{1}{k}} = \frac{\sqrt{1+\frac{3}{\infty}+\frac{1}{\infty}}+1}{3+\frac{1}{\infty}} = \frac{\sqrt{1+0+0}+1}{3+0} = \frac{1+1}{3} = \frac{2}{3}. \end{aligned}$$

Riešené príklady – 190, 191

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + n + 1} - n}$$

[190]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + n} - n}$$

[191]

Riešené príklady – 190, 191

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n}$$

[190]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n} \cdot \frac{\sqrt{n^2+n+1}+n}{\sqrt{n^2+n+1}+n} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n+1}-n] \cdot [\sqrt{n^2+n+1}+n] = (n^2+n+1) - n^2 = n+1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+n}{n+1} \\ &= \left[n \in N. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1 \right]}{n \left(1+\frac{1}{n} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+1}{1+\frac{1}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n}$$

[191]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n}-n] \cdot [\sqrt{n^2+n}+n] = (n^2+n) - n^2 = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n}+n}{n} = \left[n \in N. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \left[\sqrt{1+\frac{1}{n}}+1 \right] \end{aligned}$$

Riešené príklady – 190, 191

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n} = \frac{1}{2}$$

[190]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-n} \cdot \frac{\sqrt{n^2+n+1}+n}{\sqrt{n^2+n+1}+n} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n+1}-n] \cdot [\sqrt{n^2+n+1}+n] = (n^2+n+1) - n^2 = n+1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+n}{n+1} \\ &= \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1 \right]}{n \left(1 + \frac{1}{n} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n} + \frac{1}{n^2}} + 1}{1 + \frac{1}{n}} \\ &= \frac{\sqrt{1 + \frac{1}{\infty} + \frac{1}{\infty}} + 1}{1 + \frac{1}{\infty}} = \frac{\sqrt{1+0+0}+1}{1+0} = \frac{1+1}{1} = 2. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n} = \frac{1}{2}$$

[191]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n}-n} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2+n}-n] \cdot [\sqrt{n^2+n}+n] = (n^2+n) - n^2 = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n}+n}{n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \left[\sqrt{1 + \frac{1}{n}} + 1 \right] \\ &= \sqrt{1 + \frac{1}{\infty}} + 1 = \sqrt{1+0} + 1 = 2. \end{aligned}$$

Riešené príklady – 192, 193, 194

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p} - \sqrt{n+q}}$$

pre $p, q \in \mathbb{R}, p \neq q$.

[192]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n}}$$

[193]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n-1}}$$

[194]

Riešené príklady – 192, 193, 194

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p} - \sqrt{n+q}}$$

pre $p, q \in \mathbb{R}, p \neq q$.

[192]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p} - \sqrt{n+q}} \cdot \frac{\sqrt{n+p} + \sqrt{n+q}}{\sqrt{n+p} + \sqrt{n+q}} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+p} - \sqrt{n+q}] \cdot [\sqrt{n+p} + \sqrt{n+q}] = (n+p) - (n+q) = p-q. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+p} + \sqrt{n+q}}{p-q} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n}}$$

[193]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n}} \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n}] \cdot [\sqrt{n+1} + \sqrt{n}] = (n+1) - n = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+1} + \sqrt{n}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n+1} + \sqrt{n}] \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n-1}}$$

[194]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} - \sqrt{n-1}} \cdot \frac{\sqrt{n+1} + \sqrt{n-1}}{\sqrt{n+1} + \sqrt{n-1}} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n+1} - \sqrt{n-1}] \cdot [\sqrt{n+1} + \sqrt{n-1}] = (n+1) - (n-1) = 2. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+1} + \sqrt{n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} [\sqrt{n+1} + \sqrt{n-1}] \end{aligned}$$

Riešené príklady – 192, 193, 194

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p}-\sqrt{n+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q. \quad [192]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p}-\sqrt{n+q}} \cdot \frac{\sqrt{n+p}+\sqrt{n+q}}{\sqrt{n+p}+\sqrt{n+q}} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n+p} - \sqrt{n+q}] \cdot [\sqrt{n+p} + \sqrt{n+q}] = (n+p) - (n+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+p}+\sqrt{n+q}}{p-q} = \frac{\infty+\infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n}} = \infty \quad [193]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n}} \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n+1} - \sqrt{n}] \cdot [\sqrt{n+1} + \sqrt{n}] = (n+1) - n = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}+\sqrt{n}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n+1} + \sqrt{n}] = \infty + \infty = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n-1}} = \infty \quad [194]$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1}-\sqrt{n-1}} \cdot \frac{\sqrt{n+1}+\sqrt{n-1}}{\sqrt{n+1}+\sqrt{n-1}} = \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n+1} - \sqrt{n-1}] \cdot [\sqrt{n+1} + \sqrt{n-1}] = (n+1) - (n-1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}+\sqrt{n-1}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} [\sqrt{n+1} + \sqrt{n-1}] = \frac{1}{2} [\infty + \infty] = \frac{1}{2} \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 195

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p} - \sqrt{an+q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[195]

Riešené príklady – 195

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p} - \sqrt{an+q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[195]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p} - \sqrt{an+q}} \cdot \frac{\sqrt{an+p} + \sqrt{an+q}}{\sqrt{an+p} + \sqrt{an+q}} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{an+p} - \sqrt{an+q}] \cdot [\sqrt{an+p} + \sqrt{an+q}] = (an+p) - (an+q) = p - q. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{an+p} + \sqrt{an+q}}{p - q} \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = an \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}} \cdot \frac{\sqrt{k+p} + \sqrt{k+q}}{\sqrt{k+p} + \sqrt{k+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p - q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p} + \sqrt{k+q}}{p - q} \end{aligned}$$

$$\bullet = \left[\text{Subst. } k = an \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}}$$

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{\alpha + \frac{p}{n}} - \sqrt{\alpha + \frac{q}{n}} \right]}$$

Riešené príklady – 195

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p} - \sqrt{an+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q, a > 0.$$

[195]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{an+p} - \sqrt{an+q}} \cdot \frac{\sqrt{an+p} + \sqrt{an+q}}{\sqrt{an+p} + \sqrt{an+q}} = \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{an+p} - \sqrt{an+q}] \cdot [\sqrt{an+p} + \sqrt{an+q}] = (an+p) - (an+q) = p-q. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{an+p} + \sqrt{an+q}}{p-q} = \frac{\infty + \infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = an \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}} \cdot \frac{\sqrt{k+p} + \sqrt{k+q}}{\sqrt{k+p} + \sqrt{k+q}} \\ &= \left[\begin{array}{l} (\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p-q. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p} + \sqrt{k+q}}{p-q} \\ &= \frac{\infty + \infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = an \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p} - \sqrt{k+q}} \\ &= \left[\text{Pr. 192: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p} - \sqrt{n+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q. \right] = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{\alpha + \frac{p}{n}} - \sqrt{\alpha + \frac{q}{n}}]} = \frac{1}{\sqrt{\infty} \cdot [\sqrt{\alpha + \frac{p}{\infty}} - \sqrt{\alpha + \frac{q}{\infty}}]} = \frac{1}{\sqrt{\infty} \cdot [\sqrt{\alpha + 0} - \sqrt{\alpha + 0}]} \\ &= \frac{1}{\sqrt{\infty} \cdot (\sqrt{\alpha} - \sqrt{\alpha})} = \frac{1}{\infty \cdot 0} = \frac{1}{??}. \quad \left[\text{Postup nemôžeme použiť.} \right] \end{aligned}$$

Riešené príklady – 196, 197

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24} - \sqrt{2n+33}}$$

[196]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{3n} - \sqrt{2n}}$$

[197]

Riešené príklady – 196, 197

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24} - \sqrt{2n+33}}$$

[196]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24} - \sqrt{2n+33}} \cdot \frac{\sqrt{2n+24} + \sqrt{2n+33}}{\sqrt{2n+24} + \sqrt{2n+33}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{2n+24} - \sqrt{2n+33}] \cdot [\sqrt{2n+24} + \sqrt{2n+33}] = (2n+24) - (2n+33) = 24 - 33 = -9. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{2n+24} + \sqrt{2n+33}}{-9} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = 2n + 24 \mid n \rightarrow \infty \right. \\ &\quad \left. k + 9 = 2n + 33 \mid k \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k+9}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k+9}} \cdot \frac{\sqrt{k} + \sqrt{k+9}}{\sqrt{k} + \sqrt{k+9}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k} - \sqrt{k+9}] \cdot [\sqrt{k} + \sqrt{k+9}] = k - (k+9) = -9. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k} + \sqrt{k+9}}{-9} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{3n} - \sqrt{2n}}$$

[197]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{3} - \sqrt{2}]}$$

$$\bullet = \frac{1}{\lim_{n \rightarrow \infty} [\sqrt{3n} - \sqrt{2n}]}$$

Riešené príklady – 196, 197

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24} - \sqrt{2n+33}} = -\infty$$

[196]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{2n+24} - \sqrt{2n+33}} \cdot \frac{\sqrt{2n+24} + \sqrt{2n+33}}{\sqrt{2n+24} + \sqrt{2n+33}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{2n+24} - \sqrt{2n+33}] \cdot [\sqrt{2n+24} + \sqrt{2n+33}] = (2n+24) - (2n+33) = 24 - 33 = -9. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{2n+24} + \sqrt{2n+33}}{-9} = \frac{\infty + \infty}{-9} = -\frac{\infty}{9} = -\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = 2n + 24 \begin{cases} n \rightarrow \infty \\ k \rightarrow \infty \end{cases} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k+9}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k+9}} \cdot \frac{\sqrt{k} + \sqrt{k+9}}{\sqrt{k} + \sqrt{k+9}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k} - \sqrt{k+9}] \cdot [\sqrt{k} + \sqrt{k+9}] = k - (k+9) = -9. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k} + \sqrt{k+9}}{-9} = \frac{\infty + \infty}{-9} = -\frac{\infty}{9} = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{3n} - \sqrt{2n}} = 0$$

[197]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{3} - \sqrt{2}]} = \frac{1}{\infty \cdot [\sqrt{3} - \sqrt{2}]} = \frac{1}{\infty} = 0.$$

$$\bullet = \frac{1}{\lim_{n \rightarrow \infty} [\sqrt{3n} - \sqrt{2n}]} = \left[\text{Pr. 134: } \lim_{n \rightarrow \infty} [\sqrt{3n} - \sqrt{2n}] = \infty. \right] = \frac{1}{\infty} = 0.$$

Riešené príklady – 198

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^a + p} - \sqrt{n^a + q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[198]

Riešené príklady – 198

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^a+p}-\sqrt{n^a+q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[198]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^a+p}-\sqrt{n^a+q}} \cdot \frac{\sqrt{n^a+p}+\sqrt{n^a+q}}{\sqrt{n^a+p}+\sqrt{n^a+q}} \\ &= [(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R} \Rightarrow [\sqrt{n^a+p} - \sqrt{n^a+q}] \cdot [\sqrt{n^a+p} + \sqrt{n^a+q}] = (n^a+p) - (n^a+q) = p-q.] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^a+p}+\sqrt{n^a+q}}{p-q} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^a \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}} \cdot \frac{\sqrt{k+p}+\sqrt{k+q}}{\sqrt{k+p}+\sqrt{k+q}} \\ &= [(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R} \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p-q.] \end{aligned}$$

$$= \left[\text{Subst. } k = n^a \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^a} \cdot \left[\sqrt{1+\frac{p}{n^a}} - \sqrt{1+\frac{q}{n^a}} \right]}$$

Riešené príklady – 198

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+p}-\sqrt{n^2+q}} = \text{sgn}(p-q)\infty \text{ pre } p, q \in R, p \neq q, a > 0.$$

[198]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+p}-\sqrt{n^2+q}} \cdot \frac{\sqrt{n^2+p}+\sqrt{n^2+q}}{\sqrt{n^2+p}+\sqrt{n^2+q}} \\ &= [(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^2+p} - \sqrt{n^2+q}] \cdot [\sqrt{n^2+p} + \sqrt{n^2+q}] = (n^2+p) - (n^2+q) = p-q.] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+p}+\sqrt{n^2+q}}{p-q} = \frac{\infty+\infty}{p-q} = \frac{\infty}{p-q} = \text{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^a \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}} \cdot \frac{\sqrt{k+p}+\sqrt{k+q}}{\sqrt{k+p}+\sqrt{k+q}} \\ &= [(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{k+p} - \sqrt{k+q}] \cdot [\sqrt{k+p} + \sqrt{k+q}] = (k+p) - (k+q) = p-q.] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{k+p}+\sqrt{k+q}}{p-q} = \frac{\infty+\infty}{p-q} = \frac{\infty}{p-q} = \text{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^a \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+p}-\sqrt{k+q}} \\ &= \left[\text{Pr. 192: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+p}-\sqrt{n+q}} = \text{sgn}(p-q)\infty \text{ pre } p, q \in R, p \neq q. \right] = \text{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2} \cdot [\sqrt{1+\frac{p}{n^2}} - \sqrt{1+\frac{q}{n^2}}]} = \frac{1}{\sqrt{\infty} \cdot [\sqrt{1+\frac{p}{\infty}} - \sqrt{1+\frac{q}{\infty}}]} = \frac{1}{\sqrt{\infty} \cdot [\sqrt{1+0} - \sqrt{1+0}]} \\ &= \frac{1}{\infty \cdot (1-1)} = \frac{1}{\infty \cdot 0} = \frac{1}{??}. \quad [\text{Postup nemôžeme použiť.}] \end{aligned}$$

Riešené príklady – 199, 200

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}}$$

[199]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}}-\sqrt{\sqrt{n+1}}}$$

[200]

Riešené príklady – 199, 200

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}}$$

[199]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}} \cdot \frac{\sqrt{n^2+2}+\sqrt{n^2+1}}{\sqrt{n^2+2}+\sqrt{n^2+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2+2} - \sqrt{n^2+1}] \cdot [\sqrt{n^2+2} + \sqrt{n^2+1}] = (n^2+2) - (n^2+1) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+2}+\sqrt{n^2+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n^2+2} + \sqrt{n^2+1}] \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^2 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p}+\sqrt{k+q}}{1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}}-\sqrt{\sqrt{n+1}}}$$

[200]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}}-\sqrt{\sqrt{n+1}}} \cdot \frac{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}}{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] \cdot [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] = (\sqrt{n+2}) - (\sqrt{n+1}) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}}{1} = \lim_{n \rightarrow \infty} [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt{n} \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p}+\sqrt{k+q}}{1} \end{aligned}$$

Riešené príklady – 199, 200

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}} = \infty$$

[199]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+2}-\sqrt{n^2+1}} \cdot \frac{\sqrt{n^2+2}+\sqrt{n^2+1}}{\sqrt{n^2+2}+\sqrt{n^2+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^2+2} - \sqrt{n^2+1}] \cdot [\sqrt{n^2+2} + \sqrt{n^2+1}] = (n^2+2) - (n^2+1) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+2}+\sqrt{n^2+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n^2+2} + \sqrt{n^2+1}] = \infty + \infty = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^2 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p}+\sqrt{k+q}}{1} = \frac{\infty+\infty}{1} = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}}-\sqrt{\sqrt{n+1}}} = \infty$$

[200]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n+2}}-\sqrt{\sqrt{n+1}}} \cdot \frac{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}}{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{\sqrt{n+2}} - \sqrt{\sqrt{n+1}}] \cdot [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] = (\sqrt{n+2}) - (\sqrt{n+1}) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{\sqrt{n+2}}+\sqrt{\sqrt{n+1}}}{1} = \lim_{n \rightarrow \infty} [\sqrt{\sqrt{n+2}} + \sqrt{\sqrt{n+1}}] = \infty + \infty = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt{n} \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2} - \sqrt{k+1}] \cdot [\sqrt{k+2} + \sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+p}+\sqrt{k+q}}{1} = \frac{\infty+\infty}{1} = \infty. \end{aligned}$$

Riešené príklady – 201, 202

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1}}$$

[201]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1}}$$

[202]

Riešené príklady – 201, 202

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1}}$$

[201]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1}} \cdot \frac{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}}{\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^4 + n^3 - n - 2} - \sqrt{n^4 + n^3 - n^2 - 1}] \cdot [\sqrt{n^4 + n^3 - n - 2} + \sqrt{n^4 + n^3 - n^2 - 1}] = (n^4 + n^3 - n - 2) - (n^4 + n^3 - n^2 - 1) = n^2 - n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^2 - n - 1} \\ &= \left[n \in \mathbb{N} \Rightarrow n^2 = \sqrt{n^4} \right] = \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}} \right]}{n^2 \left(1 - \frac{1}{n} - \frac{1}{n^2} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{n} - \frac{1}{n^3} - \frac{2}{n^4}} + \sqrt{1 + \frac{1}{n} - \frac{1}{n^2} - \frac{1}{n^4}}}{1 - \frac{1}{n} - \frac{1}{n^2}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1}}$$

[202]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1}} \cdot \frac{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}}{\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3 - n - 2} - \sqrt{n^3 - n^2 - 1}] \cdot [\sqrt{n^3 - n - 2} + \sqrt{n^3 - n^2 - 1}] = (n^3 - n - 2) - (n^3 - n^2 - 1) = n^2 - n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^2 - n - 1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}} \right]}{n^2 \left(1 - \frac{1}{n} - \frac{1}{n^2} \right)} = \left[n \in \mathbb{N} \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \sqrt{n} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{1 - \frac{1}{n^2} - \frac{2}{n^3}} + \sqrt{1 - \frac{1}{n} - \frac{1}{n^3}}}{\sqrt{n} \left(1 - \frac{1}{n} - \frac{1}{n^2} \right)} \end{aligned}$$

Riešené příklady – 201, 202

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4+n^3-n-2}-\sqrt{n^4+n^3-n^2-1}} = 2$$

[201]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4+n^3-n-2}-\sqrt{n^4+n^3-n^2-1}} \cdot \frac{\sqrt{n^4+n^3-n-2}+\sqrt{n^4+n^3-n^2-1}}{\sqrt{n^4+n^3-n-2}+\sqrt{n^4+n^3-n^2-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^4+n^3-n-2}-\sqrt{n^4+n^3-n^2-1}] \cdot [\sqrt{n^4+n^3-n-2}+\sqrt{n^4+n^3-n^2-1}] = (n^4+n^3-n-2)-(n^4+n^3-n^2-1) = n^2-n-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4+n^3-n-2}+\sqrt{n^4+n^3-n^2-1}}{n^2-n-1} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt{n^4}. \right] = \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt{1+\frac{1}{n}-\frac{1}{n^3}-\frac{2}{n^4}} + \sqrt{1+\frac{1}{n}-\frac{1}{n^2}-\frac{1}{n^4}} \right]}{n^2 \left(1-\frac{1}{n}-\frac{1}{n^2} \right)} = \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{1}{n}-\frac{1}{n^3}-\frac{2}{n^4}} + \sqrt{1+\frac{1}{n}-\frac{1}{n^2}-\frac{1}{n^4}}}{1-\frac{1}{n}-\frac{1}{n^2}} \\ &= \frac{\sqrt{1+\frac{1}{\infty}-\frac{1}{\infty}-\frac{2}{\infty}} + \sqrt{1+\frac{1}{\infty}-\frac{1}{\infty}-\frac{1}{\infty}}}{1-\frac{1}{\infty}-\frac{1}{\infty}} = \frac{\sqrt{1+0-0-0} + \sqrt{1+0-0-0}}{1+0-0} = \frac{1+1}{1} = 2. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-n-2}-\sqrt{n^3-n^2-1}} = 0$$

[202]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-n-2}-\sqrt{n^3-n^2-1}} \cdot \frac{\sqrt{n^3-n-2}+\sqrt{n^3-n^2-1}}{\sqrt{n^3-n-2}+\sqrt{n^3-n^2-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3-n-2}-\sqrt{n^3-n^2-1}] \cdot [\sqrt{n^3-n-2}+\sqrt{n^3-n^2-1}] = (n^3-n-2)-(n^3-n^2-1) = n^2-n-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3-n-2}+\sqrt{n^3-n^2-1}}{n^2-n-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1-\frac{1}{n^2}-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}} \right]}{n^2 \left(1-\frac{1}{n}-\frac{1}{n^2} \right)} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n}} = \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{1-\frac{1}{n^2}-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}}}{\sqrt{n} \left(1-\frac{1}{n}-\frac{1}{n^2} \right)} \\ &= \frac{\sqrt{1-\frac{1}{\infty}-\frac{2}{\infty}} + \sqrt{1-\frac{1}{\infty}-\frac{1}{\infty}}}{\infty \left(1-\frac{1}{\infty}-\frac{1}{\infty} \right)} = \frac{\sqrt{1-0-0} + \sqrt{1-0-0}}{\infty(1+0-0)} = \frac{1+1}{\infty} = \frac{2}{\infty} = 0. \end{aligned}$$

Riešené príklady – 203, 204

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}}$$

[203]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}}$$

[204]

Riešené príklady – 203, 204

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}}$$

[203]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}} \cdot \frac{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}}{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3-2}-\sqrt{n^3-n^2-1}] \cdot [\sqrt{n^3-2}+\sqrt{n^3-n^2-1}] = (n^3-2) - (n^3-n^2-1) = n^2-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}}{n^2-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}} \right]}{n^2 \left(1 - \frac{1}{n^2} \right)} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}}}{\sqrt{n} \left(1 - \frac{1}{n^2} \right)} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}}$$

[204]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}} \cdot \frac{\sqrt{n^3-2}+\sqrt{n^3-n-1}}{\sqrt{n^3-2}+\sqrt{n^3-n-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3-2}-\sqrt{n^3-n-1}] \cdot [\sqrt{n^3-2}+\sqrt{n^3-n-1}] = (n^3-2) - (n^3-n-1) = n-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3-2}+\sqrt{n^3-n-1}}{n-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n^2}-\frac{1}{n^3}} \right]}{n \left(1 - \frac{1}{n} \right)} = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n^2}-\frac{1}{n^3}} \right]}{1 - \frac{1}{n}} \end{aligned}$$

Riešené príklady – 203, 204

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}} = 0$$

[203]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n^2-1}} \cdot \frac{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}}{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3-2} - \sqrt{n^3-n^2-1}] \cdot [\sqrt{n^3-2} + \sqrt{n^3-n^2-1}] = (n^3-2) - (n^3-n^2-1) = n^2-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3-2}+\sqrt{n^3-n^2-1}}{n^2-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}} \right]}{n^2 \left(1 - \frac{1}{n^2} \right)} = \left[n \in \mathbb{N}. \Rightarrow \frac{n^2}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \sqrt{n}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^3}}}{\sqrt{n} \left(1 - \frac{1}{n^2} \right)} \\ &= \frac{\sqrt{1-\frac{2}{\infty}} + \sqrt{1-\frac{1}{\infty}-\frac{1}{\infty}}}{\infty \left(1 - \frac{1}{\infty} \right)} = \frac{\sqrt{1-0} + \sqrt{1-0-0}}{\infty(1-0)} = \frac{1+1}{\infty} = \frac{2}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}} = \infty$$

[204]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^3-2}-\sqrt{n^3-n-1}} \cdot \frac{\sqrt{n^3-2}+\sqrt{n^3-n-1}}{\sqrt{n^3-2}+\sqrt{n^3-n-1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^3-2} - \sqrt{n^3-n-1}] \cdot [\sqrt{n^3-2} + \sqrt{n^3-n-1}] = (n^3-2) - (n^3-n-1) = n-1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n^3-2}+\sqrt{n^3-n-1}}{n-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^3} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n^2}-\frac{1}{n^3}} \right]}{n \left(1 - \frac{1}{n} \right)} = \left[n \in \mathbb{N}. \Rightarrow \frac{n}{\sqrt{n^3}} = \frac{\sqrt{n^2}}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{n} \left[\sqrt{1-\frac{2}{n^3}} + \sqrt{1-\frac{1}{n^2}-\frac{1}{n^3}} \right]}{1 - \frac{1}{n}} \\ &= \frac{\infty \left[\sqrt{1-\frac{2}{\infty}} + \sqrt{1-\frac{1}{\infty}-\frac{1}{\infty}} \right]}{1 - \frac{1}{\infty}} = \frac{\infty \left[\sqrt{1-0} + \sqrt{1-0-0} \right]}{1-0} = \frac{\infty(1+1)}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

Riešené príklady – 205, 206

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5+2} - \sqrt{n^5+1}}$$

[205]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{\sqrt{n}+2} - \sqrt[5]{\sqrt{n}+1}}$$

[206]

Riešené príklady – 205, 206

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5+2}-\sqrt{n^5+1}}$$

[205]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5+2}-\sqrt{n^5+1}} \cdot \frac{\sqrt{n^5+2}+\sqrt{n^5+1}}{\sqrt{n^5+2}+\sqrt{n^5+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{n^5+2}-\sqrt{n^5+1}] \cdot [\sqrt{n^5+2}+\sqrt{n^5+1}] = (n^5+2) - (n^5+1) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5+2}+\sqrt{n^5+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n^5+2} + \sqrt{n^5+1}] \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2}-\sqrt{k+1}] \cdot [\sqrt{k+2}+\sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+2}+\sqrt{k+1}}{1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{5n+2}-\sqrt[5]{5n+1}}$$

[206]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{5n+2}-\sqrt[5]{5n+1}} \cdot \frac{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}}{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt[5]{5n+2}-\sqrt[5]{5n+1}] \cdot [\sqrt[5]{5n+2}+\sqrt[5]{5n+1}] = (5n+2) - (5n+1) = 2-1 = 1. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt[5]{5n+2} + \sqrt[5]{5n+1}] \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow [\sqrt{k+2}-\sqrt{k+1}] \cdot [\sqrt{k+2}+\sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+2}+\sqrt{k+1}}{1} \end{aligned}$$

Riešené príklady – 205, 206

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5+2}-\sqrt{n^5+1}} = \infty$$

[205]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^5+2}-\sqrt{n^5+1}} \cdot \frac{\sqrt{n^5+2}+\sqrt{n^5+1}}{\sqrt{n^5+2}+\sqrt{n^5+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt{n^5+2}-\sqrt{n^5+1}] \cdot [\sqrt{n^5+2}+\sqrt{n^5+1}] = (n^5+2) - (n^5+1) = 2-1 = 1. \end{array} \right. \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^5+2}+\sqrt{n^5+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt{n^5+2} + \sqrt{n^5+1}] = \infty + \infty = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = n^5 \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt{k+2}-\sqrt{k+1}] \cdot [\sqrt{k+2}+\sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+2}+\sqrt{k+1}}{1} = \frac{\infty+\infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{5n+2}-\sqrt[5]{5n+1}} = \infty$$

[206]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{5n+2}-\sqrt[5]{5n+1}} \cdot \frac{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}}{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}} = \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt[5]{5n+2}-\sqrt[5]{5n+1}] \cdot [\sqrt[5]{5n+2}+\sqrt[5]{5n+1}] = (5n+2) - (5n+1) = 2-1 = 1. \end{array} \right. \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[5]{5n+2}+\sqrt[5]{5n+1}}{1} = \lim_{n \rightarrow \infty} [\sqrt[5]{5n+2} + \sqrt[5]{5n+1}] = \infty + \infty = \infty. \end{aligned}$$

$$\begin{aligned} \bullet &= \left[\text{Subst. } k = 5n \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k+2}-\sqrt{k+1}} \cdot \frac{\sqrt{k+2}+\sqrt{k+1}}{\sqrt{k+2}+\sqrt{k+1}} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in R. \\ \Rightarrow [\sqrt{k+2}-\sqrt{k+1}] \cdot [\sqrt{k+2}+\sqrt{k+1}] = (k+2) - (k+1) = 2-1 = 1. \end{array} \right] = \lim_{n \rightarrow \infty} \frac{\sqrt{k+2}+\sqrt{k+1}}{1} = \frac{\infty+\infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

Riešené príklady – 207, 208

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1} - \sqrt{n^2+n+2}}$$

[207]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1} - \sqrt{n-1}}$$

[208]

Riešené príklady – 207, 208

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

[207]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2+n+1}-\sqrt{n^2+n+2}] \cdot [\sqrt{n^2+n+1}+\sqrt{n^2+n+2}] = (n^2+n+1) - (n^2+n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^2+n+1 \mid n \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k}-\sqrt{k+1}} \cdot \frac{\sqrt{k}+\sqrt{k+1}}{\sqrt{k}+\sqrt{k+1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k}-\sqrt{k+1}] \cdot [\sqrt{k}+\sqrt{k+1}] = k - (k+1) = -1. \right] = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{k}+\sqrt{k+1}} - 1 \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n-1}}$$

[208]

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{n+1+\frac{1}{n}} - \sqrt{1-\frac{1}{n}} \right]}$$

Riešené príklady – 207, 208

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

[207]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^2+n+1}-\sqrt{n^2+n+2}] \cdot [\sqrt{n^2+n+1}+\sqrt{n^2+n+2}] = (n^2+n+1) - (n^2+n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty+\infty+1}+\sqrt{\infty+\infty+2}}{-1} = \frac{\infty+\infty}{-1} = \frac{\infty}{-1} = -\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^2 + n + 1 \mid n \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k}-\sqrt{k+1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k}-\sqrt{k+1}} \cdot \frac{\sqrt{k}+\sqrt{k+1}}{\sqrt{k}+\sqrt{k+1}} \\ &= \left[(\alpha-\beta)(\alpha+\beta) = \alpha^2-\beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{k}-\sqrt{k+1}] \cdot [\sqrt{k}+\sqrt{k+1}] = k - (k+1) = -1. \right] = \lim_{n \rightarrow \infty} \frac{-1}{\sqrt{k}+\sqrt{k+1}} - 1 = \frac{\infty+\infty}{-1} = \frac{\infty}{-1} = -\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n-1}} = 0$$

[208]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}]} = \frac{1}{\infty \cdot [\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}]} \\ &= \frac{1}{\infty \cdot [\sqrt{\infty+1+0}-\sqrt{1-0}]} = \frac{1}{\infty \cdot (\infty-1)} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 209

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}}$$

[209]

Riešené príklady – 209

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}}$$

[209]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}} \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}] = (n^2 - n + 1) - (n^2 - n - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{2} = \lim_{n \rightarrow \infty} \frac{\sqrt{n(n-1)+1} + \sqrt{n(n-1)-1}}{2} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^2 - n + 1 = n(n-1) + 1 \mid n \rightarrow \infty \right. \\ &\quad \left. n^2 - n - 1 = n(n-1) - 1 = k - 2 \mid k \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k-2}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k-2}} \cdot \frac{\sqrt{k} + \sqrt{k-2}}{\sqrt{k} + \sqrt{k-2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{k} - \sqrt{k-2}] \cdot [\sqrt{k} + \sqrt{k-2}] = k - (k-2) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{k} + \sqrt{k-2}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} [\sqrt{k} + \sqrt{k-2}] \end{aligned}$$

Riešené príklady – 209

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}} = \infty$$

[209]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}} \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - n - 1}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}] = (n^2 - n + 1) - (n^2 - n - 1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - n - 1}}{2} = \lim_{n \rightarrow \infty} \frac{\sqrt{n(n-1)+1} + \sqrt{n(n-1)-1}}{2} \\ &= \frac{\sqrt{\infty(\infty-1)+1} + \sqrt{\infty(\infty-1)-1}}{2} = \frac{\sqrt{\infty+1} + \sqrt{\infty-1}}{2} = \frac{\infty + \infty}{2} = \frac{\infty}{2} = \infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^2 - n + 1 = n(n-1) + 1 \mid n \rightarrow \infty \right. \\ &\quad \left. n^2 - n - 1 = n(n-1) - 1 = k - 2 \mid k \rightarrow \infty \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k-2}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{k} - \sqrt{k-2}} \cdot \frac{\sqrt{k} + \sqrt{k-2}}{\sqrt{k} + \sqrt{k-2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{k} - \sqrt{k-2}] \cdot [\sqrt{k} + \sqrt{k-2}] = k - (k-2) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{k} + \sqrt{k-2}}{2} = \frac{1}{2} \lim_{n \rightarrow \infty} [\sqrt{k} + \sqrt{k-2}] \\ &= \frac{1}{2} [\sqrt{\infty} + \sqrt{\infty-1}] = \frac{1}{2} [\infty + \infty] = \frac{1}{2} \infty = \infty. \end{aligned}$$

Riešené príklady – 210, 211

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}}$$

[210]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + 1} - \sqrt{n + 1}}$$

[211]

Riešené príklady – 210, 211

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}}$$

[210]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}} \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}] = (n^2 - n + 1) - (n^2 - 3n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{2n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}} \right]}{n \left(2 - \frac{1}{n} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}}{2 - \frac{1}{n}} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + 1} - \sqrt{n + 1}}$$

[211]

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{n^3 + \frac{1}{n}} - \sqrt{1 + \frac{1}{n}} \right]}$$

Riešené príklady – 210, 211

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}} = 1$$

[210]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}} \cdot \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^2 - n + 1} - \sqrt{n^2 - 3n + 2}] \cdot [\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}] = (n^2 - n + 1) - (n^2 - 3n + 2) = 2n - 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - n + 1} + \sqrt{n^2 - 3n + 2}}{2n - 1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt{n^2} = \sqrt{n \cdot n}. \right] = \lim_{n \rightarrow \infty} \frac{n \left[\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}} \right]}{n(2 - \frac{1}{n})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} + \sqrt{1 - \frac{3}{n} + \frac{2}{n^2}}}{2 - \frac{1}{n}} = \frac{2 - \frac{1}{\infty}}{\sqrt{1 - \frac{1}{\infty} + \frac{1}{\infty}} + \sqrt{1 - \frac{3}{\infty} + \frac{2}{\infty}}} = \frac{2 - 0}{\sqrt{1 - 0 + 0} + \sqrt{1 - 0 + 0}} \\ &= \frac{2}{1 + 1} = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^4 + 1} - \sqrt{n + 1}} = 0$$

[211]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{n^3 + \frac{1}{n}} - \sqrt{1 + \frac{1}{n}}]} = \frac{1}{\infty \cdot [\sqrt{\infty + \frac{1}{\infty}} - \sqrt{1 + \frac{1}{\infty}}]} \\ &= \frac{1}{\infty \cdot [\sqrt{\infty + 0} - \sqrt{1 + 0}]} = \frac{1}{\infty \cdot (\infty - 1)} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené příklady – 212, 213, 214

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n-3}}$$

[212]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n}+1}-\sqrt{n-3}}$$

[213]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n^3-3}}$$

[214]

Riešené príklady – 212, 213, 214

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n-3}}$$

[212]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{n+\frac{1}{n}} - \sqrt{1-\frac{3}{n}} \right]}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n}+1}-\sqrt{n-3}}$$

[213]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n}} \cdot \left[\sqrt{1+\frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n}-\frac{3}{\sqrt{n}}} \right]} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n} \cdot \left[\sqrt{1+\frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n}-\frac{3}{\sqrt{n}}} \right]}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n^3-3}}$$

[214]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2} \cdot \left[\sqrt{1+\frac{1}{n^2}} - \sqrt{n-\frac{3}{n^2}} \right]} = \lim_{n \rightarrow \infty} \frac{1}{n \left[\sqrt{1+\frac{1}{n^2}} - \sqrt{n-\frac{3}{n^2}} \right]}$$

Riešené príklady – 212, 213, 214

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n-3}} = 0$$

[212]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{n + \frac{1}{n}} - \sqrt{1 - \frac{3}{n}} \right]} = \frac{1}{\infty \cdot \left[\sqrt{\infty + \frac{1}{\infty}} - \sqrt{1 - \frac{3}{\infty}} \right]} = \frac{1}{\infty \cdot \left[\sqrt{\infty + 0} - \sqrt{1 - 0} \right]} \\ &= \frac{1}{\infty \cdot (\infty - 1)} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n}+1}-\sqrt{n-3}} = 0$$

[213]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\sqrt{n}} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right]} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n} \cdot \left[\sqrt{1 + \frac{1}{\sqrt{n}}} - \sqrt{\sqrt{n} - \frac{3}{\sqrt{n}}} \right]} \\ &= \frac{1}{\infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \sqrt{\infty - \frac{3}{\infty}} \right]} = \frac{1}{\infty \cdot \left[\sqrt{1 + 0} - \sqrt{\infty - 0} \right]} = \frac{1}{\infty \cdot (1 - \infty)} = \frac{1}{-\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+1}-\sqrt{n^3-3}} = 0$$

[214]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2} \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right]} = \lim_{n \rightarrow \infty} \frac{1}{n \cdot \left[\sqrt{1 + \frac{1}{n^2}} - \sqrt{n - \frac{3}{n^2}} \right]} \\ &= \frac{1}{\infty \cdot \left[\sqrt{1 + \frac{1}{\infty}} - \sqrt{\infty - \frac{3}{\infty}} \right]} = \frac{1}{\infty \cdot \left[\sqrt{1 + 0} - \sqrt{\infty - 0} \right]} = \frac{1}{\infty \cdot (1 - \infty)} = \frac{1}{-\infty} = 0. \end{aligned}$$

Riešené príklady – 215

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n+1}$$

[215]

Riešené príklady – 215

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n+1}$$

[215]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-(n-1)} \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2 \right] \right. \\ &\quad \left. = (n^3+1) - (n-1)^3 = (n^3+1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 3n + 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{3n^2 - 3n + 2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 \right]}{n^2 \left(3 - \frac{3}{n} + \frac{2}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}{3 - \frac{3}{n} + \frac{2}{n^2}} \end{aligned}$$

Riešené príklady – 215

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n+1} = 1$$

[215]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-(n-1)} \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2} \\
 &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2 \right] \right. \\
 &\quad \left. = (n^3+1) - (n-1)^3 = (n^3+1) - (n^3 - 3n^2 + 3n - 1) = 3n^2 - 3n + 2. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot (n-1) + (n-1)^2}{3n^2 - 3n + 2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 \right]}{n^2 \left(3 - \frac{3}{n} + \frac{2}{n^2}\right)} \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}{3 - \frac{3}{n} + \frac{2}{n^2}} \\
 &= \frac{\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right) + \left(1 - \frac{1}{\infty}\right)^2}{3 - \frac{3}{\infty} + \frac{2}{\infty}} \\
 &= \frac{\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} \cdot (1-0) + (1-0)^2}{3-0+0} = \frac{1+1 \cdot 1+1}{3} = \frac{1+1+1}{3} = \frac{3}{3} = 1.
 \end{aligned}$$

Riešené příklady – 216, 217

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

[216]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^4+1}-n}$$

[217]

Riešené príklady – 216, 217

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

[216]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+1} - n] \cdot [\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2] = (n^3+1) - n^3 = 1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{1} \\ &= \lim_{n \rightarrow \infty} \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right] \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^4+1}-n}$$

[217]

$$\bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} \right] = \lim_{n \rightarrow \infty} \frac{1}{n \cdot \left[\sqrt[3]{n + \frac{1}{n^3}} - 1 \right]}$$

Riešené príklady – 216, 217

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} = \infty$$

[216]

$$\begin{aligned} & \bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2} \\ & = \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ & \quad \left. \Rightarrow [\sqrt[3]{n^3+1} - n] \cdot [\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2] = (n^3+1) - n^3 = 1. \right] = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2}{1} \\ & = \lim_{n \rightarrow \infty} \left[\sqrt[3]{(n^3+1)^2} + \sqrt[3]{n^3+1} \cdot n + n^2 \right] = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ & = \lim_{n \rightarrow \infty} n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^3}} + 1 \right] = \infty \left[\sqrt[3]{\left(1 + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{1}{\infty}} + 1 \right] \\ & \quad \infty \left[\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right] = \infty(1+1+1) = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^4+1}-n} = 0$$

[217]

$$\begin{aligned} & \bullet = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] = \lim_{n \rightarrow \infty} \frac{1}{n \cdot \left[\sqrt[3]{n + \frac{1}{n^3}} - 1 \right]} = \frac{1}{\infty \cdot \left[\sqrt[3]{\infty + \frac{1}{\infty}} - 1 \right]} \\ & = \frac{1}{\infty \cdot \left[\sqrt[3]{\infty + 0} - 1 \right]} = \frac{1}{\infty \cdot (\infty - 1)} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené príklady – 218

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n + 1} - n + 1}$$

[218]

Riešené príklady – 218

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-n+1}$$

[218]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-(n-1)} \cdot \frac{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+n+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2 \right] \right. \\ &\quad \left. = (n^3+n+1) - (n-1)^3 = (n^3+n+1) - (n^3-3n^2+3n-1) = 3n^2-2n+2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2}{3n^2-2n+2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 \right]}{n^2 \left(3 - \frac{2}{n} + \frac{2}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{1}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}{3 - \frac{2}{n} + \frac{2}{n^2}} \end{aligned}$$

Riešené príklady – 218

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-n+1} = 1$$

[218]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1}-(n-1)} \cdot \frac{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+n+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2 \right] \right. \\ &\quad \left. = (n^3+n+1) - (n-1)^3 = (n^3+n+1) - (n^3-3n^2+3n-1) = 3n^2-2n+2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+n+1)^2} + \sqrt[3]{n^3+n+1} \cdot (n-1) + (n-1)^2}{3n^2-2n+2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1+\frac{1}{n^2}+\frac{1}{n^3}\right)^2} + \sqrt[3]{1+\frac{1}{n^2}+\frac{1}{n^3}} \cdot \left(1-\frac{1}{n}\right) + \left(1-\frac{1}{n}\right)^2 \right]}{n^2 \left(3-\frac{2}{n}+\frac{2}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1+\frac{1}{n^2}+\frac{1}{n^3}\right)^2} + \sqrt[3]{1+\frac{1}{n^2}+\frac{1}{n^3}} \cdot \left(1-\frac{1}{n}\right) + \left(1-\frac{1}{n}\right)^2}{3-\frac{2}{n}+\frac{2}{n^2}} \\ &= \frac{\sqrt[3]{\left(1+\frac{1}{\infty}+\frac{1}{\infty}\right)^2} + \sqrt[3]{1+\frac{1}{\infty}+\frac{1}{\infty}} \cdot \left(1-\frac{1}{\infty}\right) + \left(1-\frac{1}{\infty}\right)^2}{3-\frac{2}{\infty}+\frac{2}{\infty}} \\ &= \frac{\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \cdot (1-0) + (1-0)^2}{3-0+0} = \frac{1+1 \cdot 1+1}{3} = \frac{3}{3} = 1. \end{aligned}$$

Riešené príklady – 219

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n^2 + 1} - n + 1}$$

[219]

Riešené příklady – 219

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n^2 + 1} - n + 1}$$

[219]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3 + n^2 + 1} - (n-1)} \cdot \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n-1) + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3 + n^2 + 1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n-1) + (n-1)^2 \right] \right. \\ &\quad \left. = (n^3 + n^2 + 1) - (n-1)^3 = (n^3 + n^2 + 1) - (n^3 - 3n^2 + 3n - 1) = 4n^2 - 3n + 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3 + n^2 + 1)^2} + \sqrt[3]{n^3 + n^2 + 1} \cdot (n-1) + (n-1)^2}{4n^2 - 3n + 2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 \right]}{n^2 \left(4 - \frac{3}{n} + \frac{2}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{1}{n} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{1}{n} + \frac{1}{n^3}} \cdot \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2}{4 - \frac{3}{n} + \frac{2}{n^2}} \end{aligned}$$

Riešené príklady – 219

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n^2+1}-n+1} = \frac{3}{4}$$

[219]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n^2+1}-(n-1)} \cdot \frac{\sqrt[3]{(n^3+n^2+1)^2} + \sqrt[3]{n^3+n^2+1} \cdot (n-1) + (n-1)^2}{\sqrt[3]{(n^3+n^2+1)^2} + \sqrt[3]{n^3+n^2+1} \cdot (n-1) + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{n^3+n^2+1} - (n-1) \right] \cdot \left[\sqrt[3]{(n^3+n^2+1)^2} + \sqrt[3]{n^3+n^2+1} \cdot (n-1) + (n-1)^2 \right] \right. \\ &\quad \left. = (n^3+n^2+1) - (n-1)^3 = (n^3+n^2+1) - (n^3-3n^2+3n-1) = 4n^2-3n+2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+n^2+1)^2} + \sqrt[3]{n^3+n^2+1} \cdot (n-1) + (n-1)^2}{4n^2-3n+2} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3} = \sqrt[3]{n^3} \cdot n = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1+\frac{1}{n}+\frac{1}{n^3}\right)^2} + \sqrt[3]{1+\frac{1}{n}+\frac{1}{n^3}} \cdot \left(1-\frac{1}{n}\right) + \left(1-\frac{1}{n}\right)^2 \right]}{n^2 \left(4-\frac{3}{n}+\frac{2}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1+\frac{1}{n}+\frac{1}{n^3}\right)^2} + \sqrt[3]{1+\frac{1}{n}+\frac{1}{n^3}} \cdot \left(1-\frac{1}{n}\right) + \left(1-\frac{1}{n}\right)^2}{4-\frac{3}{n}+\frac{2}{n^2}} \\ &= \frac{\sqrt[3]{\left(1+\frac{1}{\infty}+\frac{1}{\infty}\right)^2} + \sqrt[3]{1+\frac{1}{\infty}+\frac{1}{\infty}} \cdot \left(1-\frac{1}{\infty}\right) + \left(1-\frac{1}{\infty}\right)^2}{4-\frac{3}{\infty}+\frac{2}{\infty}} \\ &= \frac{\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \cdot (1-0) + (1-0)^2}{4-0+0} = \frac{1+1 \cdot 1+1}{4} = \frac{3}{4}. \end{aligned}$$

Riešené príklady – 220, 221

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}}$$

pre $p, q \in \mathbb{R}, p \neq q$.

[220]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1} - \sqrt[3]{n+2}}$$

[221]

Riešené príklady – 220, 221

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}}$$

pre $p, q \in R, p \neq q$.

[220]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} \cdot \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n+p} - \sqrt[3]{n+q}] \cdot [\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}] = (n+p) - (n+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{p - q} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1} - \sqrt[3]{n+2}}$$

[221]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1} - \sqrt[3]{n+2}} \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{n+1} - \sqrt[3]{n+2}] \cdot [\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}] = (n+1) - (n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{-1} \end{aligned}$$

Riešené príklady – 220, 221

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} = \operatorname{sgn}(p-q)\infty \quad \text{pre } p, q \in \mathbb{R}, p \neq q.$$

[220]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} \cdot \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \quad \text{pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n+p} - \sqrt[3]{n+q}] \cdot [\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}] = (n+p) - (n+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n+p)^2} + \sqrt[3]{n+p} \cdot \sqrt[3]{n+q} + \sqrt[3]{(n+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1} - \sqrt[3]{n+2}} = -\infty$$

[221]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+1} - \sqrt[3]{n+2}} \cdot \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \quad \text{pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{n+1} - \sqrt[3]{n+2}] \cdot [\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}] = (n+1) - (n+2) = -1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n+1)^2} + \sqrt[3]{n+1} \cdot \sqrt[3]{n+2} + \sqrt[3]{(n+2)^2}}{-1} = \frac{\infty + \infty \cdot \infty + \infty}{-1} = \frac{\infty}{-1} = -\infty. \end{aligned}$$

Riešené príklady – 222, 223

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

[222]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n+3}}$$

[223]

Riešené príklady – 222, 223

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}}$$

[222]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{n-1} - \sqrt[3]{n-2}] \cdot [\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}] = (n-1) - (n-2) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n+3}}$$

[223]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n+3}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[3]{n-1} - \sqrt[3]{n+3}] \cdot [\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}] = (n-1) - (n+3) = -4. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{-4} \end{aligned}$$

Riešené príklady – 222, 223

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} = \infty$$

[222]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n-2}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{n-1} - \sqrt[3]{n-2}] \cdot [\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}] = (n-1) - (n-2) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n-2} + \sqrt[3]{(n-2)^2}}{1} = \frac{\infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n+3}} = -\infty$$

[223]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n-1} - \sqrt[3]{n+3}} \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{n-1} - \sqrt[3]{n+3}] \cdot [\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}] = (n-1) - (n+3) = -4. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n-1} \cdot \sqrt[3]{n+3} + \sqrt[3]{(n+3)^2}}{-4} = \frac{\infty + \infty \cdot \infty + \infty}{-4} = \frac{\infty}{-4} = -\infty. \end{aligned}$$

Riešené príklady – 224

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^a + p} - \sqrt[3]{n^a + q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[224]

Riešené príklady – 224

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}}$$

pre $p, q \in \mathbb{R}, p \neq q, a > 0$.

[224]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}} \cdot \frac{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}}{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}] \cdot [\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}] = (n^3+p) - (n^3+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}}{p - q} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^3 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{k+p} - \sqrt[3]{k+q}] \cdot [\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}] = (k+p) - (k+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} \end{aligned}$$

$$= \left[\text{Subst. } k = n^3 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}}$$

Riešené príklady – 224

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q, a > 0.$$

[224]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}} \cdot \frac{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}}{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}] \cdot [\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}] = (n^3+p) - (n^3+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+p)^2} + \sqrt[3]{n^3+p} \cdot \sqrt[3]{n^3+q} + \sqrt[3]{(n^3+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^3 \begin{cases} n \rightarrow \infty \\ k \rightarrow \infty \end{cases} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{k+p} - \sqrt[3]{k+q}] \cdot [\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}] = (k+p) - (k+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^3 \begin{cases} n \rightarrow \infty \\ k \rightarrow \infty \end{cases} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \left[\text{Pr. 220: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+p} - \sqrt[3]{n^3+q}} = \operatorname{sgn}(p - q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q. \right] = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

Riešené príklady – 225

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}}$$

pre $p, q \in R, p \neq q$.

[225]

Riešené príklady – 225

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}}$$

pre $p, q \in R, p \neq q$.

[225]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}} \cdot \frac{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}}{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \\ &\Rightarrow [\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}] \cdot [\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}] = (n^5+p) - (n^5+q) = p - q. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}}{p - q} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \\ &\Rightarrow [\sqrt[3]{k+p} - \sqrt[3]{k+q}] \cdot [\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}] = (k+p) - (k+q) = p - q. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} \end{aligned}$$

$$= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}}$$

Riešené príklady – 225

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in R, p \neq q.$$

[225]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}} \cdot \frac{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}}{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \\ &\Rightarrow [\sqrt[3]{n^5+p} - \sqrt[3]{n^5+q}] \cdot [\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}] = (n^5+p) - (n^5+q) = p - q. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^5+p)^2} + \sqrt[3]{n^5+p} \cdot \sqrt[3]{n^5+q} + \sqrt[3]{(n^5+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{cases} n \rightarrow \infty \\ k \rightarrow \infty \end{cases} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \\ &\Rightarrow [\sqrt[3]{k+p} - \sqrt[3]{k+q}] \cdot [\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}] = (k+p) - (k+q) = p - q. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{cases} n \rightarrow \infty \\ k \rightarrow \infty \end{cases} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \left[\text{Pr. 220: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} = \operatorname{sgn}(p - q)\infty \text{ pre } p, q \in R, p \neq q. \right] = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

Riešené příklady – 226

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}}$$

[226]

Riešené príklady – 226

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}}$$

[226]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}} \cdot \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}] \cdot [\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}] = (n^5+2) - (n^5+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{1} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{k+2} - \sqrt[3]{k+1}] \cdot [\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{1} \end{aligned}$$

$$= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}}$$

Riešené príklady – 226

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}} = \infty$$

[226]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}} \cdot \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^5+2} - \sqrt[3]{n^5+1}] \cdot [\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}] = (n^5+2) - (n^5+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^5+2)^2} + \sqrt[3]{n^5+2} \cdot \sqrt[3]{n^5+1} + \sqrt[3]{(n^5+1)^2}}{1} = \frac{\infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{k+2} - \sqrt[3]{k+1}] \cdot [\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{1} = \frac{\infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = n^5 \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} = \left[\text{Pr. 220: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} = \text{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q. \right] \\ &= \text{sgn}(2-1)\infty = \text{sgn}1 \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 227

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{5n+p} - \sqrt[3]{5n+q}}$$

pre $p, q \in \mathbb{R}, p \neq q, \alpha > 0$.

[227]

Riešené príklady – 227

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}}}$$

pre $p, q \in \mathbb{R}, p \neq q, \alpha > 0$.

[227]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}}} \cdot \frac{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}}{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}}] \cdot [\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}] = (\sqrt[5]{n+p}) - (\sqrt[5]{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}}{p - q} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[3]{k+p} - \sqrt[3]{k+q}] \cdot [\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}] = (k+p) - (k+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} \end{aligned}$$

$$= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}}$$

Riešené príklady – 227

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q, \alpha > 0.$$

[227]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}}} \cdot \frac{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}}{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n+p}} - \sqrt[3]{\sqrt[5]{n+q}} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2} \right] = (\sqrt[5]{n+p}) - (\sqrt[5]{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(\sqrt[5]{n+p})^2} + \sqrt[3]{\sqrt[5]{n+p}} \cdot \sqrt[3]{\sqrt[5]{n+q}} + \sqrt[3]{(\sqrt[5]{n+q})^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} \cdot \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[3]{k+p} - \sqrt[3]{k+q} \right] \cdot \left[\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2} \right] = (k+p) - (k+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+p)^2} + \sqrt[3]{k+p} \cdot \sqrt[3]{k+q} + \sqrt[3]{(k+q)^2}}{p - q} = \frac{\infty + \infty \cdot \infty + \infty}{p - q} = \frac{\infty}{p - q} = \operatorname{sgn}(p - q)\infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+p} - \sqrt[3]{k+q}} = \left[\text{Pr. 220: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} = \operatorname{sgn}(p - q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q. \right] \\ &= \operatorname{sgn}(p - q)\infty. \end{aligned}$$

Riešené príklady – 228

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}}}$$

[228]

Riešené príklady – 228

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}}}$$

[228]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}}{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2} \right] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}}{1} \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \left[\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2} \right] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{1} \end{aligned}$$

$$= \left[\text{Subst. } k = \sqrt[5]{n} \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}}$$

Riešené príklady – 228

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}}} = \infty$$

[228]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}}{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{\sqrt[5]{n+2}} - \sqrt[3]{\sqrt[5]{n+1}} \right] \cdot \left[\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2} \right] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(\sqrt[5]{n+2})^2} + \sqrt[3]{\sqrt[5]{n+2}} \cdot \sqrt[3]{\sqrt[5]{n+1}} + \sqrt[3]{(\sqrt[5]{n+1})^2}}{1} = \frac{\infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} \cdot \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[3]{k+2} - \sqrt[3]{k+1} \right] \cdot \left[\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2} \right] = (k+2) - (k+1) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(k+2)^2} + \sqrt[3]{k+2} \cdot \sqrt[3]{k+1} + \sqrt[3]{(k+1)^2}}{1} = \frac{\infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \left[\text{Subst. } k = \sqrt[5]{n} \left| \begin{array}{l} n \rightarrow \infty \\ k \rightarrow \infty \end{array} \right. \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{k+2} - \sqrt[3]{k+1}} = \left[\text{Pr. 220: } \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n+p} - \sqrt[3]{n+q}} = \text{sgn}(p-q)\infty \text{ pre } p, q \in R, p \neq q. \right] \\ &= \text{sgn}(2-1)\infty = \text{sgn}1 \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}}$$

[229]

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}}$$

[229]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}] \cdot [\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}] = (n^2+3n+1) - (n^2+n+2) = 2n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{2n-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]}{n(2 - \frac{1}{n})} = \left[n \in N. \Rightarrow \frac{\sqrt[3]{n^2 \cdot n^2}}{n} = \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^3}} = \frac{\sqrt[3]{n}}{1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2})^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2})^2} \right]}{2 - \frac{1}{n}} \end{aligned}$$

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} = \infty$$

[229]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2} \right] \cdot \left[\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2} \right] = (n^2+3n+1) - (n^2+n+2) = 2n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{2n-1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{\left(1 + \frac{1}{n} + \frac{2}{n^2}\right)^2} \right]}{n\left(2 - \frac{1}{n}\right)} = \left[n \in N. \Rightarrow \frac{\sqrt[3]{n^2 \cdot n^2}}{n} = \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^3}} = \frac{\sqrt[3]{n}}{1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{1}{n} + \frac{2}{n^2}} + \sqrt[3]{\left(1 + \frac{1}{n} + \frac{2}{n^2}\right)^2} \right]}{2 - \frac{1}{n}} \\ &= \frac{\sqrt[3]{\infty} \left[\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{1}{\infty} + \frac{2}{\infty}} + \sqrt[3]{\left(1 + \frac{1}{\infty} + \frac{2}{\infty}\right)^2} \right]}{2 - \frac{1}{\infty}} \\ &= \frac{\infty \left[\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \sqrt[3]{1+0+0} + \sqrt[3]{(1+0+0)^2} \right]}{2-0} = \frac{\infty \cdot (1+1 \cdot 1+1)}{2} = \frac{\infty \cdot 3}{2} = \infty. \end{aligned}$$

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}}$$

[\[Iné riešenie 229\]](#)

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}}$$

[Iné riešenie 229]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}] \cdot [\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}] = (n^2+3n+1) - (n^2+n+2) = 2n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{\left(1 + \frac{1}{n} + \frac{2}{n^2}\right) \cdot \left(n + 1 + \frac{2}{n}\right)} \right]}{n \left(2 - \frac{1}{n}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 1 + \frac{2}{n}} + \sqrt[3]{\left(1 + \frac{1}{n} + \frac{2}{n^2}\right) \cdot \left(n + 1 + \frac{2}{n}\right)}}{2 - \frac{1}{n}} \end{aligned}$$

Riešené príklady – 229

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} = \infty$$

[Iné riešenie 229]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+n+2}] \cdot [\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}] = (n^2+3n+1) - (n^2+n+2) = 2n-1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+n+2} + \sqrt[3]{(n^2+n+2)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n+3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n+1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n+1 + \frac{2}{n})} \right]}{n(2 - \frac{1}{n})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(1 + \frac{3}{n} + \frac{1}{n^2}) \cdot (n+3 + \frac{1}{n})} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n+1 + \frac{2}{n}} + \sqrt[3]{(1 + \frac{1}{n} + \frac{2}{n^2}) \cdot (n+1 + \frac{2}{n})}}{2 - \frac{1}{n}} \\ &= \frac{\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty}) \cdot (\infty+3 + \frac{1}{\infty})} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty+1 + \frac{2}{\infty}} + \sqrt[3]{(1 + \frac{1}{\infty} + \frac{2}{\infty}) \cdot (\infty+1 + \frac{2}{\infty})}}{2 - \frac{1}{\infty}} \\ &= \frac{\sqrt[3]{(1+0+0) \cdot (\infty+3+0)} + \sqrt[3]{1+0+0} \sqrt[3]{\infty+1+0} + \sqrt[3]{(1+0+0) \cdot (\infty+1+0)}}{2-0} \\ &= \frac{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}}{2} = \frac{\infty + \infty + \infty}{2} = \frac{\infty}{2} = \infty. \end{aligned}$$

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}}$$

[230]

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}}$$

[230]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1} \right] \cdot \left[\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2} \right] = (n^2+3n+1) - (n^2+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]}{n} = \left[n \in \mathbb{N} \Rightarrow \frac{\sqrt[3]{n^2 \cdot n^2}}{n} = \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^3}} = \frac{\sqrt[3]{n}}{1} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]}{1} \end{aligned}$$

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} = \infty$$

[230]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1} \right] \cdot \left[\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2} \right] = (n^2+3n+1) - (n^2+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n^2} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]}{n} = \left[n \in N. \Rightarrow \frac{\sqrt[3]{n^2 \cdot n^2}}{n} = \frac{\sqrt[3]{n^4}}{\sqrt[3]{n^3}} = \frac{\sqrt[3]{n}}{1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right)^2} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)^2} \right]}{1} \\ &= \frac{\sqrt[3]{\infty} \left[\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right)^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{2}{\infty} + \frac{1}{\infty}} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right)^2} \right]}{1} \\ &= \frac{\infty \left[\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \sqrt[3]{1+0+0} + \sqrt[3]{(1+0+0)^2} \right]}{1} = \frac{\infty \cdot (1+1 \cdot 1+1)}{1} = \frac{\infty \cdot 3}{1} = \infty. \end{aligned}$$

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}}$$

[\[Iné riešenie 230\]](#)

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}}$$

[Iné riešenie 230]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1} \right] \cdot \left[\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2} \right] = (n^2+3n+1) - (n^2+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} \cdot n \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)} \right]}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)}}{1} \end{aligned}$$

Riešené príklady – 230

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} = \infty$$

[Iné riešenie 230]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1}} \cdot \frac{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt[3]{n^2+3n+1} - \sqrt[3]{n^2+2n+1} \right] \cdot \left[\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2} \right] = (n^2+3n+1) - (n^2+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n}{\sqrt[3]{(n^2+3n+1)^2} + \sqrt[3]{n^2+3n+1} \cdot \sqrt[3]{n^2+2n+1} + \sqrt[3]{(n^2+2n+1)^2}} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3} = \sqrt[3]{n^2 \cdot n}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 \cdot n} \left[\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)} \right]}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{3}{n} + \frac{1}{n^2}\right) \cdot \left(n + 3 + \frac{1}{n}\right)} + \sqrt[3]{1 + \frac{3}{n} + \frac{1}{n^2}} \cdot \sqrt[3]{n + 2 + \frac{1}{n}} + \sqrt[3]{\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \cdot \left(n + 2 + \frac{1}{n}\right)}}{1} \\ &= \frac{\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right) \cdot \left(\infty + 3 + \frac{1}{\infty}\right)} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty + 2 + \frac{1}{\infty}} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right) \cdot \left(\infty + 2 + \frac{1}{\infty}\right)}}{1} \\ &= \frac{\sqrt[3]{(1+0+0) \cdot (\infty+3+0)} + \sqrt[3]{1+0+0} \sqrt[3]{\infty+2+0} + \sqrt[3]{(1+0+0) \cdot (\infty+2+0)}}{1} \\ &= \frac{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}}{1} = \frac{\infty + \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}}$$

[231]

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}}$$

[231]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} \cdot \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}] \cdot [\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}] = (n^3+3n+1) - (n^3+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{n} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]}{n} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right)^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right)^2} \right]}{1} \end{aligned}$$

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} = 0$$

[231]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} \cdot \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}] \cdot [\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}] = (n^3+3n+1) - (n^3+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{n} \\ &= \lim_{n \rightarrow \infty} \frac{n^2 \left[\sqrt[3]{(1 + \frac{3}{n^2} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{(1 + \frac{2}{n^2} + \frac{1}{n^3})^2} \right]}{n} = \left[n \in \mathbb{N}. \Rightarrow n^2 = \sqrt[3]{n^6} = \sqrt[3]{n^3 \cdot n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[3]{(1 + \frac{3}{n^2} + \frac{1}{n^3})^2} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{1 + \frac{2}{n^2} + \frac{1}{n^3}} + \sqrt[3]{(1 + \frac{2}{n^2} + \frac{1}{n^3})^2} \right]}{1} \\ &= \frac{\infty \left[\sqrt[3]{(1 + \frac{3}{\infty} + \frac{1}{\infty})^2} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{1 + \frac{2}{\infty} + \frac{1}{\infty}} + \sqrt[3]{(1 + \frac{2}{\infty} + \frac{1}{\infty})^2} \right]}{1} \\ &= \frac{\infty \left[\sqrt[3]{(1+0+0)^2} + \sqrt[3]{1+0+0} \sqrt[3]{1+0+0} + \sqrt[3]{(1+0+0)^2} \right]}{1} = \frac{\infty \cdot (1+1 \cdot 1+1)}{1} = \frac{\infty \cdot 3}{1} = \infty. \end{aligned}$$

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}}$$

[\[Iné riešenie 231\]](#)

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}}$$

[Iné riešenie 231]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} \cdot \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}] \cdot [\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}] = (n^3+3n+1) - (n^3+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+3n+1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+2n+1)} \right]}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+3n+1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+2n+1)}}{1} \end{aligned}$$

Riešené príklady – 231

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} = 0$$

[Iné riešenie 231]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}} \cdot \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}} \\ &= \left[(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \alpha^3 - \beta^3 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[3]{n^3+3n+1} - \sqrt[3]{n^3+2n+1}] \cdot [\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}] = (n^3+3n+1) - (n^3+2n+1) = n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{(n^3+3n+1)^2} + \sqrt[3]{n^3+3n+1} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{(n^3+2n+1)^2}}{n} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[3]{n^3}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+3n+1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+2n+1)} \right]}{n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{\left(1 + \frac{3}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+3n+1)} + \sqrt[3]{1 + \frac{3}{n^2} + \frac{1}{n^3}} \cdot \sqrt[3]{n^3+2n+1} + \sqrt[3]{\left(1 + \frac{2}{n^2} + \frac{1}{n^3}\right) \cdot (n^3+2n+1)}}{1} \\ &= \frac{\sqrt[3]{\left(1 + \frac{3}{\infty} + \frac{1}{\infty}\right) \cdot (\infty+3\infty+1)} + \sqrt[3]{1 + \frac{3}{\infty} + \frac{1}{\infty}} \cdot \sqrt[3]{\infty+2\infty+1} + \sqrt[3]{\left(1 + \frac{2}{\infty} + \frac{1}{\infty}\right) \cdot (\infty+2\infty+1)}}{1} \\ &= \frac{\sqrt[3]{(1+0+0) \cdot (\infty+\infty+1)} + \sqrt[3]{1+0+0} \sqrt[3]{\infty+\infty+1} + \sqrt[3]{(1+0+0) \cdot (\infty+\infty+1)}}{1} \\ &= \frac{\sqrt[3]{1 \cdot \infty} + \sqrt[3]{1} \cdot \sqrt[3]{\infty} + \sqrt[3]{1 \cdot \infty}}{1} = \frac{\infty + \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

Riešené príklady – 232, 233, 234

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1} - \sqrt[3]{n-1}}$$

[232]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n+1}}$$

[233]

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1} - \sqrt[5]{n^4+1}}$$

[234]

Riešené príklady – 232, 233, 234

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+n+1} - \sqrt[3]{n-1}}$$

[232]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot \left[\sqrt{n^2+1+\frac{1}{n}} - \sqrt{1-\frac{1}{n}} \right]}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n+1}}$$

[233]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n} \cdot \left[\sqrt[4]{n^3+\frac{1}{n}} - \sqrt[4]{1+\frac{1}{n}} \right]}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1} - \sqrt[5]{n^4+1}}$$

[234]

$$\bullet = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot \left[\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}} \right]}$$

Riešené príklady – 232, 233, 234

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^3+1} - \sqrt[3]{n-1}} = 0$$

[232]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n} \cdot [\sqrt{n^2+1+\frac{1}{n}} - \sqrt{1-\frac{1}{n}}]} = \frac{1}{\infty \cdot [\sqrt{\infty+1+\frac{1}{\infty}} - \sqrt{1-\frac{1}{\infty}}]} \\ &= \frac{1}{\infty \cdot [\sqrt{\infty+1+0} - \sqrt{1-0}]} = \frac{1}{\infty \cdot (\infty-1)} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n+1}} = 0$$

[233]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n} \cdot [\sqrt[4]{n^3+\frac{1}{n}} - \sqrt[4]{1+\frac{1}{n}}]} = \frac{1}{\infty \cdot [\sqrt[4]{\infty+\frac{1}{\infty}} - \sqrt[4]{1+\frac{1}{\infty}}]} \\ &= \frac{1}{\infty \cdot [\sqrt[4]{\infty+0} - \sqrt[4]{1+0}]} = \frac{1}{\infty \cdot (\infty-1)} = \frac{1}{\infty} = 0. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^5+1} - \sqrt[5]{n^4+1}} = 0$$

[234]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[5]{n^4} \cdot [\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}]} = \frac{1}{\infty \cdot [\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}]} \\ &= \frac{1}{\infty \cdot [\sqrt[5]{\infty+0} - \sqrt[5]{1+0}]} = \frac{1}{\infty \cdot (\infty-1)} = \frac{1}{\infty} = 0. \end{aligned}$$

Riešené příklady – 235

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}}$$

[235]

Riešené príklady – 235

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}}$$

[235]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}} \cdot \frac{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}}{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}] \cdot [\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}] = (n^4+1) - (n^4-1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}}{2} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}} \cdot \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}] \cdot [\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}] = \sqrt[4]{(n^4+1)^2} - \sqrt[4]{(n^4-1)^2} = \sqrt{n^4+1} - \sqrt{n^4-1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt{n^4+1} - \sqrt{n^4-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt{n^4+1} - \sqrt{n^4-1}} \cdot \frac{\sqrt{n^4+1} + \sqrt{n^4-1}}{\sqrt{n^4+1} + \sqrt{n^4-1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^4+1} - \sqrt{n^4-1}] \cdot [\sqrt{n^4+1} + \sqrt{n^4-1}] = (n^4+1) - (n^4-1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{2} \cdot \frac{\sqrt{n^4+1} + \sqrt{n^4-1}}{1} \end{aligned}$$

Riešené príklady – 235

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}} = \infty$$

[235]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}} \cdot \frac{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}}{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}] \cdot [\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}] = (n^4+1) - (n^4-1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4+1)^3} + \sqrt[4]{(n^4+1)^2} \sqrt[4]{n^4-1} + \sqrt[4]{n^4+1} \sqrt[4]{(n^4-1)^2} + \sqrt[4]{(n^4-1)^3}}{2} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{2} = \frac{\infty}{2} = \infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}} \cdot \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^4+1} - \sqrt[4]{n^4-1}] \cdot [\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}] = \sqrt[4]{(n^4+1)^2} - \sqrt[4]{(n^4-1)^2} = \sqrt{n^4+1} - \sqrt{n^4-1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt{n^4+1} - \sqrt{n^4-1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{\sqrt{n^4+1} - \sqrt{n^4-1}} \cdot \frac{\sqrt{n^4+1} + \sqrt{n^4-1}}{\sqrt{n^4+1} + \sqrt{n^4-1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^4+1} - \sqrt{n^4-1}] \cdot [\sqrt{n^4+1} + \sqrt{n^4-1}] = (n^4+1) - (n^4-1) = 2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+1} + \sqrt[4]{n^4-1}}{2} \cdot \frac{\sqrt{n^4+1} + \sqrt{n^4-1}}{1} = \frac{\infty + \infty}{2} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{2} \cdot \frac{\infty}{1} = \infty \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 236

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}}$$

pre $p, q \in R, p \neq q, a > 0$.

[236]

Riešené príklady – 236

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}}$$

pre $p, q \in R, p \neq q, a > 0$.

[236]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} \cdot \frac{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}}{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}] \cdot [\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}] = (n^3+p) - (n^3+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}}{p - q} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} \cdot \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}] \cdot [\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}] = \sqrt[4]{(n^3+p)^2} - \sqrt[4]{(n^3+q)^2} = \sqrt{n^3+p} - \sqrt{n^3+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt{n^3+p} - \sqrt{n^3+q}} \cdot \frac{\sqrt{n^3+p} + \sqrt{n^3+q}}{\sqrt{n^3+p} + \sqrt{n^3+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^3+p} - \sqrt{n^3+q}] \cdot [\sqrt{n^3+p} + \sqrt{n^3+q}] = (n^3+p) - (n^3+q) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{p - q} \cdot \frac{\sqrt{n^3+p} + \sqrt{n^3+q}}{1} \end{aligned}$$

Riešené príklady – 236

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q, a > 0.$$

[236]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} \cdot \frac{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}}{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}] \cdot [\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}] = (n^3+p) - (n^3+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^3+p)^3} + \sqrt[4]{(n^3+p)^2} \sqrt[4]{n^3+q} + \sqrt[4]{n^3+p} \sqrt[4]{(n^3+q)^2} + \sqrt[4]{(n^3+q)^3}}{p-q} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} \cdot \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}] \cdot [\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}] = \sqrt[4]{(n^3+p)^2} - \sqrt[4]{(n^3+q)^2} = \sqrt{n^3+p} - \sqrt{n^3+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{\sqrt[4]{n^3+p} - \sqrt[4]{n^3+q}} \cdot \frac{\sqrt{n^3+p} + \sqrt{n^3+q}}{\sqrt{n^3+p} + \sqrt{n^3+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^3+p} - \sqrt{n^3+q}] \cdot [\sqrt{n^3+p} + \sqrt{n^3+q}] = (n^3+p) - (n^3+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^3+p} + \sqrt[4]{n^3+q}}{p-q} \cdot \frac{\sqrt{n^3+p} + \sqrt{n^3+q}}{1} = \frac{\infty + \infty}{p-q} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{p-q} \cdot \frac{\infty}{1} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

Riešené príklady – 237

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}}$$

pre $p, q \in R, p \neq q$.

[237]

Riešené príklady – 237

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}}$$

pre $p, q \in R, p \neq q$.

[237]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} \cdot \frac{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}}{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}] \cdot [\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}] = (n^5+p) - (n^5+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}}{p-q} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} \cdot \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}] \cdot [\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}] = \sqrt[4]{(n^5+p)^2} - \sqrt[4]{(n^5+q)^2} = \sqrt{n^5+p} - \sqrt{n^5+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt{n^5+p} - \sqrt{n^5+q}} \cdot \frac{\sqrt{n^5+p} + \sqrt{n^5+q}}{\sqrt{n^5+p} + \sqrt{n^5+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow [\sqrt{n^5+p} - \sqrt{n^5+q}] \cdot [\sqrt{n^5+p} + \sqrt{n^5+q}] = (n^5+p) - (n^5+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{p-q} \cdot \frac{\sqrt{n^5+p} + \sqrt{n^5+q}}{1} \end{aligned}$$

Riešené príklady – 237

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q.$$

[237]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} \cdot \frac{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}}{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}] \cdot [\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}] = (n^5+p) - (n^5+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^5+p)^3} + \sqrt[4]{(n^5+p)^2} \sqrt[4]{n^5+q} + \sqrt[4]{n^5+p} \sqrt[4]{(n^5+q)^2} + \sqrt[4]{(n^5+q)^3}}{p-q} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} \cdot \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}] \cdot [\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}] = \sqrt[4]{(n^5+p)^2} - \sqrt[4]{(n^5+q)^2} = \sqrt{n^5+p} - \sqrt{n^5+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt[4]{n^5+p} - \sqrt[4]{n^5+q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{\sqrt{n^5+p} - \sqrt{n^5+q}} \cdot \frac{\sqrt{n^5+p} + \sqrt{n^5+q}}{\sqrt{n^5+p} + \sqrt{n^5+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^5+p} - \sqrt{n^5+q}] \cdot [\sqrt{n^5+p} + \sqrt{n^5+q}] = (n^5+p) - (n^5+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+p} + \sqrt[4]{n^5+q}}{p-q} \cdot \frac{\sqrt{n^5+p} + \sqrt{n^5+q}}{1} = \frac{\infty + \infty}{p-q} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{p-q} \cdot \frac{\infty}{1} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

Riešené příklady – 238

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}}$$

[238]

Riešené príklady – 238

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}}$$

[238]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}} \cdot \frac{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}}{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}] \cdot [\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}] = (n^5+2) - (n^5+1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}}{1} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}} \cdot \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}] \cdot [\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}] = \sqrt[4]{(n^5+2)^2} - \sqrt[4]{(n^5+1)^2} = \sqrt{n^5+2} - \sqrt{n^5+1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt{n^5+2} - \sqrt{n^5+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt{n^5+2} - \sqrt{n^5+1}} \cdot \frac{\sqrt{n^5+2} + \sqrt{n^5+1}}{\sqrt{n^5+2} + \sqrt{n^5+1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^5+2} - \sqrt{n^5+1}] \cdot [\sqrt{n^5+2} + \sqrt{n^5+1}] = (n^5+2) - (n^5+1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{1} \cdot \frac{\sqrt{n^5+2} + \sqrt{n^5+1}}{1} \end{aligned}$$

Riešené príklady – 238

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}} = \infty$$

[238]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}} \cdot \frac{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}}{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}] \cdot [\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}] = (n^5+2) - (n^5+1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^5+2)^3} + \sqrt[4]{(n^5+2)^2} \sqrt[4]{n^5+1} + \sqrt[4]{n^5+2} \sqrt[4]{(n^5+1)^2} + \sqrt[4]{(n^5+1)^3}}{1} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}} \cdot \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^5+2} - \sqrt[4]{n^5+1}] \cdot [\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}] = \sqrt[4]{(n^5+2)^2} - \sqrt[4]{(n^5+1)^2} = \sqrt{n^5+2} - \sqrt{n^5+1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt{n^5+2} - \sqrt{n^5+1}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{\sqrt{n^5+2} - \sqrt{n^5+1}} \cdot \frac{\sqrt{n^5+2} + \sqrt{n^5+1}}{\sqrt{n^5+2} + \sqrt{n^5+1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{n^5+2} - \sqrt{n^5+1}] \cdot [\sqrt{n^5+2} + \sqrt{n^5+1}] = (n^5+2) - (n^5+1) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^5+2} + \sqrt[4]{n^5+1}}{1} \cdot \frac{\sqrt{n^5+2} + \sqrt{n^5+1}}{1} = \frac{\infty + \infty}{1} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{1} \cdot \frac{\infty}{1} = \infty \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 239

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}}$$

pre $p, q \in \mathbb{R}, p \neq q$.

[239]

Riešené príklady – 239

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}}$$

pre $p, q \in \mathbb{R}, p \neq q$.

[239]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}} \cdot \frac{\sqrt[4]{(5\sqrt{n+p})^3} + \sqrt[4]{(5\sqrt{n+p})^2} \sqrt[4]{5\sqrt{n+q}} + \sqrt[4]{5\sqrt{n+p}} \sqrt[4]{(5\sqrt{n+q})^2} + \sqrt[4]{(5\sqrt{n+q})^3}}{\sqrt[4]{(5\sqrt{n+p})^3} + \sqrt[4]{(5\sqrt{n+p})^2} \sqrt[4]{5\sqrt{n+q}} + \sqrt[4]{5\sqrt{n+p}} \sqrt[4]{(5\sqrt{n+q})^2} + \sqrt[4]{(5\sqrt{n+q})^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}] \cdot [\sqrt[4]{(5\sqrt{n+p})^3} + \sqrt[4]{(5\sqrt{n+p})^2} \sqrt[4]{5\sqrt{n+q}} + \sqrt[4]{5\sqrt{n+p}} \sqrt[4]{(5\sqrt{n+q})^2} + \sqrt[4]{(5\sqrt{n+q})^3}] = (5\sqrt{n+p}) - (5\sqrt{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(5\sqrt{n+p})^3} + \sqrt[4]{(5\sqrt{n+p})^2} \sqrt[4]{5\sqrt{n+q}} + \sqrt[4]{5\sqrt{n+p}} \sqrt[4]{(5\sqrt{n+q})^2} + \sqrt[4]{(5\sqrt{n+q})^3}}{p - q} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}} \cdot \frac{\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}}{\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}] \cdot [\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}] = \sqrt[4]{(5\sqrt{n+p})^2} - \sqrt[4]{(5\sqrt{n+q})^2} = \sqrt{5\sqrt{n+p}} - \sqrt{5\sqrt{n+q}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}}{\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}}{\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}} \cdot \frac{\sqrt{5\sqrt{n+p}} + \sqrt{5\sqrt{n+q}}}{\sqrt{5\sqrt{n+p}} + \sqrt{5\sqrt{n+q}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{5\sqrt{n+p}} - \sqrt[4]{5\sqrt{n+q}}] \cdot [\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}] = (5\sqrt{n+p}) - (5\sqrt{n+q}) = p - q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5\sqrt{n+p}} + \sqrt[4]{5\sqrt{n+q}}}{p - q} \cdot \frac{\sqrt{5\sqrt{n+p}} + \sqrt{5\sqrt{n+q}}}{1} \end{aligned}$$

Riešené príklady – 239

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}} = \operatorname{sgn}(p-q)\infty \text{ pre } p, q \in \mathbb{R}, p \neq q.$$

[239]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}} \cdot \frac{\sqrt[4]{(5n+p)^3} + \sqrt[4]{(5n+p)^2} \sqrt[4]{5n+q} + \sqrt[4]{5n+p} \sqrt[4]{(5n+q)^2} + \sqrt[4]{(5n+q)^3}}{\sqrt[4]{(5n+p)^3} + \sqrt[4]{(5n+p)^2} \sqrt[4]{5n+q} + \sqrt[4]{5n+p} \sqrt[4]{(5n+q)^2} + \sqrt[4]{(5n+q)^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{5n+p} - \sqrt[4]{5n+q}] \cdot [\sqrt[4]{(5n+p)^3} + \sqrt[4]{(5n+p)^2} \sqrt[4]{5n+q} + \sqrt[4]{5n+p} \sqrt[4]{(5n+q)^2} + \sqrt[4]{(5n+q)^3}] = (5n+p) - (5n+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(5n+p)^3} + \sqrt[4]{(5n+p)^2} \sqrt[4]{5n+q} + \sqrt[4]{5n+p} \sqrt[4]{(5n+q)^2} + \sqrt[4]{(5n+q)^3}}{p-q} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{p-q} = \frac{\infty}{p-q} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}} \cdot \frac{\sqrt[4]{5n+p} + \sqrt[4]{5n+q}}{\sqrt[4]{5n+p} + \sqrt[4]{5n+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{5n+p} - \sqrt[4]{5n+q}] \cdot [\sqrt[4]{5n+p} + \sqrt[4]{5n+q}] = \sqrt[4]{(5n+p)^2} - \sqrt[4]{(5n+q)^2} = \sqrt{5n+p} - \sqrt{5n+q}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5n+p} + \sqrt[4]{5n+q}}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5n+p} + \sqrt[4]{5n+q}}{\sqrt[4]{5n+p} - \sqrt[4]{5n+q}} \cdot \frac{\sqrt{5n+p} + \sqrt{5n+q}}{\sqrt{5n+p} + \sqrt{5n+q}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{5n+p} - \sqrt[4]{5n+q}] \cdot [\sqrt[4]{5n+p} + \sqrt[4]{5n+q}] = (5n+p) - (5n+q) = p-q. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{5n+p} + \sqrt[4]{5n+q}}{p-q} \cdot \frac{\sqrt{5n+p} + \sqrt{5n+q}}{1} = \frac{\infty + \infty}{p-q} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{p-q} \cdot \frac{\infty}{1} = \operatorname{sgn}(p-q)\infty. \end{aligned}$$

Riešené príklady – 240

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}}$$

[240]

Riešené příklady – 240

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}}$$

[240]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}}{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}] \cdot [\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}}{1} \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}] \cdot [\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}] = \sqrt[4]{(\sqrt[5]{n+2})^2} - \sqrt[4]{(\sqrt[5]{n+1})^2} = \sqrt{\sqrt[5]{n+2}} - \sqrt{\sqrt[5]{n+1}}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt{\sqrt[5]{n+2}} - \sqrt{\sqrt[5]{n+1}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt{\sqrt[5]{n+2}} - \sqrt{\sqrt[5]{n+1}}} \cdot \frac{\sqrt{\sqrt[5]{n+2}} + \sqrt{\sqrt[5]{n+1}}}{\sqrt{\sqrt[5]{n+2}} + \sqrt{\sqrt[5]{n+1}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt{\sqrt[5]{n+2}} - \sqrt{\sqrt[5]{n+1}}] \cdot [\sqrt{\sqrt[5]{n+2}} + \sqrt{\sqrt[5]{n+1}}] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{1} \cdot \frac{\sqrt{\sqrt[5]{n+2}} + \sqrt{\sqrt[5]{n+1}}}{1} \end{aligned}$$

Riešené příklady – 240

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} = \infty$$

[240]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}}{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}] \cdot [\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(\sqrt[5]{n+2})^3} + \sqrt[4]{(\sqrt[5]{n+2})^2} \sqrt[4]{\sqrt[5]{n+1}} + \sqrt[4]{\sqrt[5]{n+2}} \sqrt[4]{(\sqrt[5]{n+1})^2} + \sqrt[4]{(\sqrt[5]{n+1})^3}}{1} \\ &= \frac{\infty + \infty \cdot \infty + \infty \cdot \infty + \infty}{1} = \frac{\infty}{1} = \infty. \end{aligned}$$

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}] \cdot [\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}] = \sqrt[4]{(\sqrt[5]{n+2})^2} - \sqrt[4]{(\sqrt[5]{n+1})^2} = \sqrt[5]{n+2} - \sqrt[5]{n+1}. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}} \cdot \frac{\sqrt[5]{n+2} + \sqrt[5]{n+1}}{\sqrt[5]{n+2} + \sqrt[5]{n+1}} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{\sqrt[5]{n+2}} - \sqrt[4]{\sqrt[5]{n+1}}] \cdot [\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}] = (\sqrt[5]{n+2}) - (\sqrt[5]{n+1}) = 1. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\sqrt[5]{n+2}} + \sqrt[4]{\sqrt[5]{n+1}}}{1} \cdot \frac{\sqrt[5]{n+2} + \sqrt[5]{n+1}}{1} = \frac{\infty + \infty}{1} \cdot \frac{\infty + \infty}{1} = \frac{\infty}{1} \cdot \frac{\infty}{1} = \infty \cdot \infty = \infty. \end{aligned}$$

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n + 1} - n + 1}$$

[241]

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-n+1}$$

[241]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-(n-1)} \cdot \frac{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \Rightarrow \left[\sqrt[4]{n^4+n+1} - (n-1) \right] \cdot \left[\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3 \right] \\ &\quad \quad \quad = (n^4+n+1) - (n-1)^4 = (n^4+n+1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \left. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3}{4n^3 - 6n^2 + 5n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4 \cdot n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n^3} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n^3} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3}{4 - \frac{6}{n} + \frac{5}{n^2}} \end{aligned}$$

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-n+1} = 1$$

[241]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-(n-1)} \cdot \frac{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3} \\
 &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow [\sqrt[4]{n^4+n+1} - (n-1)] \cdot [\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3] \\ &= (n^4+n+1) - (n-1)^4 = (n^4+n+1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 6n^2 + 5n. \end{aligned} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4+n+1)^3} + \sqrt[4]{(n^4+n+1)^2} \cdot (n-1) + \sqrt[4]{n^4+n+1} \cdot (n-1)^2 + (n-1)^3}{4n^3 - 6n^2 + 5n} \\
 &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^4 \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(1+\frac{1}{n^3}+\frac{1}{n^4})^3} + \sqrt[4]{(1+\frac{1}{n^3}+\frac{1}{n^4})^2} \cdot (1-\frac{1}{n}) + \sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}} \cdot (1-\frac{1}{n})^2 + (1-\frac{1}{n})^3}{4 - \frac{6}{n} + \frac{5}{n^2}} \\
 &= \frac{\sqrt[4]{(1+\frac{1}{\infty}+\frac{1}{\infty})^3} + \sqrt[4]{(1+\frac{1}{\infty}+\frac{1}{\infty})^2} \cdot (1-\frac{1}{\infty}) + \sqrt[4]{1+\frac{1}{\infty}+\frac{1}{\infty}} \cdot (1-\frac{1}{\infty})^2 + (1-\frac{1}{\infty})^3}{4 - \frac{6}{\infty} + \frac{5}{\infty}} \\
 &= \frac{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} \cdot (1-0) + \sqrt[4]{1+0+0} \cdot (1-0)^2 + (1-0)^3}{4-0+0} = \frac{1+1 \cdot 1+1 \cdot 1+1}{4} = \frac{4}{4} = 1.
 \end{aligned}$$

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n + 1} - n + 1}$$

[\[Iné riešenie 241\]](#)

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-n+1}$$

[Iné riešenie 241]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-(n-1)} \cdot \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt[4]{n^4+n+1}+(n-1)} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt[4]{n^4+n+1}-(n-1) \right] \cdot \left[\sqrt[4]{n^4+n+1}+(n-1) \right] = \sqrt[4]{(n^4+n+1)^2} - (n-1)^2 = \sqrt{n^4+n+1} - (n-1)^2. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt{n^4+n+1}-(n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt{n^4+n+1}-(n-1)^2} \cdot \frac{\sqrt{n^4+n+1}+(n-1)^2}{\sqrt{n^4+n+1}+(n-1)^2} \\ &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^4+n+1}-(n-1)^2 \right] \cdot \left[\sqrt{n^4+n+1}+(n-1)^2 \right] = (n^4+n+1) - (n-1)^4 = (n^4+n+1) - (n^4-4n^3+6n^2-4n+1) = 4n^3-6n^2+5n. \end{array} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{4n^3-6n^2+5n} \cdot \frac{\sqrt{n^4+n+1}+(n-1)^2}{1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}} + \left(1-\frac{1}{n}\right) \right]}{n^3 \left(4-\frac{6}{n}+\frac{5}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt{\left(1+\frac{1}{n^3}+\frac{1}{n^4}\right)^2} + \left(1-\frac{1}{n}\right)^2 \right]}{1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}} + \left(1-\frac{1}{n}\right)}{4-\frac{6}{n}+\frac{5}{n^2}} \cdot \frac{\sqrt{\left(1+\frac{1}{n^3}+\frac{1}{n^4}\right)^2} + \left(1-\frac{1}{n}\right)^2}{1} \end{aligned}$$

Riešené príklady – 241

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-n+1} = 1$$

[Iné riešenie 241]

$$\begin{aligned}
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4+n+1}-(n-1)} \cdot \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt[4]{n^4+n+1}+(n-1)} \\
 &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt[4]{n^4+n+1} - (n-1) \right] \cdot \left[\sqrt[4]{n^4+n+1} + (n-1) \right] = \sqrt{(n^4+n+1)^2} - (n-1)^2 = \sqrt{n^4+n+1} - (n-1)^2. \end{array} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt{n^4+n+1}-(n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{\sqrt{n^4+n+1}-(n-1)^2} \cdot \frac{\sqrt{n^4+n+1}+(n-1)^2}{\sqrt{n^4+n+1}+(n-1)^2} \\
 &= \left[\begin{array}{l} (\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ \Rightarrow \left[\sqrt{n^4+n+1} - (n-1)^2 \right] \cdot \left[\sqrt{n^4+n+1} + (n-1)^2 \right] = (n^4+n+1) - (n-1)^4 = (n^4+n+1) - (n^4-4n^3+6n^2-4n+1) = 4n^3-6n^2+5n. \end{array} \right] \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4+n+1}+(n-1)}{4n^3-6n^2+5n} \cdot \frac{\sqrt{n^4+n+1}+(n-1)^2}{1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\
 &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}} + \left(1-\frac{1}{n}\right) \right]}{n^3 \left(4-\frac{6}{n}+\frac{5}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt[4]{\left(1+\frac{1}{n^3}+\frac{1}{n^4}\right)^2} + \left(1-\frac{1}{n}\right)^2 \right]}{1} \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1+\frac{1}{n^3}+\frac{1}{n^4}} + \left(1-\frac{1}{n}\right)}{4-\frac{6}{n}+\frac{5}{n^2}} \cdot \frac{\sqrt[4]{\left(1+\frac{1}{n^3}+\frac{1}{n^4}\right)^2} + \left(1-\frac{1}{n}\right)^2}{1} = \frac{\sqrt[4]{1+\frac{1}{\infty}+\frac{1}{\infty}} + \left(1-\frac{1}{\infty}\right)}{4-\frac{6}{\infty}+\frac{5}{\infty}} \cdot \frac{\sqrt[4]{\left(1+\frac{1}{\infty}+\frac{1}{\infty}\right)^2} + \left(1-\frac{1}{\infty}\right)^2}{1} \\
 &= \frac{\sqrt[4]{1+0+0} + (1-0)}{4-0+0} \cdot \frac{\sqrt[4]{(1+0+0)^2} + (1-0)^2}{1} = \frac{1+1}{4} \cdot \frac{1+1}{1} = \frac{2 \cdot 2}{4} = 1.
 \end{aligned}$$

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1}$$

[242]

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1}$$

[242]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - (n-1)} \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow [\sqrt[4]{n^4 + n^2 + 1} - (n-1)] \cdot [\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3] \\ &= (n^4 + n^2 + 1) - (n-1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3}{4n^3 - 5n^2 + 4n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3 \right]}{n^3 \left(4 - \frac{5}{n} + \frac{4}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3}{4 - \frac{5}{n} + \frac{4}{n^2}} \end{aligned}$$

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1} = 1$$

[242]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - (n-1)} \cdot \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n-1) \right] \cdot \left[\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3 \right] \\ &\quad \quad \quad = (n^4 + n^2 + 1) - (n-1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \left. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4 + n^2 + 1)^3} + \sqrt[4]{(n^4 + n^2 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^2 + 1} \cdot (n-1)^2 + (n-1)^3}{4n^3 - 5n^2 + 4n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^3 = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3 \right]}{n^3 \left(4 - \frac{5}{n} + \frac{4}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3}{4 - \frac{5}{n} + \frac{4}{n^2}} \\ &= \frac{\sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^2} \cdot \left(1 - \frac{1}{\infty}\right) + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot \left(1 - \frac{1}{\infty}\right)^2 + \left(1 - \frac{1}{\infty}\right)^3}{4 - \frac{5}{\infty} + \frac{4}{\infty}} \\ &= \frac{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} \cdot (1-0) + \sqrt[4]{1+0+0} \cdot (1-0)^2 + (1-0)^3}{4-0+0} = \frac{1+1 \cdot 1 + 1 \cdot 1 + 1}{4} = \frac{4}{4} = 1. \end{aligned}$$

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1}$$

[\[Iné riešenie 242\]](#)

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1}$$

[Iné riešenie 242]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - (n-1)} \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt[4]{n^4 + n^2 + 1} + (n-1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^2 + 1} - (n-1) \right] \cdot \left[\sqrt[4]{n^4 + n^2 + 1} + (n-1) \right] = \sqrt[4]{(n^4 + n^2 + 1)^2} - (n-1)^2 = \sqrt{n^4 + n^2 + 1} - (n-1)^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt{n^4 + n^2 + 1} - (n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt{n^4 + n^2 + 1} - (n-1)^2} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n-1)^2}{\sqrt{n^4 + n^2 + 1} + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt{n^4 + n^2 + 1} - (n-1)^2 \right] \cdot \left[\sqrt{n^4 + n^2 + 1} + (n-1)^2 \right] = (n^4 + n^2 + 1) - (n-1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{4n^3 - 5n^2 + 4n} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n-1)^2}{1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right) \right]}{n^3 \left(4 - \frac{5}{n} + \frac{4}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2 + \left(1 - \frac{1}{n}\right)^2} \right]}{1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right)}{4 - \frac{5}{n} + \frac{4}{n^2}} \cdot \frac{\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2 + \left(1 - \frac{1}{n}\right)^2}}{1} \end{aligned}$$

Riešené príklady – 242

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - n + 1} = 1$$

[Iné riešenie 242]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^2 + 1} - (n-1)} \cdot \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt[4]{n^4 + n^2 + 1} + (n-1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow [\sqrt[4]{n^4 + n^2 + 1} - (n-1)] \cdot [\sqrt[4]{n^4 + n^2 + 1} + (n-1)] = \sqrt[4]{(n^4 + n^2 + 1)^2} - (n-1)^2 = \sqrt{n^4 + n^2 + 1} - (n-1)^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt{n^4 + n^2 + 1} - (n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{\sqrt{n^4 + n^2 + 1} - (n-1)^2} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n-1)^2}{\sqrt{n^4 + n^2 + 1} + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow [\sqrt{n^4 + n^2 + 1} - (n-1)^2] \cdot [\sqrt{n^4 + n^2 + 1} + (n-1)^2] = (n^4 + n^2 + 1) - (n-1)^4 = (n^4 + n^2 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 4n^3 - 5n^2 + 4n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^2 + 1} + (n-1)}{4n^3 - 5n^2 + 4n} \cdot \frac{\sqrt{n^4 + n^2 + 1} + (n-1)^2}{1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right) \right]}{n^3 \left(4 - \frac{5}{n} + \frac{4}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} + \left(1 - \frac{1}{n}\right)^2 \right]}{1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right)}{4 - \frac{5}{n} + \frac{4}{n^2}} \cdot \frac{\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} + \left(1 - \frac{1}{n}\right)^2}{1} = \frac{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + \left(1 - \frac{1}{\infty}\right)}{4 - \frac{5}{\infty} + \frac{4}{\infty}} \cdot \frac{1}{\sqrt{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^2} + \left(1 - \frac{1}{\infty}\right)^2} \cdot 1 \\ &= \frac{\sqrt[4]{1+0+0} + (1-0)}{4-0+0} \cdot \frac{\sqrt{(1+0+0)^2} + (1-0)^2}{1} = \frac{1+1}{4} \cdot \frac{1+1}{1} = \frac{2 \cdot 2}{4} = \frac{4}{4} = 1. \end{aligned}$$

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1}$$

[243]

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1}$$

[243]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - (n-1)} \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3} \\ &= \left[\begin{aligned} &(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \\ &\Rightarrow [\sqrt[4]{n^4 + n^3 + 1} - (n-1)] \cdot [\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3] \\ &= (n^4 + n^3 + 1) - (n-1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \end{aligned} \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3}{5n^3 - 6n^2 + 4n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left[\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3 \right]}{n^3 \left(5 - \frac{6}{n} + \frac{4}{n^2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^3} + \sqrt[4]{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} \cdot \left(1 - \frac{1}{n}\right) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot \left(1 - \frac{1}{n}\right)^2 + \left(1 - \frac{1}{n}\right)^3}{5 - \frac{6}{n} + \frac{4}{n^2}} \end{aligned}$$

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1} = \frac{4}{5}$$

[243]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - (n-1)} \cdot \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3}{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3} \\ &= \left[(\alpha - \beta)(\alpha^3 + \alpha^2\beta + \alpha\beta^2 + \beta^3) = \alpha^4 - \beta^4 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n-1) \right] \cdot \left[\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3 \right] \\ &\quad \quad \quad = (n^4 + n^3 + 1) - (n-1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \left. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(n^4 + n^3 + 1)^3} + \sqrt[4]{(n^4 + n^3 + 1)^2} \cdot (n-1) + \sqrt[4]{n^4 + n^3 + 1} \cdot (n-1)^2 + (n-1)^3}{5n^3 - 6n^2 + 4n} \\ &= \left[n \in \mathbb{N}. \Rightarrow n^3 = \sqrt[4]{n^{12}} = \sqrt[4]{n^4 \cdot n^4 \cdot n^4} = \sqrt[4]{n^4} \cdot n^3 = \sqrt[4]{n^4} \cdot n^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n^3 \left[\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} \cdot (1 - \frac{1}{n}) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot (1 - \frac{1}{n})^2 + (1 - \frac{1}{n})^3 \right]}{n^3(5 - \frac{6}{n} + \frac{4}{n^2})} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^3} + \sqrt[4]{(1 + \frac{1}{n} + \frac{1}{n^4})^2} \cdot (1 - \frac{1}{n}) + \sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} \cdot (1 - \frac{1}{n})^2 + (1 - \frac{1}{n})^3}{5 - \frac{6}{n} + \frac{4}{n^2}} \\ &= \frac{\sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^3} + \sqrt[4]{(1 + \frac{1}{\infty} + \frac{1}{\infty})^2} \cdot (1 - \frac{1}{\infty}) + \sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} \cdot (1 - \frac{1}{\infty})^2 + (1 - \frac{1}{\infty})^3}{5 - \frac{6}{\infty} + \frac{4}{\infty}} \\ &= \frac{\sqrt[4]{(1+0+0)^3} + \sqrt[4]{(1+0+0)^2} \cdot (1-0) + \sqrt[4]{1+0+0} \cdot (1-0)^2 + (1-0)^3}{5-0+0} = \frac{1+1 \cdot 1 + 1 \cdot 1 + 1}{5} = \frac{4}{5}. \end{aligned}$$

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1}$$

[\[Iné riešenie 243\]](#)

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1}$$

[Iné riešenie 243]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - (n-1)} \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt[4]{n^4 + n^3 + 1} + (n-1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n-1) \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + (n-1) \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - (n-1)^2 = \sqrt{n^4 + n^3 + 1} - (n-1)^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt{n^4 + n^3 + 1} - (n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt{n^4 + n^3 + 1} - (n-1)^2} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n-1)^2}{\sqrt{n^4 + n^3 + 1} + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in R. \right. \\ &\quad \left. \Rightarrow \left[\sqrt{n^4 + n^3 + 1} - (n-1)^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + (n-1)^2 \right] = (n^4 + n^3 + 1) - (n-1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{5n^3 - 6n^2 + 4n} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n-1)^2}{1} = \left[n \in N. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right) \right]}{n^3 \left(5 - \frac{6}{n} + \frac{4}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2 + \left(1 - \frac{1}{n}\right)^2} \right]}{1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right)}{5 - \frac{6}{n} + \frac{4}{n^2}} \cdot \frac{\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2 + \left(1 - \frac{1}{n}\right)^2}}{1} \end{aligned}$$

Riešené príklady – 243

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - n + 1} = \frac{4}{5}$$

[Iné riešenie 243]

$$\begin{aligned} &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{n^4 + n^3 + 1} - (n-1)} \cdot \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt[4]{n^4 + n^3 + 1} + (n-1)} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \Rightarrow \left[\sqrt[4]{n^4 + n^3 + 1} - (n-1) \right] \cdot \left[\sqrt[4]{n^4 + n^3 + 1} + (n-1) \right] = \sqrt[4]{(n^4 + n^3 + 1)^2} - (n-1)^2 = \sqrt{n^4 + n^3 + 1} - (n-1)^2. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt{n^4 + n^3 + 1} - (n-1)^2} = \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{\sqrt{n^4 + n^3 + 1} - (n-1)^2} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n-1)^2}{\sqrt{n^4 + n^3 + 1} + (n-1)^2} \\ &= \left[(\alpha - \beta)(\alpha + \beta) = \alpha^2 - \beta^2 \text{ pre } \alpha, \beta \in \mathbb{R}. \right. \\ &\quad \left. \Rightarrow \left[\sqrt{n^4 + n^3 + 1} - (n-1)^2 \right] \cdot \left[\sqrt{n^4 + n^3 + 1} + (n-1)^2 \right] = (n^4 + n^3 + 1) - (n-1)^4 = (n^4 + n^3 + 1) - (n^4 - 4n^3 + 6n^2 - 4n + 1) = 5n^3 - 6n^2 + 4n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{n^4 + n^3 + 1} + (n-1)}{5n^3 - 6n^2 + 4n} \cdot \frac{\sqrt{n^4 + n^3 + 1} + (n-1)^2}{1} = \left[n \in \mathbb{N}. \Rightarrow n = \sqrt[4]{n^4}, n^2 = \sqrt[4]{n^4 \cdot n^4} = n \cdot n. \right] \\ &= \lim_{n \rightarrow \infty} \frac{n \left[\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right) \right]}{n^3 \left(5 - \frac{6}{n} + \frac{4}{n^2}\right)} \cdot \frac{n^2 \left[\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} + \left(1 - \frac{1}{n}\right)^2 \right]}{1} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt[4]{1 + \frac{1}{n} + \frac{1}{n^4}} + \left(1 - \frac{1}{n}\right)}{5 - \frac{6}{n} + \frac{4}{n^2}} \cdot \frac{\sqrt{\left(1 + \frac{1}{n} + \frac{1}{n^4}\right)^2} + \left(1 - \frac{1}{n}\right)^2}{1} = \frac{\sqrt[4]{1 + \frac{1}{\infty} + \frac{1}{\infty}} + \left(1 - \frac{1}{\infty}\right)}{5 - \frac{6}{\infty} + \frac{4}{\infty}} \cdot \frac{\sqrt{\left(1 + \frac{1}{\infty} + \frac{1}{\infty}\right)^2} + \left(1 - \frac{1}{\infty}\right)^2}{1} \\ &= \frac{\sqrt[4]{1+0+0} + (1-0)}{5-0+0} \cdot \frac{\sqrt{(1+0+0)^2} + (1-0)^2}{1} = \frac{1+1}{5} \cdot \frac{1+1}{1} = \frac{2 \cdot 2}{5} = \frac{4}{5}. \end{aligned}$$

Riešené príklady – 244

$$\lim_{n \rightarrow \infty} n \left[\ln n - \ln(n+2) \right]$$

[Iné riešenie 244]

Riešené príklady – 244

$$\lim_{n \rightarrow \infty} n \left[\ln n - \ln(n+2) \right]$$

[Iné riešenie 244]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{2}{n} \right)^n \right]^{-1} \end{aligned}$$

$$\begin{aligned} \bullet &= - \lim_{n \rightarrow \infty} n \left[\ln(n+2) - \ln n \right] = - \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n+2}{n} \right] = - \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^n \\ &= - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^{n+2-2} \\ &= \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{-2}{n+2} \right)^{n+2} \cdot \left(1 + \frac{-2}{n+2} \right)^{-2} \right] \end{aligned}$$

$$\bullet = \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{-2}{n+2} \right)^{n+2} \right]^{\frac{n}{n+2}}$$

Riešené príklady – 244

$$\lim_{n \rightarrow \infty} n \left[\ln n - \ln(n+2) \right] = -2$$

[Iné riešenie 244]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^{-n} = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^{-n} \\ &= \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{2}{n} \right)^n \right]^{-1} = \left[\text{ViL: } \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n} \right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}. \right] = \ln(e^2)^{-1} = \ln e^{-2} = -2. \end{aligned}$$

$$\begin{aligned} \bullet &= - \lim_{n \rightarrow \infty} n \left[\ln(n+2) - \ln n \right] = - \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n+2}{n} \right] = - \lim_{n \rightarrow \infty} \ln \left(\frac{n+2}{n} \right)^n \\ &= - \lim_{n \rightarrow \infty} \ln \left(1 + \frac{2}{n} \right)^n = \left[\text{ViL: } \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n} \right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}. \right] = - \ln e^2 = -2. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{-2}{n+2} \right)^{n+2-2} \\ &= \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{-2}{n+2} \right)^{n+2} \cdot \left(1 + \frac{-2}{n+2} \right)^{-2} \right] = \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } k = n+2 \mid \begin{smallmatrix} n \rightarrow \infty \\ k \rightarrow \infty \end{smallmatrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-2}{k} \right)^k = e^{-2}. \right] \\ &\quad \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{-2} = \left(1 + \frac{-2}{\infty} \right)^{-2} = (1+0)^{-2} = 1^{-2} = 1. \right] = \ln [e^{-2} \cdot 1] = \ln e^{-2} = -2. \end{aligned}$$

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n}{n+2} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left(\frac{n+2-2}{n+2} \right)^n = \lim_{n \rightarrow \infty} \ln \left[\left(1 + \frac{-2}{n+2} \right)^{n+2} \right]^{\frac{n}{n+2}} \\ &= \left[\lim_{n \rightarrow \infty} \left(1 + \frac{-2}{n+2} \right)^{n+2} = \left[\text{Subst. } k = n+2 \mid \begin{smallmatrix} n \rightarrow \infty \\ k \rightarrow \infty \end{smallmatrix} \right] = \lim_{k \rightarrow \infty} \left(1 + \frac{-2}{k} \right)^k = e^{-2}. \right] \left[\lim_{n \rightarrow \infty} \frac{n}{n+2} = \lim_{n \rightarrow \infty} \frac{n}{n(1+\frac{2}{n})} = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1. \right] \\ &= \ln [e^{-2}]^1 = \ln e^{-2} = -2. \end{aligned}$$

Riešené příklady – 245, 246, 247

$$\lim_{n \rightarrow \infty} n [\ln(n+3) - \ln n]$$

[245]

$$\lim_{n \rightarrow \infty} n^2 [\ln(n+3) - \ln n]$$

[246]

$$\lim_{n \rightarrow \infty} \sqrt{n} [\ln(n+3) - \ln n]$$

[247]

Riešené příklady – 245, 246, 247

$$\lim_{n \rightarrow \infty} n \left[\ln(n+3) - \ln n \right]$$

[245]

$$\bullet = \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n$$

$$\lim_{n \rightarrow \infty} n^2 \left[\ln(n+3) - \ln n \right]$$

[246]

$$\bullet = \lim_{n \rightarrow \infty} \left[n \cdot n \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} n \cdot \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right)^n$$

$$\lim_{n \rightarrow \infty} \sqrt{n} \left[\ln(n+3) - \ln n \right]$$

[247]

$$\bullet = \lim_{n \rightarrow \infty} \left[\frac{n}{\sqrt{n}} \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \cdot \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \cdot \ln \left(1 + \frac{3}{n} \right)^n$$

Riešené příklady – 245, 246, 247

$$\lim_{n \rightarrow \infty} n \left[\ln(n+3) - \ln n \right] = 3$$

[245]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{3}{n} \right)^n \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n} \right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}. \right] = \ln e^3 = 3. \end{aligned}$$

$$\lim_{n \rightarrow \infty} n^2 \left[\ln(n+3) - \ln n \right] = \infty$$

[246]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[n \cdot n \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} n \cdot \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} n \cdot \ln \left(1 + \frac{3}{n} \right)^n \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n} \right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}. \right] = \infty \cdot \ln e^3 = \infty \cdot 3 = \infty. \end{aligned}$$

$$\lim_{n \rightarrow \infty} \sqrt{n} \left[\ln(n+3) - \ln n \right] = 0$$

[247]

$$\begin{aligned} \bullet &= \lim_{n \rightarrow \infty} \left[\frac{n}{\sqrt{n}} \cdot \ln \frac{n+3}{n} \right] = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \cdot \ln \left(\frac{n+3}{n} \right)^n = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \cdot \ln \left(1 + \frac{3}{n} \right)^n \\ &= \left[\text{ViL: } \lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n} \right)^n = e^\alpha \text{ pre } \alpha \in \mathbb{R}. \right] = \frac{1}{\infty} \cdot \ln e^3 = 0 \cdot 3 = 0. \end{aligned}$$

Riešené príklady – 248

$$\lim_{n \rightarrow \infty} a_n \quad \text{pre} \quad \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots \right\}. \quad [248]$$

Riešené príklady – 248

$$\lim_{n \rightarrow \infty} a_n \quad \text{pre} \quad \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots \right\}.$$

[248]

• $\{a_n\}_{n=1}^{\infty}$ je zadaná rekurentne: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$, $n \in \mathbb{N}$, t. j. $a_{n+1}^2 = 2 + a_n = a_n + 2$, $n \in \mathbb{N}$.

[Je zrejmé, že $a_n \geq \sqrt{2} > 0$, $n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a$. $\Rightarrow$ • $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$.

[Je zrejmé, že $a \geq \sqrt{2} > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (a_n + 2) = a + 2.$$

$$\Rightarrow \bullet 0 = a^2 - a - 2 = (a - 2)(a + 1). \Rightarrow a = -1, \text{ resp. } a = 2.$$

[Riešime kvadratickú rovnicu.]

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 2 \text{ (pokiaľ limita existuje).}$$

[Koreň $a = -1 < 0$ nevyhovuje.]

Riešené príklady – 248

$$\lim_{n \rightarrow \infty} a_n = 2 \text{ pre } \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots \right\}.$$

[248]

• $\{a_n\}_{n=1}^{\infty}$ je zadaná rekurentne: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2+a_n}$, $n \in \mathbb{N}$, t. j. $a_{n+1}^2 = 2 + a_n = a_n + 2$, $n \in \mathbb{N}$.

[Je zrejmé, že $a_n \geq \sqrt{2} > 0$, $n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a \Rightarrow \bullet \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$.

[Je zrejmé, že $a \geq \sqrt{2} > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (a_n + 2) = a + 2.$$

$$\Rightarrow \bullet 0 = a^2 - a - 2 = (a - 2)(a + 1). \Rightarrow a = -1, \text{ resp. } a = 2.$$

[Riešime kvadratickú rovnicu.]

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 2 \text{ (pokiaľ limita existuje).}$$

[Koreň $a = -1 < 0$ nevyhovuje.]

• Ešte musíme ukázať, že $\lim_{n \rightarrow \infty} a_n$ existuje.

[Ukážeme, že $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora a rastúca.]

Pre všetky $n \in \mathbb{N}$ platí $a_n < 2$, t. j. $a_n - 2 < 0$ (ohraničená zhora).

[Dôkaz matematickou indukciou.]

Krok 1. $a_1 = \sqrt{2} < 2$.

Krok 2. Pre $k \in \mathbb{N}$ platí $a_k < 2 \Rightarrow$ Pre $k+1$ platí $a_{k+1} = \sqrt{2+a_k} < \sqrt{2+2} = \sqrt{4} = 2$.

Pre všetky $n \in \mathbb{N}$ platí $a_n < a_{n+1}$ (rastúca).

[Priamy dôkaz.]

Dôkaz. $a_n^2 - a_{n+1}^2 = a_n^2 - (a_n + 2) = a_n^2 - a_n - 2 = (a_n - 2)(a_n + 1) = [n \in \mathbb{N} \Rightarrow \sqrt{2} \leq a_n < 2.] < 0 \Rightarrow a_n^2 < a_{n+1}^2 \Rightarrow a_n < a_{n+1}.$

Riešené príklady – 249

$$\lim_{n \rightarrow \infty} a_n$$

pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 1, a_{n+1} = \sqrt{a_n + 6}, n \in \mathbb{N}$.

[249]

Riešené príklady – 249

$\lim_{n \rightarrow \infty} a_n$ pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 1, a_{n+1} = \sqrt{a_n + 6}, n \in \mathbb{N}$.

[249]

• $\{a_n\}_{n=1}^{\infty}$ zadaná explicitne: $\{1, \sqrt{1+6}, \sqrt{1+\sqrt{1+6}}, \sqrt{1+\sqrt{1+\sqrt{1+6}}}, \dots\}$. Navyše platí $a_{n+1}^2 = a_n + 6, n \in \mathbb{N}$.

[Je zrejmé, že $a_n \geq 1 > 0, n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a$. $\Rightarrow$ • $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$.

[Je zrejmé, že $a \geq 1 > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (a_n + 6) = a + 6.$$

$$\Rightarrow \bullet 0 = a^2 - a - 6 = (a - 3)(a + 2). \Rightarrow a = -2, \text{ resp. } a = 3.$$

[Riešime kvadratickú rovnicu.]

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \text{ (pokiaľ limita existuje).}$$

[Koreň $a = -2 < 0$ nevyhovuje.]

Riešené príklady – 249

$\lim_{n \rightarrow \infty} a_n = 3$ pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 1, a_{n+1} = \sqrt{a_n + 6}, n \in \mathbb{N}$.

[249]

• $\{a_n\}_{n=1}^{\infty}$ zadaná explicitne: $\{1, \sqrt{1+6}, \sqrt{1+\sqrt{1+6}}, \sqrt{1+\sqrt{1+\sqrt{1+6}}}, \dots\}$. Navyše platí $a_{n+1}^2 = a_n + 6, n \in \mathbb{N}$. [Je zrejmé, že $a_n \geq 1 > 0, n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a$. $\Rightarrow$ • $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$. [Je zrejmé, že $a \geq 1 > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (a_n + 6) = a + 6.$$

$$\Rightarrow \bullet 0 = a^2 - a - 6 = (a - 3)(a + 2). \Rightarrow a = -2, \text{ resp. } a = 3. \quad [\text{Riešime kvadratickú rovnicu.}]$$

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \quad (\text{pokiaľ limita existuje}). \quad [\text{Koreň } a = -2 < 0 \text{ nevyhovuje.}]$$

• Ešte musíme ukázať, že $\lim_{n \rightarrow \infty} a_n$ existuje. [Ukážeme, že $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora a rastúca.]

Pre všetky $n \in \mathbb{N}$ platí $a_n < 3$, t. j. $a_n - 3 < 0$ (ohraničená zhora). [Dôkaz matematickou indukciou.]

Krok 1. $a_1 = 1 < 3$.

Krok 2. Pre $k \in \mathbb{N}$ platí $a_k < 3$. $\Rightarrow$ Pre $k+1$ platí $a_{k+1} = \sqrt{a_k + 6} < \sqrt{3 + 6} = \sqrt{9} = 3$.

Pre všetky $n \in \mathbb{N}$ platí $a_n < a_{n+1}$ (rastúca). [Priamy dôkaz.]

Dôkaz. $a_n^2 - a_{n+1}^2 = a_n^2 - (a_n + 6) = a_n^2 - a_n - 6$
 $= (a_n - 3)(a_n + 2) = [n \in \mathbb{N} \Rightarrow 1 \leq a_n < 3.] < 0. \Rightarrow a_n^2 < a_{n+1}^2. \Rightarrow a_n < a_{n+1}.$

Riešené príklady – 250

$$\lim_{n \rightarrow \infty} a_n$$

pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 2, a_{n+1} = \sqrt{2a_n + 3}, n \in \mathbb{N}$.

[250]

Riešené príklady – 250

$\lim_{n \rightarrow \infty} a_n$ pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 2, a_{n+1} = \sqrt{2a_n + 3}, n \in \mathbb{N}$.

[250]

• $\{a_n\}_{n=1}^{\infty}$ zadaná explicitne: $\{2, \sqrt{4+3}, \sqrt{2\sqrt{4+3}+3}, \sqrt{2\sqrt{2\sqrt{4+3}+3}+3}, \dots\}$. Navyše platí $a_{n+1}^2 = 2a_n + 3, n \in \mathbb{N}$. [Je zrejmé, že $a_n \geq 2 > 0, n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a$. $\Rightarrow$ • $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$. [Je zrejmé, že $a \geq 2 > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (2a_n + 3) = 2a + 3.$$

$$\Rightarrow \bullet 0 = a^2 - 2a - 3 = (a - 3)(a + 1). \Rightarrow a = -1, \text{ resp. } a = 3. \quad [\text{Riešime kvadratickú rovnicu.}]$$

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \quad (\text{pokiaľ limita existuje}). \quad [\text{Koreň } a = -1 < 0 \text{ nevyhovuje.}]$$

Riešené príklady – 250

$\lim_{n \rightarrow \infty} a_n = 3$ pre $\{a_n\}_{n=1}^{\infty}$ zadanú rekurentne $a_1 = 2, a_{n+1} = \sqrt{2a_n + 3}, n \in \mathbb{N}$.

[250]

• $\{a_n\}_{n=1}^{\infty}$ zadaná explicitne: $\{2, \sqrt{4+3}, \sqrt{2\sqrt{4+3}+3}, \sqrt{2\sqrt{2\sqrt{4+3}+3}+3}, \dots\}$. Navyše platí $a_{n+1}^2 = 2a_n + 3, n \in \mathbb{N}$. [Je zrejmé, že $a_n \geq 2 > 0, n \in \mathbb{N}$.]

• Označme $\lim_{n \rightarrow \infty} a_n = a$. $\Rightarrow$ • $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_{n+1} = a$. [Je zrejmé, že $a \geq 2 > 0$.]

$$\Rightarrow \bullet a^2 = \lim_{n \rightarrow \infty} a_n^2 = \lim_{n \rightarrow \infty} a_{n+1}^2 = \lim_{n \rightarrow \infty} (2a_n + 3) = 2a + 3.$$

$$\Rightarrow \bullet 0 = a^2 - 2a - 3 = (a - 3)(a + 1). \Rightarrow a = -1, \text{ resp. } a = 3. \quad [\text{Riešime kvadratickú rovnicu.}]$$

$$\Rightarrow \bullet a = \lim_{n \rightarrow \infty} a_n = 3 \quad (\text{pokiaľ limita existuje}). \quad [\text{Koreň } a = -1 < 0 \text{ nevyhovuje.}]$$

• Ešte musíme ukázať, že $\lim_{n \rightarrow \infty} a_n$ existuje. [Ukážeme, že $\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora a rastúca.]

Pre všetky $n \in \mathbb{N}$ platí $a_n < 3$, t. j. $a_n - 3 < 0$ (ohraničená zhora). [Dôkaz matematickou indukciou.]

Krok 1. $a_1 = 2 < 3$.

Krok 2. Pre $k \in \mathbb{N}$ platí $a_k < 3$. $\Rightarrow$ Pre $k+1$ platí $a_{k+1} = \sqrt{2a_k + 3} < \sqrt{2 \cdot 3 + 3} = \sqrt{9} = 3$.

Pre všetky $n \in \mathbb{N}$ platí $a_n < a_{n+1}$ (rastúca). [Priamy dôkaz.]

Dôkaz. $a_n^2 - a_{n+1}^2 = a_n^2 - (2a_n + 3) = a_n^2 - 2a_n - 3$
 $= (a_n - 3)(a_n + 1) = [n \in \mathbb{N} \Rightarrow 2 \leq a_n < 3.] < 0. \Rightarrow a_n^2 < a_{n+1}^2. \Rightarrow a_n < a_{n+1}.$

Riešené príklady – 251, 252

$$\lim_{n \rightarrow \infty} a_n \quad \text{pre} \quad \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots \right\}. \quad [251]$$

Vyjadrite ako zlomok periodické číslo $0,2\overline{3} = 0,233\,333\,33\dots$ [252]

Riešené príklady – 251, 252

$$\lim_{n \rightarrow \infty} a_n \quad \text{pre} \quad \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots \right\}. \quad [251]$$

Pre všetky $n \in \mathbb{N}$ platí: $\bullet a_1 = \sqrt{2}$, $\bullet a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $\bullet a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, ..., $\bullet a_4 = \sqrt{\sqrt[8]{2}} = \sqrt[16]{2} = \sqrt[2^4]{2}$, ..., $\bullet a_n = \sqrt[2^n]{2}$, ...

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[2^n]{2} = \left[\text{Subst. } k = 2^n \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \sqrt[k]{2}$$

$$\text{Vyjadrite ako zlomok periodické číslo } 0,2\overline{3} = 0,233\,333\,33\dots \quad [252]$$

Periodické číslo môžeme vyjadriť ako súčet nekonečného geometrického radu:

$$\begin{aligned} \bullet 0,2\overline{3} &= 0,233\,33\dots = 0,2 + 0,03 + 0,003 + 0,000\,3 + 0,000\,03 + \dots \\ &= \frac{2}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \frac{3}{10^5} + \dots = \frac{2}{10} + \frac{3}{10^2} \cdot \left[1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \right] \end{aligned}$$

Použijeme základnú vlastnosť periodického čísla (periodické opakovanie číslic za desatinnou čiarkou):

$$\left. \begin{aligned} \bullet x &= 0,2\overline{3} = 0,233\,33\dots \\ \bullet 10x &= 2,\overline{3} = 2,333\,33\dots \\ \bullet 100x &= 23,\overline{3} = 23,333\,33\dots \end{aligned} \right\} \Rightarrow \bullet 90x = 100x - 10x = 23,\overline{3}\dots - 2,\overline{3}\dots = 21.$$

Riešené príklady – 251, 252

$$\lim_{n \rightarrow \infty} a_n = 1 \text{ pre } \{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots \right\}. \quad [251]$$

Pre všetky $n \in \mathbb{N}$ platí: $\bullet a_1 = \sqrt{2}$, $\bullet a_2 = \sqrt{\sqrt{2}} = \sqrt[4]{2} = \sqrt[2^2]{2}$, $\bullet a_3 = \sqrt{\sqrt[4]{2}} = \sqrt[8]{2} = \sqrt[2^3]{2}$, ..., $\bullet a_4 = \sqrt{\sqrt[8]{2}} = \sqrt[16]{2} = \sqrt[2^4]{2}$, ..., $\bullet a_n = \sqrt[2^n]{2}$, ...

$$\bullet = \lim_{n \rightarrow \infty} \sqrt[2^n]{2} = \left[\text{Subst. } k = 2^n \begin{matrix} n \rightarrow \infty \\ k \rightarrow \infty \end{matrix} \right] = \lim_{k \rightarrow \infty} \sqrt[k]{2} = \left[\text{ViL: } \lim_{\alpha \rightarrow \infty} \sqrt[\alpha]{\alpha} = 1 \text{ pre } \alpha > 0. \right] = 1.$$

Vyjadrite ako zlomok periodické číslo $0,2\bar{3} = 0,233\,333\,33\dots = \frac{7}{30}$ [252]

Periodické číslo môžeme vyjadriť ako súčet nekonečného geometrického radu:

$$\begin{aligned} \bullet 0,2\bar{3} &= 0,233\,33\dots = 0,2 + 0,03 + 0,003 + 0,000\,3 + 0,000\,03 + \dots \\ &= \frac{2}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \frac{3}{10^5} + \dots = \frac{2}{10} + \frac{3}{10^2} \cdot \left[1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots \right] \\ &= \left[\text{Geometrický rad, } q \in (-1; 1): \right. \\ &\quad \left. 1 + q + q^2 + \dots + q^n + \dots = \frac{1}{1-q}. \right] = \frac{2}{10} + \frac{3}{10^2} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{2}{10} + \frac{3}{10^2 \cdot \frac{9}{10}} = \frac{2}{10} + \frac{3}{10 \cdot 9} = \frac{2}{10} + \frac{3}{90} = \frac{6}{30} + \frac{1}{30} = \frac{7}{30}. \end{aligned}$$

Použijeme základnú vlastnosť periodického čísla (periodické opakovanie číslic za desatinnou čiarkou):

$$\left. \begin{aligned} \bullet x &= 0,2\bar{3} = 0,233\,33\dots \\ \bullet 10x &= 2,\bar{3} = 2,333\,33\dots \\ \bullet 100x &= 23,\bar{3} = 23,333\,33\dots \end{aligned} \right\} \Rightarrow \bullet 90x = 100x - 10x = 23,\bar{3}\dots - 2,\bar{3}\dots = 21.$$

$$\Rightarrow \bullet x = \frac{21}{90} = \frac{3 \cdot 7}{3 \cdot 30} = \frac{7}{30}.$$

Riešené príklady – 253

Vyjadrite ako zlomok periodické číslo $32,17\overline{71} = 32,177\,171\,717\,17\dots$

[253]

Riešené príklady – 253

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,177\,171\,717\,17\ldots$

[253]

Periodické číslo môžeme vyjadriť ako súčet nekonečného geometrického radu:

$$\begin{aligned}
 \bullet \quad 32,1\overline{771} &= 32,177\,171\,717\,17\ldots \\
 &= 32,17 + 0,00\,71 + 0,00\,00\,71 + 0,00\,00\,00\,71 + 0,00\,00\,00\,00\,71 + \ldots \\
 &= \frac{3\,217}{100} + \frac{71}{100^2} + \frac{71}{100^3} + \frac{71}{100^4} + \frac{71}{100^5} + \ldots \\
 &= \frac{3\,217}{100} + \frac{71}{100^2} \cdot \left[1 + \frac{1}{100} + \frac{1}{100^2} + \frac{1}{100^3} + \ldots \right]
 \end{aligned}$$

Použijeme základnú vlastnosť periodického čísla (periodické opakovanie číslic za desatinnou čiarkou):

$$\left. \begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,177\,171\,717\,17\ldots \\
 \bullet \quad 100x &= 3\,217,\overline{71} = 3\,217,71\,71\,71\,71\ldots \\
 \bullet \quad 10\,000x &= 321\,771,\overline{71} = 321\,771,71\,71\,71\,71\ldots
 \end{aligned} \right\} \Rightarrow$$

Riešené príklady – 253

Vyjadrite ako zlomok periodické číslo $32,1\overline{771} = 32,17717171717\ldots = \frac{318554}{9900}$

[253]

Periodické číslo môžeme vyjadriť ako súčet nekonečného geometrického radu:

$$\begin{aligned}
 \bullet \quad 32,1\overline{771} &= 32,17717171717\ldots \\
 &= 32,17 + 0,0071 + 0,000071 + 0,00000071 + 0,0000000071 + \ldots \\
 &= \frac{3217}{100} + \frac{71}{100^2} + \frac{71}{100^3} + \frac{71}{100^4} + \frac{71}{100^5} + \ldots \\
 &= \frac{3217}{100} + \frac{71}{100^2} \cdot \left[1 + \frac{1}{100} + \frac{1}{100^2} + \frac{1}{100^3} + \ldots \right] \\
 &= \left[\text{Geometrický rad, } q \in (-1; 1): \right. \\
 &\quad \left. 1 + q + q^2 + \ldots + q^n + \ldots = \frac{1}{1-q} \right] = \frac{3217}{100} + \frac{71}{100^2} \cdot \frac{1}{1 - \frac{1}{100}} = \frac{3217}{100} + \frac{71}{100^2 \cdot \frac{99}{100}} = \frac{3217}{100} + \frac{71}{100 \cdot 99} \\
 &= \frac{3217 \cdot 99 + 71}{100 \cdot 99} = \frac{318483 + 71}{9900} = \frac{318554}{9900}
 \end{aligned}$$

Použijeme základnú vlastnosť periodického čísla (periodické opakovanie číslic za desatinnou čiarkou):

$$\begin{aligned}
 \bullet \quad x &= 32,1\overline{771} = 32,1771717171\ldots \\
 \bullet \quad 100x &= 3217,\overline{71} = 3217,71717171\ldots \\
 \bullet \quad 10000x &= 321771,\overline{71} = 321771,71717171\ldots
 \end{aligned} \quad \Rightarrow$$

$$\Rightarrow \bullet \quad 9900x = 10000x - 100x = 321771,\overline{71} - 3217,\overline{71} = 321771 - 3217 = 318554.$$

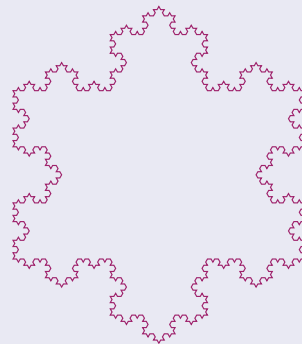
$$\Rightarrow \bullet \quad x = \frac{318554}{9900}$$

Riešené příklady – 254

Vypočítajte dĺžku d

obvodu Kochovej vločky.

[254]



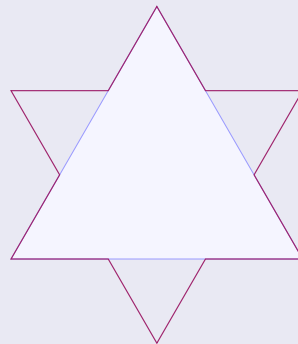
Riešené príklady – 254

Vypočítajte dĺžku d

obvodu **Kochovej vločky.**

[254]

- Každú úsečku rozdelíme na tri rovnaké úsečky.
- Stredná z úsečiek sa zdvojnásobí na dve strany rovnostranného trojuholníka.



Riešené príklady – 254

Vypočítajte dĺžku d

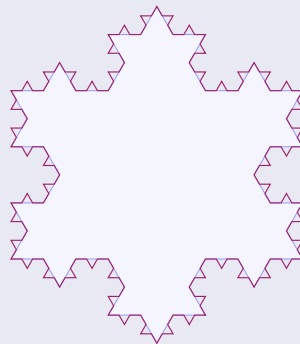
obvodu **Kochovej vložky.**

[254]

- Každú úsečku rozdelíme na tri rovnaké úsečky.
- Stredná z úsečiek sa zdvojnásobí na dve strany rovnostranného trojuholníka.



- Označme obvod počiatočného trojuholníka d_t .
- Pôvodná úsečka sa zväčší o tretinu, t. j. bude $\frac{4}{3}d_t$.
- Obvod vložky d sa po každom kroku zväčší $\frac{4}{3}$ -krát.



Riešené príklady – 254

Vypočítajte dĺžku $d = \infty$ obvodu Kochovej vložky.

[254]

- Každú úsečku rozdelíme na tri rovnaké úsečky.
- Stredná z úsečiek sa zdvojnásobí na dve strany rovnostranného trojuholníka.



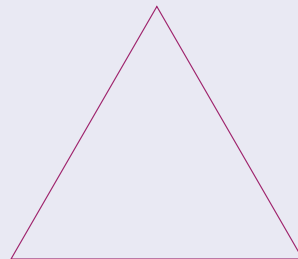
- Označme obvod počiatočného trojuholníka d_t .
- Pôvodná úsečka sa zväčší o tretinu, t. j. bude $\frac{4}{3}d_t$.
- Obvod vložky d sa po každom kroku zväčší $\frac{4}{3}$ -krát.

Po prvom kroku bude $d = \frac{4}{3}d_t$.

Po druhom kroku bude $d = \left(\frac{4}{3}\right)^2 d_t$.

Po n -tom kroku bude $d = \left(\frac{4}{3}\right)^n d_t$.

Pre $n \rightarrow \infty$ bude $d = \lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n d_t = \infty \cdot d_t = \infty$.



Koniec riešených príkladov 2. časti

Ďakujem za pozornosť.